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ANALYSIS AND MODELING OF COMPLEX LIVING SYSTEMS

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Modeling and analytical approximation of rhinoceros beetle infestation in coconut plantations via homotopy perturbation method

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In this paper, a non-linear mathematical model is formulated to examine the impact of rhinoceros beetles on coconut plantations through the interactions among healthy coconut trees, infected trees, and the beetle population. The model consists of three coupled non-linear ordinary differential equations (ODEs) with a Holling type II functional response to describe beetle-induced infection. Positivity and boundedness analyses ensure the biological feasibility of the system. The basic reproduction number (R_0) is derived using the next-generation matrix method to determine the threshold for beetle persistence. Local and global stability analyses of the equilibrium points are performed to investigate the long-term behavior of the system. The biological interpretation of equilibrium points explains the conditions for pest eradication and persistence. Optimal control strategies are incorporated to identify effective measures for reducing beetle infestation. The homotopy perturbation method (HPM) is employed to obtain approximate analytical solutions of the non-linear system. Sensitivity analysis identifies the parameters that strongly influence pest dynamics. Numerical simulations validate the analytical findings and show that reducing the contact rate between beetles and healthy trees can effectively control infestation and protect coconut plantations.

Keywords: mathematical modeling, homotopy perturbation method, non-linear ordinary differential equations, pest control model, numerical simulation

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1. Introduction

Cocos nucifera (coconut tree) is an economically and nutritionally cultivated palm that provides various benefits, from its fruit to its trunk. India ranks as the third-largest producer of coconuts and their by-products worldwide [Sreejith et al., 2013]. The livelihoods of families and industries that are dependent on the coconut economy are greatly affected by diseases that lowered coconut plantation yields. An important and natural feature of the coconut fruit was its antiviral properties [Varghese, Jacob, 2017]. According to the research conducted by Dayrit, the coconut product market grew rapidly due to the presence of antiviral compounds, which were found to be effective in treating COVID-19 [Dayrit, Mary, 2020]. The members of the International Coconut Community (ICC), with India as a founding member, designated September 2nd as World Coconut Day to commemorate coconut production [Ijinu et al., 2011]. Indonesia, India, and the Philippines are the leading coconut-producing countries. In major coconut-growing states such as Kerala and Tamil Nadu, approximately 50 % of coconut production is used for food and religious purposes, 35 % for copra, 11 % for tender coconuts, and 2 % each for seeds and value-added products. Moreover, Coconut is widely recognized as a functional food due to its health benefits [Nadanasabapathy, Kumar, 2013].

Oryctes rhinoceros, commonly known as the coconut rhinoceros beetle (CRB) or the Asiatic coconut palm rhinoceros beetle, is a large insect measuring approximately 4–5 cm in length and belongs to the subfamily Dynastinae. It primarily attacks coconut and oil palms [Martin, 2004; Karthick et al., 2018]. This beetle causes serious damage to young fronds, which can eventually lead to the death of the palm. It typically breeds in decaying palm trunks or organic matter such as sawdust and compost piles [Elango et al., 2019].

The concept and development of epidemiological models have been widely discussed in [Allen et al., 2019], highlighting their importance in representing the dynamics of interacting populations and enhancing the understanding of relationships among variables and their biological significance. In agriculture, pest control plays a crucial role in ensuring high crop yields and maintaining plant health. To address non-linear problems, several analytical techniques have been developed to obtain approximate analytical solutions. Among these, HPM was proposed as a semi-analytical approach that effectively solves non-linear problems by combining the classical perturbation method with the standard homotopy technique [He, 1998b]. In contrast to traditional perturbation methods that depend on the presence of small parameters, the HPM provides accurate approximations without requiring such assumptions. This method transforms a complex non-linear problem into a sequence of simpler subproblems, producing solutions in the form of a rapidly converging series. The convergence of the HPM solution depends critically on the appropriate selection of the initial approximation and the proper construction of the homotopy [He, 1998a; He, 2000]. The effectiveness of HPM has been demonstrated in the analytical investigation of non-linear prey – predator systems [Dhivyadharshini, Senthamarai, 2022]. Similarly, the infestation dynamics of rugose spiraling whitefly on coconut trees have been studied using delay differential equations, with approximate solutions obtained via HPM [Dhivyadharshini, Senthamarai, 2024]. In agricultural pest management, mathematical models have been developed to evaluate the role of awareness campaigns in controlling coconut tree pests and to assess their effectiveness in reducing infestation levels [Suganya, Senthamarai, 2022a]. Furthermore, mathematical modeling has been employed to analyze the ecological and economic impacts of rugose spiraling whitefly on coconut trees, providing a structured framework for understanding its consequences [Suganya, Senthamarai, 2022b]. Integrated pest management strategies for major coconut insect pests have been investigated with an emphasis on sustainable and environmentally responsible control measures [Winotai, 2014]. Additionally, mathematical models have been used to study the population dynamics and control of the coconut rhinoceros beetle, highlighting key factors such as life stages, breeding habitats, and sanitation-based management strategies [Caasia et al., 2023].

This study focuses on the direct interaction between beetles and coconut trees through a three-dimensional non-linear dynamical system, distinguishing it from earlier works centered on whitefly infestations, awareness-based control strategies, and virus-mediated beetle dynamics. The dynamics of the model have been analyzed using MATLAB software. The Pollachi area in Tamil Nadu serves as the main focus for our study, guiding the choice of parameter values. HPM is applied to obtain a simple closed-form solution capturing the non-linear dynamics of trees and beetles interactions. Error analysis and comparison with numerical simulations confirm the accuracy of the HPM solution. This analytical approach provides an efficient framework for understanding pest outbreaks and supporting effective management decisions. The validation of our solution is shown by error tables. We assessed equilibrium point stability and determined the reproduction number via a next-generation matrix. Sensitivity analysis highlighted the parameters with the greatest effect on the system.

Section 2 presents the mathematical formulation and basic properties of the ODE model. Section 3 discusses the stability analysis of the equilibrium points. Section 4 provides the biological interpretation of the equilibria. Section 5 deals with sensitivity analysis of the parameters. Section 6 formulates the optimal control problem. Section 7 presents the approximate analytical solution using HPM. Section 8 contains the results and discussion, while Section 9 concludes the study.

2. Mathematical formulation of the problem

The tree population is categorized into healthy trees H and infected trees X, while B represents the population of Rhinoceros beetles. Following the approach in [Ray, Basir, 2020], the model considers population densities measured per square meter. The corresponding mathematical formulation is outlined below.

2.1. ODE model

$$\frac{dH}{dt} = sH\left(1 - \frac{H}{K}\right) - \frac{\alpha BH}{1 + \gamma H}, \tag{1}$$

$$\frac{dX}{dt} = \frac{\alpha BH}{1 + \gamma H} - rX, \tag{2}$$

$$\frac{dB}{dt} = \rho X - \eta B, \tag{3}$$

with the following initial conditions:

$$H(0) = l > 0, \quad X(0) = m > 0, \quad B(0) = n > 0. \tag{4}$$

Table 1. Description of model parameters [Suganya, Senthamarai, 2022a]

Parameter	Description	Value	Unit	Range
s	Tree density	0.0138	m^{-2}	[0.01138–0.0178]
K	Replanting rate	0.0005	day^{-1}	[0–0.008]
α	Beetle attack rate	0.0002	$beetle^{-1}\ day^{-1}$	[0–0.002]
γ	Saturation constant	0.1	$tree^{-1}$	[0.05–0.3]
r	Recovery rate of infected trees	0.005	day^{-1}	[0–0.008]
ρ	Beetle growth rate due to infection	0.04	day^{-1}	[0.01–0.1]
η	Natural death rate of beetles	0.05	day^{-1}	[0.01–0.1]

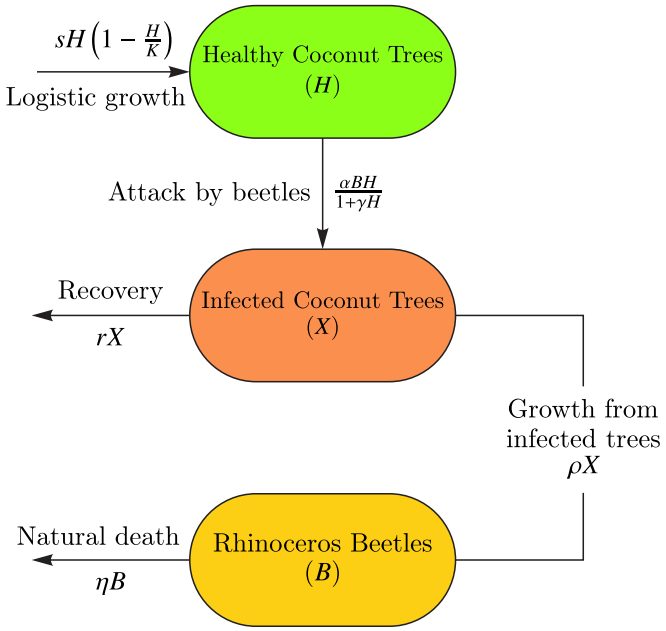


Figure 1. Schematic diagram

2.2. Basic properties of the model

2.2.1. Positivity

From Eqs. (1)–(3) we see that

$$\left.\frac{dH}{dt}\right|_{H=0,\,X>0,\,B>0} = 0, \quad \left.\frac{dX}{dt}\right|_{H>0,\,X=0,\,B>0} = \frac{\alpha BH}{1+\gamma H} \geqslant 0, \quad \left.\frac{dB}{dt}\right|_{H>0,\,X>0,\,B=0} = \rho X \geqslant 0. \tag{5}$$

Therefore, the solutions remain positive provided that the system is initiated with positive initial conditions.

2.2.2. Boundedness

Let us define the total tree population at time t as

$$N(t) = H(t) + X(t). \tag{6}$$

Adding the first two equations

$$\frac{dN}{dt} = \frac{dH}{dt} + \frac{dX}{dt} = sH\left(1-\frac{H}{K}\right) - rX, \tag{7}$$

since $H(t) \geqslant 0$, $X(t) \geqslant 0$, and $H(t) \leqslant N(t)$, we can bound

$$\frac{dN}{dt} \leqslant sN\left(1-\frac{N}{K}\right), \tag{8}$$

this is a logistic-type inequality, implying $N(t)$ remains bounded above by K. Hence:

$$\limsup_{t\to\infty} N(t) \leqslant K. \tag{9}$$

Next, for the beetle population $B(t)$, from

$$\frac{dB}{dt} = \rho X - \eta B, \tag{10}$$

since $X(t) \leqslant N(t) \leqslant K$, we have

$$\frac{dB}{dt} \leqslant \rho K - \eta B, \tag{11}$$

solving this linear differential inequality gives

$$\limsup_{t\to\infty} B(t) \leqslant \frac{\rho K}{\eta}. \tag{12}$$

We define the positively invariant region

$$\mathcal{D} = \left\{(H, X, B) \in \mathbb{R}^3_+ : 0 \leqslant H + X \leqslant K,\ 0 \leqslant B \leqslant \frac{\rho K}{\eta}\right\}. \tag{13}$$

This region traps all trajectories of the system — any initial condition inside $\mathcal{D}$ stays inside it for all future time.

3. Model analysis

3.1. Local stability of the pest free equilibrium (PFE)

At the PFE $E_1 = (K, 0, 0)$, we derive the reproduction number by constructing the next-generation matrix based on the transmission and transition matrices [Diekmann et al., 1990]:

$$R_0 = \frac{K\alpha\rho}{\eta r(K\gamma + 1)}.$$

Lemma 1. *The PFE E_0 of the model equations* (1)–(3) *is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

According to Lemma 1, when $R_0 < 1$, a minor introduction of infected individuals does not result in a major disease outbreak. However, to guarantee that disease eradication does not depend on the initial sizes of the subpopulations, it is essential to demonstrate that the PFE is globally asymptotically stable under the condition $R_0 < 1$. This is further explored in the following.

3.2. Global stability of the PFE

Theorem 1. *The PFE of the model given by Eqs.* (1)–(3)*, defined as $E_1 = (K, 0, 0)$, is globally asymptotically stable whenever the basic reproduction number $R_0 \leqslant 1$.*

Proof. To prove global stability of the PFE, we apply a comparison theorem approach. The equations for the infected components of the model equations (2) and (3) can be simplified as follows:

$$\begin{bmatrix}\frac{dX}{dt}\\ \frac{dB}{dt}\end{bmatrix} = \left(\frac{H}{N^*}\right)F\begin{bmatrix}X\\B\end{bmatrix} - V\begin{bmatrix}X\\B\end{bmatrix} = (F - V)\begin{bmatrix}X\\B\end{bmatrix} - \left(1 - \frac{H}{N^*}\right)F\begin{bmatrix}X\\B\end{bmatrix} \leqslant (F - V)\begin{bmatrix}X\\B\end{bmatrix}, \tag{14}$$

where N^* is the total population and F and V are defined as above.

Lemma 1 confirms that the PFE is locally asymptotically stable for $R_0 < 1$, or equivalently, when $\rho\left(FV^{-1}\right) < 1$. This condition implies that all eigenvalues of $F - V$ have negative real parts, as shown in [Van den Driessche, Watmough, 2002]. Thus, the linearized form of Eq. (14) satisfies a differential inequality, and by a standard comparison theorem, it follows that

$$(X, B) \to (0, 0) \quad \text{as } t \to \infty. \tag{15}$$

Substituting Eq. (15) into the model equations (1)–(3) gives

$$(H) \to (K) \quad \text{as } t \to \infty. \tag{16}$$

Therefore,

$$(H, X, B) \to (K, 0, 0) \quad \text{as } t \to \infty. \tag{17}$$

Thus, the PFE E_0 is globally asymptotically stable whenever $R_0 \leqslant 1$. This implies, from an epidemiological perspective, that the pest population can be eradicated if the basic reproduction number R_0 is reduced below unity and kept there. Hence, the condition $R_0 < 1$ serves as both a necessary and sufficient criterion for disease elimination.

3.2.1. Existence of endemic equilibrium

Let $E^* = (H^*, X^*, B^*)$ denote an arbitrary endemic equilibrium of the model equations (1)–(3), where

$$H^* = \frac{r\eta}{\alpha\rho - r\eta\gamma}, \quad X^* = \frac{\alpha B^* H^*}{r(1 + \gamma H^*)}, \quad B^* = \frac{\rho}{\eta} X^*.$$

Theorem 2. *The endemic equilibrium* (*coexistence equilibrium*) *of the system* (1)–(3) *is globally asymptotically stable in* Ω *whenever* $R_0 > 1$.

Proof. Let the endemic equilibrium of the system (1)–(3) be denoted by $E^* = (H^*, X^*, B^*)$ and suppose that $R_0 > 1$ so that E^* exists. Consider the following non-linear Goh – Volterra type Lyapunov function:

$$V = H - H^* - H^* \ln\left(\frac{H}{H^*}\right) + X - X^* - X^* \ln\left(\frac{X}{X^*}\right) + B - B^* - B^* \ln\left(\frac{B}{B^*}\right),$$

with the Lyapunov derivative

$$\dot{V} = \left(1 - \frac{H^*}{H}\right)\dot{H} + \left(1 - \frac{X^*}{X}\right)\dot{X} + \left(1 - \frac{B^*}{B}\right)\dot{B},$$

where the upper dot denotes differentiation with respect to time. Substituting the right-hand sides of Eqs. (1)–(3) into the above expression, we obtain

$$\dot{V} = \left(1 - \frac{H^*}{H}\right)\left[sH\left(1 - \frac{H}{K}\right) - \frac{\alpha BH}{1 + \gamma H}\right] + \left(1 - \frac{X^*}{X}\right)\left[\frac{\alpha BH}{1 + \gamma H} - rX\right] + \left(1 - \frac{B^*}{B}\right)[\rho X - \eta B]. \tag{18}$$

Now, using the equilibrium relations at E^*,

$$sH^*\left(1 - \frac{H^*}{K}\right) = \frac{\alpha B^* H^*}{1 + \gamma H^*}, \tag{19}$$

$$\frac{\alpha B^* H^*}{1 + \gamma H^*} = rX^*, \tag{20}$$

$$\rho X^* = \eta B^*. \tag{21}$$

Substituting these steady-state expressions into $\dot{V}$ simplifies each term. After cancellation and rearrangement, we conclude that $\dot{V} \leqslant 0$ and $\dot{V} = 0$ only at the endemic equilibrium point $E^* = (H^*, X^*, B^*)$. Hence, V is a Lyapunov function in Ω, and by LaSalle's invariance principle [Hale, 1969], every solution of the system (1)–(3) with initial conditions in $\Omega \setminus \Omega_0$ approaches the unique endemic equilibrium E^* as $t \to \infty$, whenever $R_0 > 1$.

4. Biological interpretation of equilibrium points

The equilibrium points of the system provide important insights into the long-term dynamics of the interaction between coconut trees and rhinoceros beetles. Each equilibrium has a clear biological meaning, which helps in understanding pest persistence or eradication.

4.1. PFE: $E_1 = (K, 0, 0)$

At this equilibrium, the entire tree population is healthy and reaches its carrying capacity K, while both the infected tree population and beetle population are absent. Biologically, this represents a completely pest-free environment where no infestation occurs. This situation can be achieved when the basic reproduction number satisfies $R_0 < 1$. Under this condition, the beetle population cannot invade or sustain itself in the system. Any small introduction of beetles or infection will eventually die out over time. Hence, the coconut plantation remains fully healthy and stable.

4.2. Endemic equilibrium: $E^* = (H^*, X^*, B^*)$

The endemic equilibrium represents a coexistence state where healthy trees, infected trees, and beetles all persist in the system at constant levels. Biologically, this corresponds to a situation where the infestation becomes permanent and cannot be eradicated naturally.

This equilibrium exists only when $R_0 > 1$, meaning that the beetle population can successfully invade and maintain itself. In this case:

- H^* represents the reduced number of healthy trees due to continuous infestation,
- X^* represents the sustained level of infected trees,
- B^* represents the stable beetle population supported by infected trees.

The persistence of beetles leads to a decline in the overall health of the plantation, and the system stabilizes at a compromised state rather than returning to a pest-free condition.

4.3. Threshold interpretation of R_0

The basic reproduction number R_0 acts as a threshold parameter governing the system dynamics:

- If $R_0 < 1$: The pest population dies out, and the system approaches the pest-free equilibrium.
- If $R_0 > 1$: The pest persists, and the system approaches the endemic equilibrium.

Thus, R_0 determines whether the infestation can be controlled or will become endemic in the coconut plantation.

4.4. Biological implications

From a practical perspective, the results suggest that effective pest management strategies should aim to reduce R_0 below unity. This can be achieved by:

- decreasing the beetle attack rate α,
- increasing the natural death rate of beetles η,
- improving recovery mechanisms of infected trees (increasing r),
- reducing beetle reproduction linked to infection (lowering ρ).

Maintaining these control measures ensures that the system remains in the pest-free state, thereby protecting coconut plantations from long-term damage.

5. Sensitivity analysis of parameters

Here, we discuss the impact of varying parameter values on the functional value of the reproduction number R_0 [Dhivyadharshini, Senthamarai, 2025]. Since the critical parameters may act as thresholds for disease or infestation persistence, identifying them is essential. The sensitivity indices of R_0 with respect to the parameters s, K, α, γ, r, η, ρ are represented algebraically as follows:

$$\frac{\partial R_0}{\partial s} = 0, \quad \frac{\partial R_0}{\partial K} = \frac{\alpha\rho}{\eta r(K\gamma+1)^2}, \quad \frac{\partial R_0}{\partial \alpha} = \frac{K\rho}{\eta r(K\gamma+1)}, \quad \frac{\partial R_0}{\partial \gamma} = \frac{-K^2\alpha\rho}{\eta r(K\gamma+1)^2},$$
$$\frac{\partial R_0}{\partial r} = \frac{-K\alpha\rho}{\eta r^2(K\gamma+1)}, \quad \frac{\partial R_0}{\partial \eta} = \frac{-K\alpha\rho}{\eta^2 r(K\gamma+1)}, \quad \frac{\partial R_0}{\partial \rho} = \frac{K\alpha}{\eta r(K\gamma+1)}.$$

In conclusion, the partial derivatives with respect to parameters K, α, ρ are positive, indicating that the basic reproduction number R_0 increases as any of these parameters increase. The elasticity is evaluated based on the proportional change in R_0 resulting from a proportional change in each parameter. We have

$$E_K = \frac{K}{R_0}\frac{\partial R_0}{\partial K} = 0.99, \quad E_\alpha = \frac{\alpha}{R_0}\frac{\partial R_0}{\partial \alpha} = 1, \quad E_\rho = \frac{\rho}{R_0}\frac{\partial R_0}{\partial \rho} = 1.$$

The expressions clearly show that E_K, E_α, E_ρ are all positive, indicating that an increase in the parameters K, α, ρ leads to an increase in the basic reproduction number R_0. Even slight changes in these parameters can produce significant shifts in R_0, emphasizing the need to identify and manage highly sensitive parameters with care.

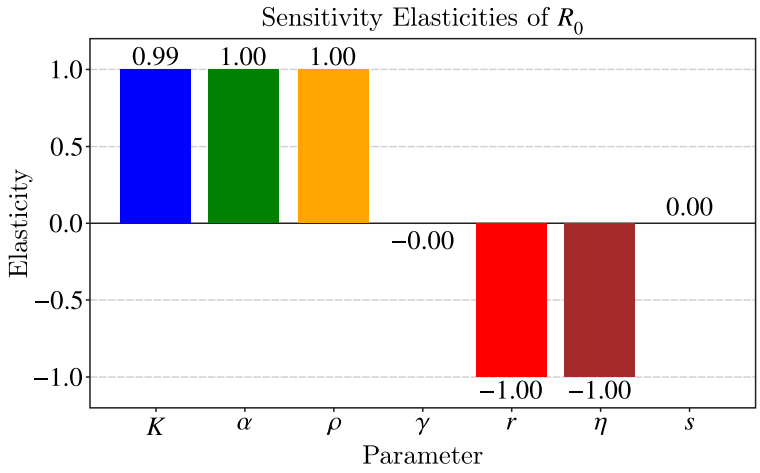


Figure 2. Elasticity analysis of R_0 with respect to model parameters

6. Optimal control problem

In this section, we formulate an optimal control problem with the aim of minimizing the infected coconut trees and rhinoceros beetle population by applying an effective control strategy. We introduce a control function $u(t)$, where $0 \leqslant u(t) \leqslant 1$, representing the effort to reduce the beetle attack rate (e. g., insecticide spraying or trapping).

The controlled system is given by

$$\frac{dH}{dt} = sH\left(1 - \frac{H}{K}\right) - (1-u)\frac{\alpha BH}{1+\gamma H}, \tag{22}$$

$$\frac{dX}{dt} = (1-u)\frac{\alpha BH}{1+\gamma H} - rX, \tag{23}$$

$$\frac{dB}{dt} = \rho X - \eta B, \tag{24}$$

with initial conditions:

$$H(0) = l, \quad X(0) = m, \quad B(0) = n.$$

The control $u(t)$ reduces the effective contact between beetles and healthy trees.

6.1. Objective functional

The objective is to minimize the infected trees and the cost of applying control. The cost functional is defined as

$$J(u) = \int_0^T \left[AX + \frac{1}{2}Cu(t)^2\right] dt, \tag{25}$$

where A represents the weight associated with infected trees and C represents the cost of implementing the control.

6.2. Hamiltonian formulation

The Hamiltonian is given by

$$\mathcal{H} = AX + \frac{1}{2}Cu^2 + \lambda_1\left[sH\left(1 - \frac{H}{K}\right) - (1-u)\frac{\alpha BH}{1+\gamma H}\right] + \lambda_2\left[(1-u)\frac{\alpha BH}{1+\gamma H} - rX\right] + \lambda_3[\rho X - \eta B], \tag{26}$$

where λ_1, λ_2, λ_3 are adjoint variables.

6.3. Optimality system

Using Pontryagin's minimum principle, the adjoint equations are

$$\frac{d\lambda_1}{dt} = -\frac{\partial \mathcal{H}}{\partial H}, \tag{27}$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial \mathcal{H}}{\partial X}, \tag{28}$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial \mathcal{H}}{\partial B}, \tag{29}$$

with transversality conditions

$$\lambda_i(T) = 0, \quad i = 1, 2, 3.$$

6.4. Characterization of optimal control

The optimal control $u^*(t)$ is obtained from

$$\frac{\partial \mathcal{H}}{\partial u} = 0,$$

which gives

$$u^*(t) = \frac{\alpha BH}{C(1+\gamma H)}(\lambda_2 - \lambda_1). \tag{30}$$

Considering the bounds $0 \leqslant u(t) \leqslant 1$, the optimal control is

$$u^*(t) = \max\left\{0, \min\left(1, \frac{\alpha BH}{C(1+\gamma H)}(\lambda_2 - \lambda_1)\right)\right\}. \tag{31}$$

The optimal control $u(t)$ reduces the effective beetle attack rate. When the difference $(\lambda_2 - \lambda_1)$ is large, stronger control is applied, indicating a higher need to suppress infection spread.

The optimality system is solved using the forward-backward sweep method, where the state equations are solved forward in time and adjoint equations backward in time, iteratively updating the control until convergence.

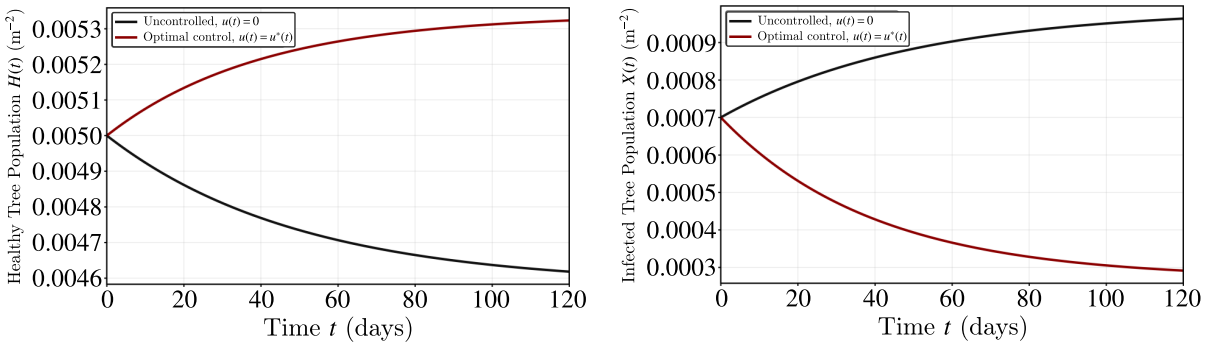


Figure 3. Profiles of the tree populations versus time (120 days) (a) healthy tree population $H(t)$ and (b) infected tree population $X(t)$, for the uncontrolled case, $u(t) = 0$, and the optimal control case, $u(t) = u^*(t)$, with other parameters as given in Table 1

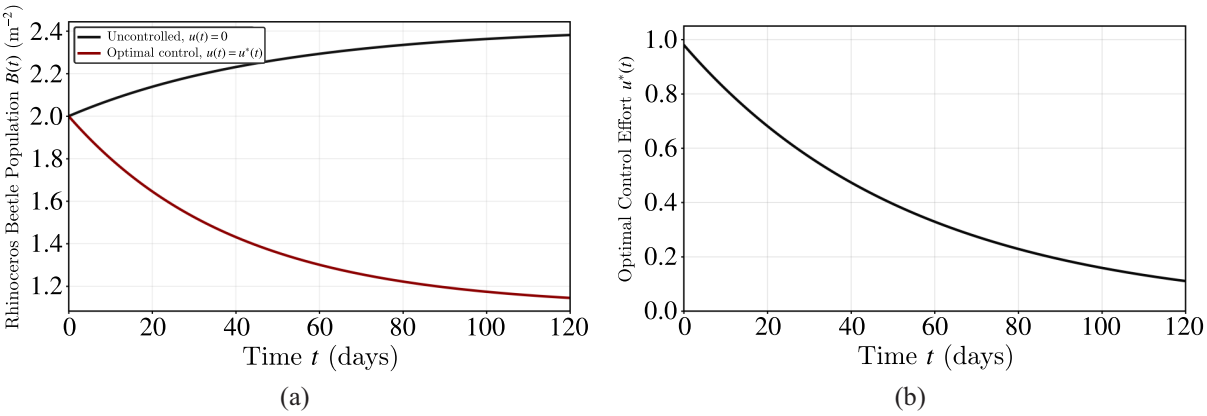


Figure 4. Profiles of the optimal control strategy and rhinoceros beetle population versus time (120 days) (a) optimal control effort $u^*(t)$ and (b) rhinoceros beetle population $B(t)$ for the uncontrolled case, $u(t) = 0$, and the optimal control case, $u(t) = u^*(t)$, with other parameters as given in Table 1

7. Approximate analytical solution of ODE by using HPM

The analytical solutions of the system of ODEs (1)–(3) are obtained using the HPM. According to the HPM framework, the linear and non-linear components of the equations can be effectively

separated to facilitate the solution process:

$$[1-P]\left[\frac{dH}{dt}-sH\right]+P\left[\frac{dH}{dt}-sH+\frac{sH^2}{K}-\alpha BH+\alpha B\gamma H^2\right]=0, \tag{32}$$

$$[1-P]\left[\frac{dX}{dt}+rX\right]+P\left[\frac{dX}{dt}+rX-\alpha BH+\alpha\gamma BH^2\right]=0, \tag{33}$$

$$[1-P]\left[\frac{dB}{dt}+\eta B\right]+P\left[\frac{dB}{dt}+\eta B-\rho X\right]=0. \tag{34}$$

And the initial conditions are as follows:

$$H_0(0)=l,\quad X_0(0)=m,\quad B_0(0)=n, \tag{35}$$

$$H=H_0+pH_1+p^2H_2+\ldots, \tag{36}$$

$$X=X_0+pX_1+p^2X_2+\ldots, \tag{37}$$

$$B=B_0+pB_1+p^2B_2+\ldots, \tag{38}$$

$$p^0:\ \frac{dH_0}{dt}-sH_0=0, \tag{39}$$

$$p^1:\ \frac{dH_1}{dt}-sH_1+\frac{s}{K}H_0^2-\alpha B_0H_0+\alpha\gamma B_0H_0^2, \tag{40}$$

$$p^0:\ \frac{dX_0}{dt}+rX_0=0, \tag{41}$$

$$p^1:\ \frac{dX_1}{dt}+rX_1-\alpha B_0H_0+\alpha\gamma B_0H_0^2, \tag{42}$$

$$p^0:\ \frac{dB_0}{dt}+\eta B_0=0, \tag{43}$$

$$p^1:\ \frac{dB_1}{dt}+\eta B_1-\rho X_0, \tag{44}$$

$$H_0=le^{st},\quad X_0=me^{-rt},\quad B_0=ne^{-\eta t}, \tag{45}$$

$$H_1(t)=\left(\frac{-l^2}{K}-\frac{\alpha nl}{\eta}-\frac{\alpha\gamma nl^2}{-\eta+s}\right)e^{st}-\frac{l^2}{K}e^{2st}-\frac{\alpha nl}{\eta}e^{(-\eta+s)t}-\frac{\alpha\gamma nl^2}{-\eta+s}e^{(-\eta+2s)t}, \tag{46}$$

$$X_1(t)=\left(\frac{\alpha nl}{-\eta}+\frac{\alpha\gamma ln}{\eta}\right)e^{-rt}+\frac{\alpha nl}{-\eta}e^{(-\eta+s)t}+\frac{\alpha\gamma nl}{\eta}e^{(-\eta+s)t}, \tag{47}$$

$$B_1(t)=ne^{-\eta t}+\left(\frac{\rho m}{r+\eta}\right)e^{-\eta t}+\frac{\rho m}{-r+\eta}e^{-rt}, \tag{48}$$

$$H(t)=H_0+H_1+H_2, \tag{49}$$

$$X(t)=X_0+X_1+X_2, \tag{50}$$

$$B(t)=B_0+B_1+B_2, \tag{51}$$

$$H(t)=\lim_{p\to1}H(t)=H_0+H_1+H_2, \tag{52}$$

$$X(t)=\lim_{p\to1}X(t)=X_0+X_1+X_2, \tag{53}$$

$$B(t)=\lim_{p\to1}B(t)=B_0+B_1+B_2. \tag{54}$$

Final solution is,

$$H(t) = le^{st} + \left(-\frac{l^2}{K} - \frac{\alpha nl}{\eta} - \frac{\alpha\gamma nl^2}{-\eta + s}\right)e^{st} - \frac{l^2}{K}e^{2st} + \frac{\alpha nl}{\eta}e^{(-\eta+s)t} + \frac{\alpha\gamma nl^2}{-\eta + s}e^{(-\eta+2s)t}, \tag{55}$$

$$X(t) = me^{-rt} + \left(\frac{-\alpha nl}{-\eta + s + r} + \frac{\alpha\gamma nl^2}{-\eta + 2s + r}\right)e^{-rt} + \frac{\alpha nl}{-\eta + s + r}e^{(-\eta+s)t} - \frac{\alpha\gamma nl^2}{-\eta + 2s + r}e^{(-\eta+2s)t}, \tag{56}$$

$$B(t) = ne^{-\eta t} + \left(\frac{\rho m}{r + \eta}\right)e^{-\eta t} + \frac{\rho m}{-r + \eta}e^{-rt}. \tag{57}$$

8. Results and discussion

For the numerical analysis, representative values were chosen for each parameter to serve as a reference. A single coconut tree is estimated to occupy an area between 7.5 m × 7.5 m and 8.5 m × × 8.5 m. Based on this, population densities are expressed per square meter. The initial conditions are set as $H(0) = 0.005$, $X(0) = 0.0007$, and the rhinoceros beetle population is assumed to be approximately $B(0) = 2$.

The parameter values used in the numerical simulations were selected based on the biological interpretation of the model and the parameterization considered in the previous study [Suganya, Senthamarai, 2022a]. The tree density parameter was taken as $s = 0.0138$ m^{-2}, representing a realistic coconut tree density. The replanting rate was chosen as $K = 0.0005$ day^{-1}, corresponding to the gradual replacement of coconut trees in the plantation. The beetle attack rate was taken as $\alpha =$ $= 0.0002$ day^{-1}, representing the interaction between rhinoceros beetles and healthy coconut trees. The saturation constant was selected as $\gamma = 0.1$ to account for the saturation effect in the beetle attack process. The recovery rate of infected trees was taken as $r = 0.005$ day^{-1}, representing a slow recovery process. The beetle growth rate due to infection was chosen as $\rho = 0.04$ day^{-1}, while the natural death rate of beetles was taken as $\eta = 0.05$ day^{-1}. These values were selected as biologically meaningful reference values within the parameter ranges given in Table 1. The ranges were further considered to examine the sensitivity of the model dynamics to variations in the parameter values. Thus, the selected parameter set provides a consistent reference regime for the numerical investigation of rhinoceros beetle infestation and its control in coconut plantations.

To demonstrate the application of the proposed optimal-control strategy, the controlled system with $u(t) = u^*(t)$ is compared with the uncontrolled system, $u(t) = 0$. The control reduces the effective beetle attack rate through the term $(1 - u)\frac{\alpha BH}{1+\gamma H}$, thereby reducing the infection of healthy coconut trees. Consequently, the optimal control is designed to maintain a higher healthy-tree population and reduce both infected trees and rhinoceros beetles compared with the uncontrolled case. The optimal control $u^*(t)$ also indicates the required intensity of intervention over time, providing a practical framework for determining appropriate pest-management effort while accounting for its associated cost.

Figure 1 shows the model flow diagram. Figure 2 shows the elasticity analysis of R_0 with respect to model parameters. The numerical simulations were carried out using MATLAB. To demonstrate the practical application of the proposed optimal-control strategy, Figs. 3 and 4 compare the controlled system, $u(t) = u^*(t)$, with the uncontrolled system, $u(t) = 0$, over a simulation period of 120 days. Figure 3, *a* shows that the healthy tree population is maintained at a higher level under the optimal-control strategy than in the uncontrolled case. Figure 3, *b* shows that the infected tree population is reduced when the optimal control is implemented. These results indicate that the control strategy can help reduce infestation and preserve the healthy coconut tree population. Figure 4, *a* shows the rhinoceros beetle population $B(t)$ for the controlled and uncontrolled cases. The results show that the beetle population remains lower under the optimal-control strategy than in the uncontrolled case, demonstrating the effectiveness of the proposed control in suppressing beetle infestation. Figure 4, *b*

shows the optimal control profile $u^*(t)$ over the simulation period. The control effort is initially high and gradually decreases with time, indicating that a stronger intervention is required during the early stage of the infestation, whereas a lower level of intervention is sufficient at later times. Thus, the proposed optimal-control approach provides a practical framework for assessing the appropriate timing and intensity of pest-management measures while reducing the infected tree and rhinoceros beetle populations and maintaining a higher healthy tree population. Figure 5, *a* interprets that the replanting rate K increases the healthy tree population and Fig. 5, *b* the beetle attack rate α decreases the healthy tree population. From Fig. 6, *a*, we see that the beetle attack rate increases the infected tree population and Fig. 6, *b* recovery rate of infected trees decreases the infected tree population. Figure 7 shows the decrease in rhinoceros beetle population according to its natural death rate η. The figures clearly demonstrate that the analytical results are in close agreement with the numerical simulations. All parameter values used are listed in Table 1. The approximation error between the HPM and the numerical solutions of Eqs. (1), (2) and (3) is presented in Tables 2, 3, and 4. HPM provides us with a great deal of freedom in developing the solution for non-linear problems. This provides various benefits which are not available in ordinary perturbation methods. Thus, the results obtained by HPM seem to be trustworthy as they agree with numerical simulation results and the maximum error percentage is 0.3 %.

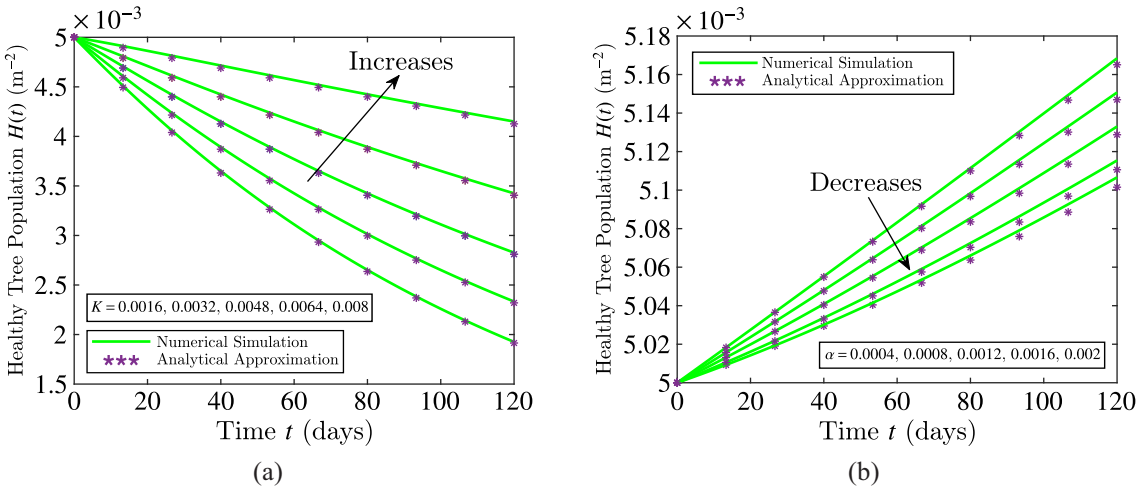


Figure 5. Profiles of the healthy tree population $H(t)$ versus time (120 days) (a) for different values of replanting rate K (b) for different values of beetle attack rate α with other parameters as given in Table 1

Table 2. A comparison of the numerical results and the corresponding HPM solution of Eq. (1) for various values of K, as in Fig. 5, *a*

	K = 0.0016			K = 0.0032			K = 0.0048		
t	Numerical	HPM	Error %	Numerical	HPM	Error %	Numerical	HPM	Error %
0.0	0.00500000	0.00499998	0.00040	0.00500	0.00500	0.00000	0.00500	0.00500	0.00000
0.2	0.00499903	0.00499998	0.01893	0.00499	0.00499	0.00000	0.00500	0.00499	0.14321
0.4	0.00499807	0.00499998	0.03821	0.00498	0.00498	0.00000	0.00500	0.00498	0.28559
0.6	0.00499711	0.00499998	0.05736	0.00498	0.00498	0.00000	0.00500	0.00498	0.42713
0.8	0.00499616	0.00499998	0.07637	0.00497	0.00497	0.00000	0.00500	0.00497	0.56785
1.0	0.00499521	0.00499998	0.09524	0.00496	0.00496	0.00000	0.00500	0.00496	0.70775
Mean Error %			0.04852			0.00000			0.35525

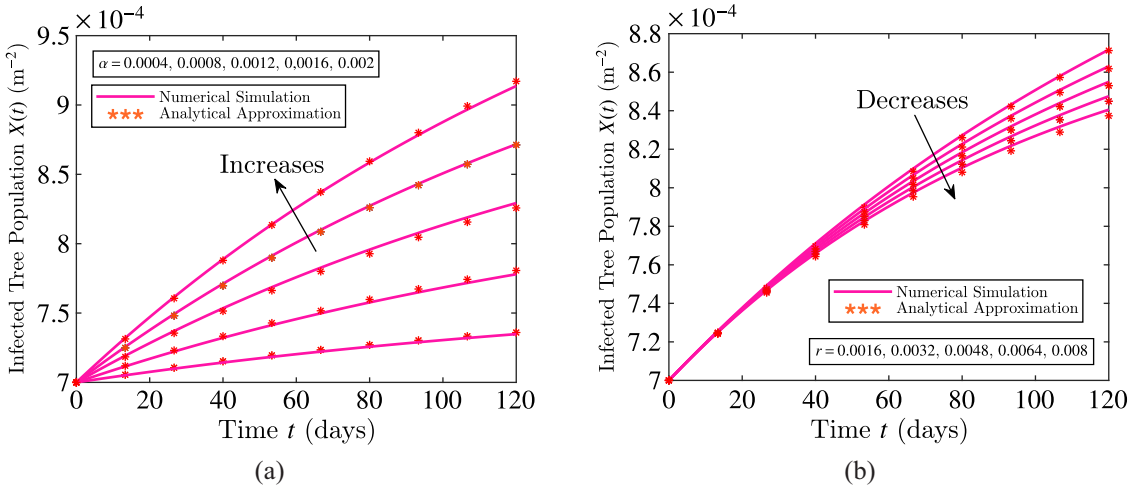


Figure 6. Profiles of the infected tree population $X(t)$ versus time (120 days) (a) for different values of growth rate of beetle attack rate α (b) for different values of recovery rate of infected trees r with other parameters as given in Table 1

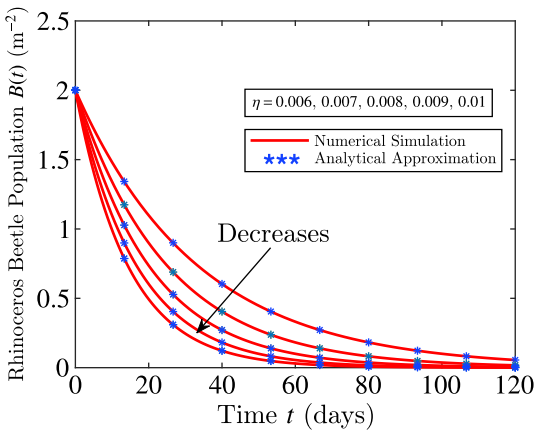


Figure 7. Profiles of the rhinoceros beetle population $B(t)$ versus time (120 days) (a) for different values of natural death rate of beetles η with other parameters as given in Table 1

Table 3. A comparison of the numerical results and the corresponding HPM solution of Eq. (2) for various values of α, as in Fig. 6, *a*

	α = 0.0004			α = 0.0008			α = 0.0012		
t	Numerical	HPM	Error %	Numerical	HPM	Error %	Numerical	HPM	Error %
0.0	0.00500019	0.00500000	0.00377	0.00500	0.00500	0.00000	0.00500	0.00500	0.00000
0.2	0.00500019	0.00500000	0.00377	0.00500	0.00500	0.00000	0.00500	0.00500	0.00000
0.4	0.00500039	0.00500000	0.00765	0.00500	0.00500	0.00000	0.00500	0.00500	0.00000
0.6	0.00500058	0.00500000	0.01152	0.00500	0.00500	0.00000	0.00500	0.00500	0.00000
0.8	0.00500077	0.00500000	0.01540	0.00500	0.00500	0.00000	0.00500	0.00500	0.00000
1.0	0.00500097	0.00500000	0.01928	0.00500	0.00500	0.00000	0.00500	0.00500	0.00000
Mean Error %			0.00925			0.00000			0.00000

9. Conclusion

This study provides a comprehensive understanding of the dynamics between rhinoceros beetles and coconut trees. The system's equilibrium points were determined and local as well as global stability conditions were analyzed to assess long-term behavior. The biological interpretation of the equilibrium

Table 4. A comparison of the numerical results and the corresponding HPM solution of Eq. (3) for various values of η, as in Fig. 7

	$\eta = 0.07$			$\eta = 0.08$			$\eta = 0.09$		
t	Numerical	HPM	Error %	Numerical	HPM	Error %	Numerical	HPM	Error %
0.0	2.00100	2.00000	0.04998	2.00000	2.00000	0.00000	2.00100	1.99900	0.09995
0.2	1.97331	1.97227	0.05262	1.98804	1.98804	0.00000	1.96950	1.96950	0.00000
0.4	1.94599	1.94492	0.05518	1.97616	1.97616	0.00000	1.93849	1.93849	0.00000
0.6	1.91906	1.91795	0.05764	1.96434	1.96434	0.00000	1.90795	1.90795	0.00000
0.8	1.89248	1.89135	0.06001	1.95259	1.95259	0.00000	1.87788	1.87788	0.00000
1.0	1.86627	1.86511	0.06228	1.94092	1.94092	0.00000	1.84828	1.84828	0.00000
Mean Error %			0.05628			0.00000			0.01666

points explained the conditions governing pest persistence and eradication in the system. Sensitivity analysis highlighted the parameters with the greatest impact on the system, identifying the contact rate as a critical factor in the spread of the infestation. An approximate analytical solution derived using the HPM showed close agreement with numerical simulations, confirming the validity of the analytical approach. The optimal control analysis further demonstrated that implementing suitable management strategies can effectively suppress beetle infestation and improve plantation health. These findings demonstrate that targeted interventions to reduce the contact rate can play a significant role in managing beetle populations and mitigating their effects on coconut plantations.

Conflict of interest

The authors declare that there is no conflict of interest.

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