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Iterative diffusion importance: advancing edge criticality evaluation in complex networks

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This paper is devoted to the problem of edge criticality identification and ranking in complex networks, which is a part of a modern research direction in the novel network science. The diffusion importance belongs to the set of acknowledged methods that help to identify the significant connections in the graph that are critical to retaining structural integrity. In the present work, we develop the Iterative Diffusion Importance algorithm that is based on the re-estimation of critical topological features at each step of the graph deconstruction. The Iterative Diffusion Importance has been compared with methods such as diffusion importance and degree product, which are two very well-known benchmark algorithms. As for benchmark networks, we tested the Iterative Diffusion Importance on three standard networks, such as Zachary's Karate Club, the American Football Network, and the Dolphins Network, which are often used for algorithm efficiency evaluation and are different in size and density. Also, we proposed a new benchmark network representing the airplane communication between Japan and the US. The numerical experiment on finding the ranking of critical edges and the following network decomposition demonstrated that the proposed Iterative Diffusion Importance exceeds the conventional diffusion importance by the efficiency for 2–35 % depending on the network complexity, the number of nodes, and the number of edges. The only drawback of the Iterative Diffusion Importance is an increase in computation complexity and hencely in the run-time, but this drawback can be easily compensated for by the preliminary planning of the network deconstruction or protection and by reducing the re-evaluation frequency of the iterative process.

Keywords: edge significance, diffusion importance, complex networks

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1. Introduction

The progression and growth of intricate networks have markedly enhanced our comprehension of diverse natural and societal systems. Whether it be scrutinizing biological networks [Newman, 2002] essential for life processes in organisms, such problems as PageRank in application to search engines [Skachkov et al., 2023], mathematical modeling of group interaction [Vasilyeva et al., 2023], social networks [Pei et al., 2015] influentially shaping human interactions and idea diffusion, or technological networks [Wan et al., 2018] fundamental to pivotal infrastructures and information flow, the field of network science provides a crucial lens for grasping the dynamics governing these complex systems. With our dependence on these networks escalating to sustain societal, economic, or environmental functionalities [Newman, 2003], the significance of comprehending their functionality, resilience, and vulnerability [Xia, Hill, 2008] to internal or external disruptions is progressively emphasized.

Network science encompasses a broad spectrum of theories, models, and analytical methods aimed at revealing the structure and dynamics of networks. Analyzing networks from various angles, the significance and strength of their connections or their response to changes [Albert, Barabási, 2002] — researchers can uncover patterns and principles that apply across different systems. This comprehensive approach is essential for devising strategies to enhance the efficiency of networks [Louzada et al., 2015], which ultimately creates more robust and adaptive systems that can withstand and recover from contemporary challenges and crises. Various metrics have been explored in the network science literature [Qian et al., 2017] to assess the significance of links within networks, including edge betweenness centrality [Girvan, Newman, 2002], degree product [Wang, Chen, 2008], diffusion importance [Liu et al., 2015], bridgeness [Cheng et al., 2010; Wu et al., 2018], topological overlap [Onnela et al., 2007], and k-path centrality [De Meo et al., 2012]. These methods provide a different perspective on the role of the edges, from facilitating information flow to influencing community structures. Our study acknowledges the value of these metrics and concentrates on diffusion importance, a method that emphasizes network topology. By focusing on this aspect, we aim to uncover the mechanisms that enhance network stability and adaptability, offering new insights for managing these complex systems to maximize their resilience.

The structure of these networks shapes our interactions and transactions, which are interconnected to the "edges" whose significance extends beyond just the simple connection. Within this framework, Diffusion Importance serves as a pivotal network metric that is used to assess the importance of individual edges or links in easing the spread of information or other kinds of transmissible elements throughout the network. This metric is fundamental for comprehending how different sectors of a network engage and communicate effectively. This helps analysts to evaluate and identify which edges are essential for maintaining overall network connectivity and functionality. These critical links ensure rapid propagation of information or influence to distant nodes. Consequently, scrutinizing diffusion importance aids in optimizing network architectures to enhance resilience, address vulnerabilities, and mitigate disruptions caused by natural phenomena or human activities. This metric holds particular significance across various domains, including social network analysis, infrastructure management, and cybersecurity.

The study aims to identify the critical structural elements that enhance network stability. By investigating the impact of Diffusion Importance on network cohesion and flexibility, we can gain deeper insights into how these systems adapt and evolve under changing conditions. In the present work, we propose a new method that is named "Iterative Diffusion Importance". This method is inspired by advanced methodologies like the Deep Link Entropy [Ozaydin, Ozaydin, 2021] and Improved Link Entropy [Lubashevskiy et al., 2023], which iteratively re-evaluate the metrics after each edge removal. The resulting edge criticality ranking is analyzed and compared with conventional Diffusion Importance using the conventional metrics of the dynamic of the largest connected component ratio [Qian et al., 2017], which has been later reformulated and reduced to the single value estimator [Lubashevskiy, Lubashevsky, 2023].

This paper is organized as follows: Introduction, Algorithm Efficiency Measure, Algorithms, Data and Numerical Simulations, Discussion and Conclusion.

2. Algorithm efficiency measure

In order to compare the efficiency of algorithms for network decomposition, we introduce a single value metric enabling us to determine which principle is better and to what extent. In general, most of the algorithms of network decomposition via removing the most significant edges may be categorized as the local-type algorithms. The outcome of each of those algorithms is the sequence of edges sorted according to their significance, and by removing those edges one-by-one, the network connectivity must gradually drop, resulting in the emergence of mutually disconnected components, and ending with all the nodes being disconnected from each other.

The standard approach to assess the efficiency of these algorithms is the dynamics of the size R_{gc} of the largest connected component vs the relative amount of removed edges ρ [Yu et al., 2018; Qian et al., 2017; Ozaydin, Ozaydin, 2021]. The size of connected components may be normalized to the total number of nodes to bring all the network decompositions to the same standard, or it can be presented in absolute values, when the comparison of networks of different sizes is not part of a numerical simulation. In the present work, we are going to use normalized values, so before any portion of edges is removed ($\rho = 0$) the value $R_{gc} = 1$ (the initial network is assumed to be a connected graph), correspondingly, at the end of the edge removing sequence, as $\rho \to 1$ (100 % of edges are removed), the value $R_{gc} \to 0$.

The popularity of the $R_{gc}(\rho)$-criterion is explained by its simplicity and perspicuity — the faster the drop of R_{gc} as ρ increases, the higher the significance of the removed edges. When for two algorithms A_1 and A_2 applied to decomposing the same network, the corresponding dependencies $R_{gc:1}(\rho)$ and $R_{gc:2}(\rho)$ are such that, e. g., $R_{gc:1}(\rho) < R_{gc:2}(\rho)$ for any $\rho > 0$, the $R_{gc}(\rho)$-criterion enables one to order the two algorithms unambiguously according to their efficiency, namely, $A_1 \succ A_2$. However, when the drop of $R_{gc}(\rho)$ for one algorithm temporally outpaces another one, e. g., $R_{gc:1}(\rho) < R_{gc:2}(\rho)$ for $\rho < \rho_b$ and $R_{gc:1}(\rho) > R_{gc:2}(\rho)$ for $\rho > \rho_b$, a more sophisticated efficiency criterion is required.

In the present work we use an integral criterion based on the $R_{gc}(\rho)$-dependence [Lubashevskiy, Lubashevsky, 2023]. Namely, treating the $R_{gc:2}(\rho)$-dependence as a continuous function of ρ, we introduce the integral

$$S_{rgc} = \int_0^1 R_{gc}(\rho)\, d\rho, \tag{1}$$

specifying the area under the curve $R_{gc}(\rho)$ for $\rho \in (0, 1)$. This area is used to measure the efficiency of network decomposition. Put differently, the smaller the value S_{rgc}, the more efficient the network decomposition.

It is worth noting that, first, a similar integral criterion is used to quantify recovery processes of large-scale disasters within the theory of resilience [Lubashevskiy, 2022]. Second, as it must, the given integral criterion leads to the same conclusion about the superiority of one of two algorithms A_1 and A_2 specifying the edge significance in the former case noted above. However, in the latter case, when two curves $R_{gc:1}(\rho)$ and $R_{gc:2}(\rho)$ outpace each other at different stages of the graph decomposition, it also enables one to compare the related algorithms.

3. Algorithms

Diffusion importance

Diffusion importance pertains to a metric within networks used for evaluating the significance of individual edges or links in facilitating the transmission of information, influence, or other transmissible entities across the network. This concept is essential for understanding how effectively different parts of a network communicate and interact. Assessing diffusion importance enables researchers to pinpoint which edges are pivotal in maintaining overall network connectivity and functionality. These critical links ensure that information or influence can swiftly traverse the network and reach distant nodes. Consequently, analyzing diffusion importance aids in optimizing network designs to bolster resilience, manage vulnerabilities, and enhance robustness against disruptions, whether natural or human-induced. Such insights are particularly valuable in diverse applications ranging from social network analysis to infrastructure management and cybersecurity.

The diffusion importance has often been used as a main or a benchmark algorithm in various papers [Qian et al., 2017]. The diffusion importance of an edge takes the disease spread process into consideration. For an edge e_{xy}, when disease spreads along it, there are two possible directions. In one direction, the disease originates from node x and spreads along e_{xy} to node y, and then spreads to the other parts of the network through node y. So does the spread mechanism in the other direction. In that sense, the diffusion importance of edge e_{xy} is defined as

$$DI = \frac{n_{x\to y} + n_{y\to x}}{2}, \tag{2}$$

where $n_{x\to y}$ is the number of links of node y connecting outside the nearest neighborhood of node x. The value of the index is inevitable to be misled by the degree of the node somehow: an edge with one high-degree node and one low-degree node may have a higher value of edge significance than its real effect when the edge is in the periphery of the network.

Degree product

Degree product is defined as one of the most used benchmark algorithms [Duan et al., 2016; Qian et al., 2017] due to its clarity and relative efficiency:

$$DP = k_x \cdot k_y, \tag{3}$$

where k_x and k_y are the degree of nodes x and y. The extended form of the DP can be expressed by the product of nodes' degrees in the power of θ, $(k_x k_y)^\theta$, where θ is a tunable positive parameter. In the present work, we are focused on the ranking of edges according to their criticality, so we set the $\theta = 1$. The computation of the index only needs the degree of each node, which is quite easy to get.

Iterative diffusion importance

Recent works in the topic of edge criticality assessment for complex networks [Ozaydin, Ozaydin, 2021; Lubashevskiy, Lubashevsky, 2023; Gao et al., 2024; Ejjbiri, Lubashevsky, 2024] underlined a blind spot long. Conventional and newly developed algorithms are based on a single assessment principle: analyze the network, assess the edges' features, and rank their criticality. The problem with this approach is in the fact that after the first edge is removed, the network cannot be considered the same; the features that have been assessed might be or even expected to be changed, so the further usage of an edge ranking that has been obtained from the first network is rather doubtful. In the present paper, we propose and introduce a new algorithm that is based on a conventional metric (Diffusion Importance), but that tackles the noted blind spot. This algorithm is named Iterative

Diffusion Importance. The gist of the algorithm is on iterative re-assessment of network features after each edge removal, so all the changes are taken into account. The iterative step-by-step algorithm and its pseudocode are the following:
Step 1. Load the network and create the empty list of edges, which will later be filled.
Step 2. Calculate the Diffusion Importance for each of the edges in the network.
Step 3. Rank the edges according to the descending values of their Diffusion Importance.
Step 4. Record the edge with the highest value of Diffusion Importance in the list and remove the edge from the network.
Step 5. If the network still contains any edges — return to step 2.
Step 6. If the network doesn't contain any edges and all the nodes are disconnected, the formed list of edges represents the recommended sequence of edge removals to maximize the drop of $R_{gc}(\rho)$.

Algorithm 1. The pseudocode of the Iterative Diffusion Importance

Input: The undirected and unweighted graph $G = (V, E)$.
Output: The suggested sequence of edges leading to the best deconstruction of the graph G: $e_{i1} \succcurlyeq \succcurlyeq e_{i2} \succcurlyeq \ldots \succcurlyeq e_{in}$, where operators $\prec$ and $\succcurlyeq$ correspond to precede and succeed.

Compose an empty list L_e to which the edges will be written.
for k = 1 : n **do**
  for i = 1 : n **do**
    $\overline{V} \leftarrow V \setminus \{v_i\}$;
    $\overline{E} \leftarrow E \setminus \{v_i, v_j \mid v_j \in N(v_i)\}$;
    for i = 1 : s **do**
      Compute the Diffusion Importance, $DI(e_i)$ by Equation (2);
    end for
  end for
  for $e_i, e_j \in E$ **do**
    if $DI(e_i) \leqslant DI(e_j)$ **then**
      $e_i \succcurlyeq e_j$;
    else
      $e_i \prec e_j$;
    end if
  end for
  Remove the most preceding edge e_i from the set of edges E, append it to the list L_e;
end for
Return: sequence of edges to be removed L_e : $\{e_{r1} \prec e_{r2} \prec \ldots \prec e_{rn}\}$

The present algorithm is based on conventional metric of Diffusion Importance the complexity of which is $O(n)$, so the complexity of Iterative Diffusion Importance is $O\left(\sum_{1}^{n} i\right)$. This complexity is significantly higher than that of Diffusion Importance, so in the case of emergency need for the criticality assessment, the original Diffusion Importance algorithm might be preferred, but considering the fact that the network deconstruction or, vice versa, deconstruction prevention is a long-planned process, the resulting complexity of Iterative Diffusion Importance is suitable.

4. Data and numerical simulations

To assess the efficiency of the newly proposed Iterative Diffusion Importance algorithm, we conducted a numerical experiment of decomposing three real-world benchmark networks and a real-world and currently functioning one. The results obtained with Iterative Diffusion Importance have been compared with the results obtained by Diffusion Importance and Degree Product. The choice of the networks is motivated by its variety in sizes (number of nodes and edges), to ensure that the comparison of results is fair. The choice of benchmark algorithms is motivated by two factors: both algorithms are often used as benchmarks in various studies, and the new Iterative Diffusion Importance algorithm must provide a better result than the original Diffusion Importance to justify its significance.

Zachary's club network

The Zachary's karate club is one of the most well-known benchmark networks [Zachary et al., 1977], which is often used as a benchmark for various analyses. It consists of 78 edges and 34 nodes, usually considered to have two communities, and represents a small-ized network, the circular layout of which is drawn (see Fig. 1).

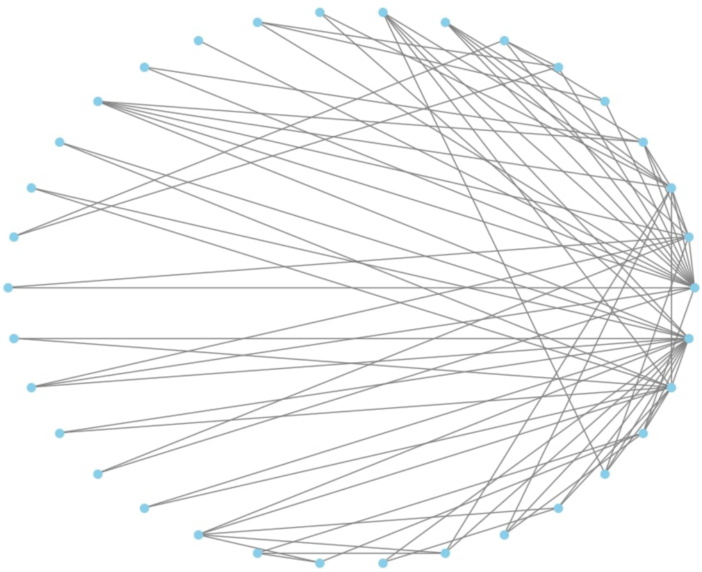

Figure 1. The circular layout of the Zachary's Karate Club network [Zachary et al., 1977]

Numerical simulation of the graph decomposition using all three methods is illustrated in Fig. 2. The figure depicts how the size R_{gc} of the largest connected component (normalized to the initial number of nodes) decreases with an increment of a fraction of removed edges sorted by the decrease of its relative significance (ρ). Three lines correspond to the decrease in size of R_{gc} when the decomposition is governed by Diffusion Importance (pink line), Degree Product (blue line), and Iterative Diffusion Importance (dashed green line) algorithms. As is seen, for the Zachary's Karate Club network, the Iterative Diffusion Importance shows significantly better performance than the conventional Diffusion Importance or Degree Product.

Dolphins network

The Dolphins network [Lusseau et al., 2003] is an undirected social network of frequent associations between 62 dolphins in a community living off Doubtful Sound, New Zealand. It has often been used as a benchmark for various studies, consists of 62 nodes, 160 links, and is usually considered to have six communities, which makes it a representative of a medium-sized network. The circular layout of Dolphins' network is shown in Fig. 3.

Numerical simulation of the graph decomposition using all three methods is illustrated in Fig. 4. The figure depicts how the size R_{gc} of the largest connected component (normalized to the

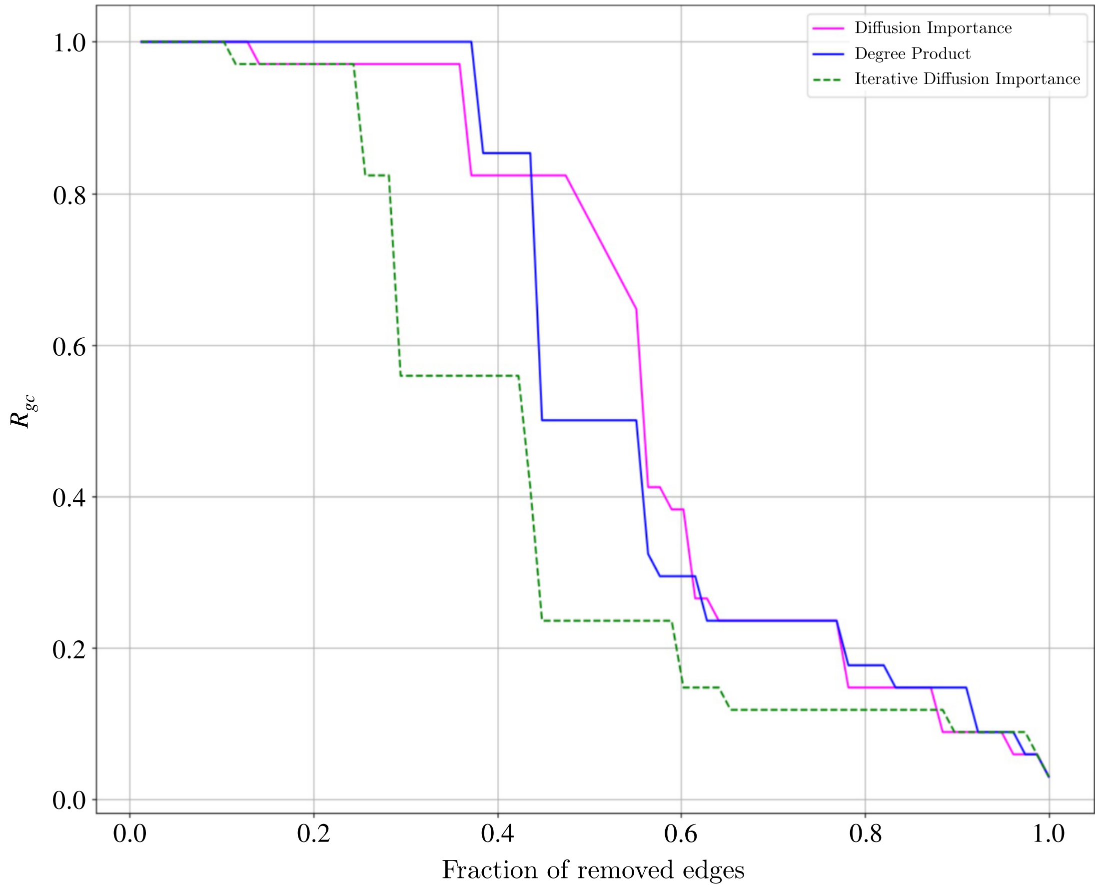

Figure 2. The graph represents decomposition of Zachary's Karate Club network [Zachary et al., 1977] within the Diffusion Importance (pink line), Degree Product (blue line), and Iterative Diffusion Importance (dashed green line). The lines represent the corresponding fraction of nodes R_{gc} belonging to the largest connected component vs the relative amount of removed nodes ρ

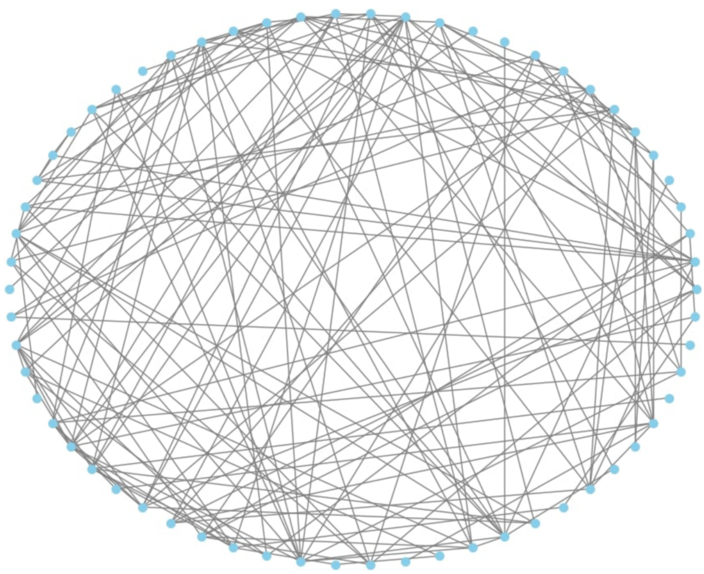

Figure 3. The circular layout of the Dolphins Network [Lusseau et al., 2003]

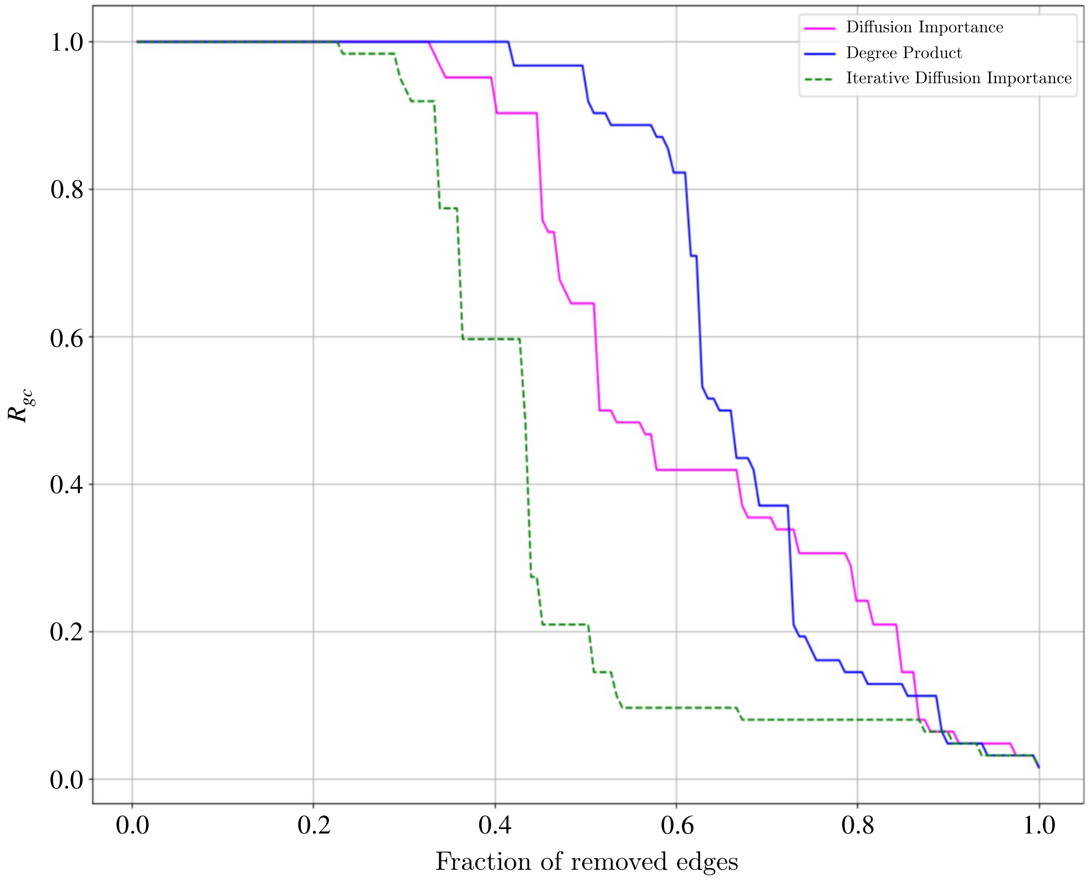

Figure 4. The graph represents decomposition of the Dolphins network [Lusseau et al., 2003] within the Diffusion Importance (pink line), Degree Product (blue line) and Iterative Diffusion Importance (dashed green line). The lines represent the corresponding fraction of nodes R_{gc} belonging to the largest connected component vs the relative amount of removed nodes ρ

initial number of nodes) decreases with an increment of a fraction of removed edges sorted by the decrease in its relative significance (ρ). Three lines correspond to the decrease in size of R_{gc} when the decomposition is governed by Diffusion Importance (pink line), Degree Product (blue line), and Iterative Diffusion Importance (dashed green line) algorithms. As can be seen, for the Dolphins network, the Iterative Diffusion Importance shows a significantly better performance than the conventional Diffusion Importance or Degree Product.

American college football network

The American College Football network [Girvan, Newman, 2002] is the network of American football games between Division IA colleges during the regular season in the fall of 2000. The football network can be regarded as a representative of big networks: it consists of 115 nodes and 613 edges. The circular layout of the American College Football network is represented in Fig. 5.

Numerical simulation of the graph decomposition using all three methods is illustrated in Fig. 6. The figure depicts how the size R_{gc} of the largest connected component (normalized to the initial number of nodes) decreases with an increment of a fraction of removed edges sorted by the decrease in its relative significance (ρ). Three lines correspond to the decrease in size of R_{gc} when the decomposition is governed by Diffusion Importance (pink line), Degree Product (blue line), and Iterative Diffusion Importance (dashed green line) algorithms. As can be seen, for the American College

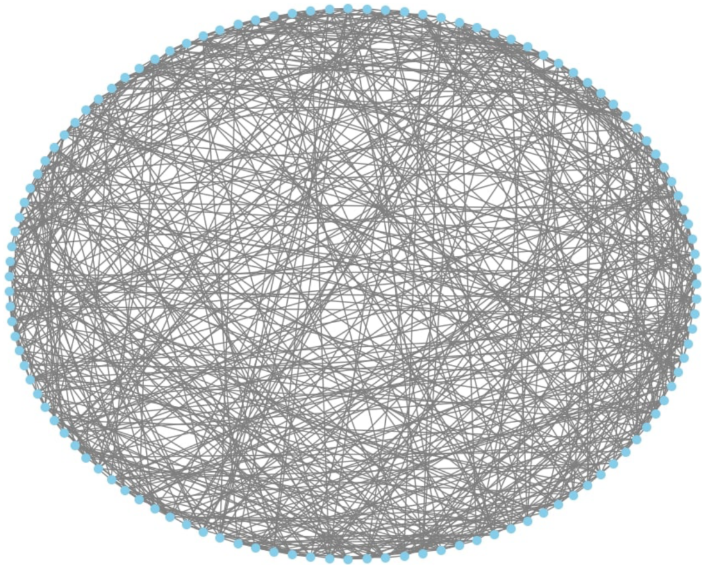

Figure 5. The circular layout of the American College Football Network [Girvan, Newman, 2002]

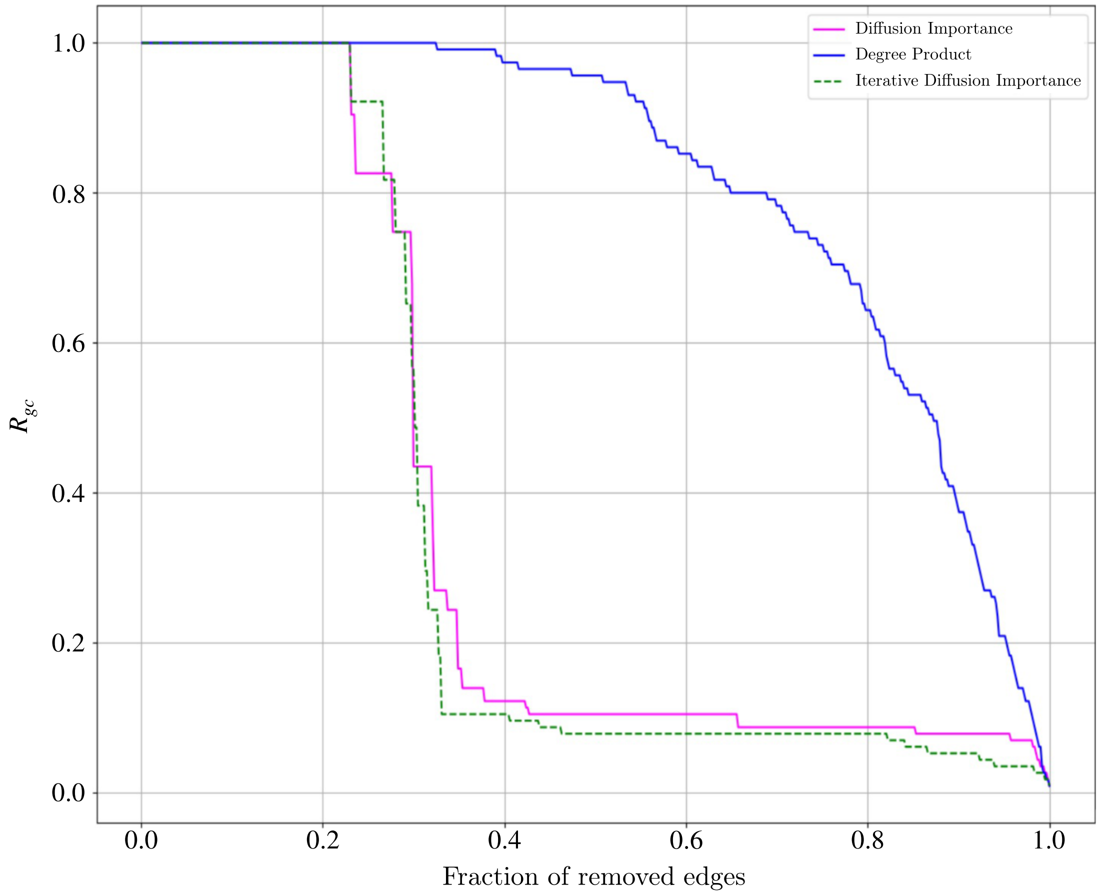

Figure 6. The graph represents decomposition of the American College Football Network [Girvan, Newman, 2002] within the Diffusion Importance (pink line), Degree Product (blue line) and Iterative Diffusion Importance (dashed green line). The lines represent the corresponding fraction of nodes R_{gc} belonging to the largest connected component vs the relative amount of removed nodes ρ

Football network, the Iterative Diffusion Importance shows a significantly better performance than the conventional Diffusion Importance or Degree Product.

US – Japan post-COVID airplane communication network

The fourth numerical experiment has been conducted on a current real-life network of airplane communication between the US and Japan. The COVID period demonstrated how important and at the same time vulnerable the international logistics and communication are, so even after the termination of transportation restrictions, we remain attentive to critical elements of the existing routes. Understanding of critical edges enables us to point out the attention on the individual connection between and within countries, the absence or temporal suspension of which could collapse the whole trade and exchange of goods between countries or regions. The following graph has been collected by the analysis of airplane communication in key airports of the US and Japan from April to May 2024. The graph consists of 36 nodes, 56 edges, and can be considered as a benchmark network alternative to Zachary's karate club. The adjacency list is presented in Table 1, and the match between indexes and airports is shown in Table 2. The circular layout of the given network is represented by Fig. 7

Table 1. Table represents the adjacency list for the analyzed graph

S	Target	S	Target
1	6, 18, 34	18	20, 21, 22, 27, 30
2	18, 33	20	33, 34
3	34	21	34
4	6	22	34
5	19, 33, 34	23	34
6	7, 11, 13, 14, 20, 36	24	33
7	34	25	33
8	33, 34	26	33, 34
9	34	27	34
10	33	28	34
12	13, 14	29	34
13	18, 34	30	33, 34
14	18, 33, 34	31	34
15	33, 34	32	33, 34
16	18	33	35
17	33, 34		

Numerical simulation of the graph decomposition using three methods is illustrated in Fig. 8. The figure depicts how the size R_{gc} of the largest connected component (normalized to the initial number of nodes) decreases with an increment of a fraction of removed edges sorted by the decrease in its relative significance (ρ). Three lines correspond to the decrease in size of R_{gc} when the decomposition is governed by Diffusion Importance (pink line), Degree Product (blue line), and Iterative Diffusion Importance (dashed green line) algorithms. As can be seen, for the US – Japan post-COVID Airplane Communication network, the Iterative Diffusion Importance shows a significantly better performance than the conventional Diffusion Importance or Degree Product.

5. Discussion

The deconstruction of the four analyzed networks of small, average and large size clearly demonstrated that the proposed Iterative Diffusion Importance algorithm provides a good ranking of edge significance for the connectivity of the network, identifying the critical link, the removal of which collapses the network integrity. We used as benchmark methods the original Diffusion Importance [Liu

Table 2. Match between the ID used in the table 1 and airports

ID	Airport	ID	Airport
1	Anchorage	19	Komatsu Airport
2	Atlanta	20	Los Angeles
3	Boston	21	Louisville
4	Charleston (SC)	22	Memphis
5	Chicago – O'Hare	23	Miami
6	Chubu Centrair International Airport	24	Minneapolis/St. Paul
7	Cincinnati	25	New York – JFK
8	Dallas/Fort Worth	26	Newark
9	Denver	27	Oakland
10	Detroit	28	Saipan
11	Everett	29	San Diego
12	Fukuoka Airport	30	San Francisco
13	Guam	31	San Jose (CA)
14	Honolulu	32	Seattle/Tacoma
15	Houston – Intercontinental	33	Tokyo Haneda International Airport
16	Indianapolis	34	Tokyo Narita International Airport
17	Kailua – Kona	35	Washington – Dulles
18	Kansai International Airport	36	Wichita – McConnell

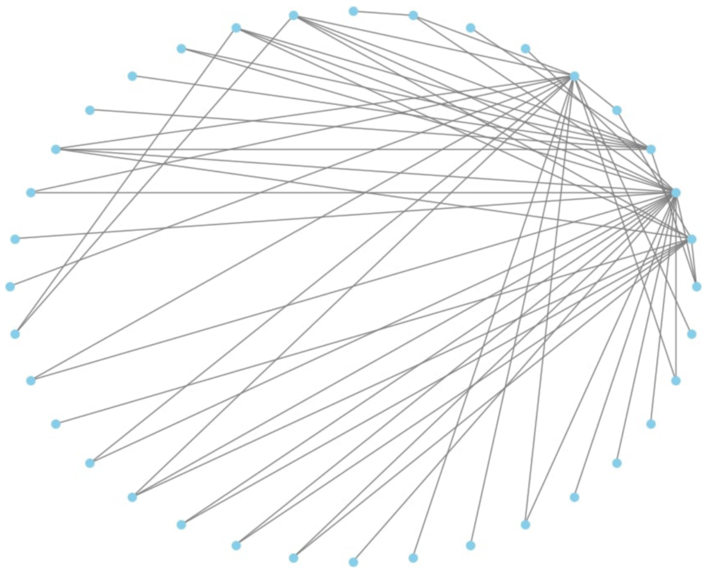

Figure 7. The circular layout of the US – Japan post-COVID Airplane Communication Network

et al., 2015], Degree Product [Duan et al., 2016], which are often considered as good methods for edge criticality assessment. In Table 3, we can see that these two methods are competitive with each other, because for some networks the first one gives a better result, and for other networks, the second. But the last column of Table 3 demonstrates that considering the integral area S_{rgc} under the curve $R_{gc}(\rho)$ as an efficiency metric, the Iterative Diffusion Importance is superior and outperforms both benchmark methods.

The superiority of the Iterative Diffusion Importance over the conventional Diffusion Importance is explained by its iterative nature. After any edge removal, the network under consideration cannot be considered as the same as the original, so the ranking of critical edges that has been obtained by the Diffusion Importance obviously might become invalid. In the case of the Iterative Diffusion

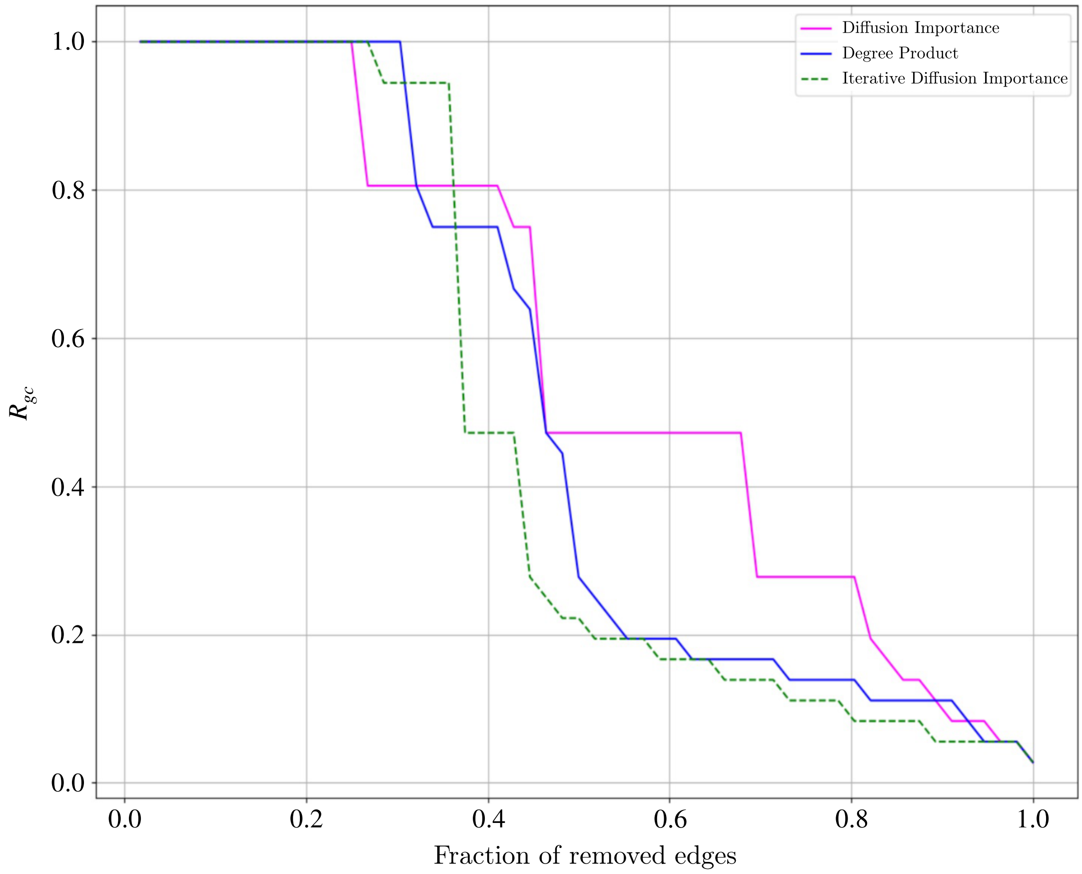

Figure 8. The graph represents decomposition of the US – Japan post-COVID Airplane Communication Network within the Diffusion Importance (pink line), Degree Product (blue line) and Iterative Diffusion Importance (dashed green line). The lines represent the corresponding fraction of nodes R_{gc} belonging to the largest connected component vs the relative amount of removed nodes ρ

Table 3. The integral area S_{rgc} under the curve $R_{gc}(\rho)$, see Exp. (1), for all three methods applied to decomposing the analyzed networks. The first column provides the list of names of analyzed networks, the remaining second, third, and fourth columns represent S_{rgc}-values for the Diffusion Importance, Degree Product, and the Iterative Diffusion Importance, respectively

Network\Method	DI	DP	IDI
Karate club	0.59	0.57	0.44
Dolphins	0.61	0.67	0.45
Football	0.36	0.81	0.35
Airports	0.57	0.50	0.46

Importance, after each modification of the analyzed network, we re-assess the topological metric and re-evaluate edge criticality ranking, taking into account all the previous interventions into the network structure. It resulted in an improvement of the efficiency from 2 % to 35 % depending on the network.

The superiority of the Iterative Diffusion Importance over the Degree Product is also demonstrated in Table 3. It is shown that even for networks where Degree Product is provides a better output than the conventional Diffusion Importance, the Iterative Diffusion Importance underlines the edges to be removed in such a way that the integral drop of $R_{gc}(\rho)$ is faster and lower, which enables us to state that the Iterative Diffusion Importance is not just a slight improvement of a conventional Diffusion Importance, but a new and noteworthy algorithm.

However, we want to underline the existing drawback of the Iterative Diffusion Importance. As explained in Section 2, the computational complexity of the Iterative Diffusion Importance is relatively higher than of the other two benchmark methods. The iterative nature of the algorithm leads to the recalculation of topological features over and over after every edge removal. Reducing re-evaluation frequency could decrease complexity but might also reduce the effectiveness of edge criticality assessment. Balancing update frequency, computational complexity, and resulting efficiency requires further investigation. So, in the case when the edge criticality ranking must be done within a short time and the duration of the computation is a critical point by itself, the other methods might be preferable. But in a real-life scenario, the network criticality assessment is not a question of emergency computation, so in cases where efficiency is more important than speed, the Iterative Diffusion Importance remains a better option.

As a more general statement resulting from the conducted numerical experiments, the iterative re-assessment of topological features of decomposing networks is a promising principle that potentially can be applied to various conventional methods and result in its improvement. We cannot state that every recognized conventional method will be enhanced by the iterative approach, because regardless of its intuitive evidence, the given hypothesis is impossible to prove. But the discovery and analysis of an iterative approach in application to other conventional methods could re-rank the methods' efficiencies and, in some cases, could detect dysfunctional/poorly designed methods, if the iterative approach doesn't improve its outcomes.

6. Conclusion

Identifying critical links in complex networks is an interesting and challenging issue in network science. Various straightforward algorithms have been proposed to cope with it, which are based on an initial assessment of topological features and ranking the edges according to this evaluation. In this paper, we have offered to look at the problem from another perspective and to take into account that any modification of the original network unavoidably leads to its topological change. We proposed to apply the iterative re-evaluation of the network features to enhance the efficiency of edge criticality assessment, and to be capable of finding a better sequence of edges, the removal of which leads to faster and better network decomposition. As a topological metric, we offered to use the Diffusion Importance, and the resulting enhanced method has been named the Iterative Diffusion Importance. We conducted a set of numerical experiments on the analysis and deconstruction of four networks: the real-life, currently functioning network of the post-COVID Airplane Communication between the US and Japan, and three well-known benchmark networks: Zachary's Karate Club, the American Football Network, and the Dolphins Network. The result of the proposed Iterative Diffusion Importance has been compared with results obtained by the Diffusion Importance and the Degree Product. The efficiency of the Iterative Diffusion Importance exceeded the efficiencies of the benchmark methods by 2–35 % depending on the complexity of the network, the number of edges, and the number of nodes.

The challenging issue of the Iterative Diffusion Importance is the increased computational complexity, but this drawback can be compensated by the preliminary planning of the network decomposition or protection, and by the reduction of the re-evaluation frequency of the iterative process. Our research does not seek to diminish the effectiveness of conventional methods but rather to showcase the additional benefits of the iterative principle for specific networks. This study enriches the existing body of knowledge by demonstrating that methodological innovation can significantly enhance the assessment of network robustness. The result of the present work opens the discussion on the potential improvement of other conventional methods by applying the iterative approach of topological features re-evaluation to them.

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