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The onset of the Darcy-ferroconvection flow model in a couple stress fluid subjected to a time-periodic magnetic field

C. Balaji[1,a], S. Maruthamanikandan[2], C. Rudresha[3,b], V. Vidyashree[4,c]

[1]Department of Mathematics, CMR Institute of Technology,
Bengaluru, 560037, India
[2]Department of Mathematics, School of Engineering, Presidency University,
Bengaluru, 560064, India
[3]Department of Mathematics, Sir MVIT,
Bengaluru, 562157, India
[4]Department of Mathematics, SJB Institute of Technology,
Bengaluru, 560060, India

E-mail: [a] balaji.c@cmrit.ac.in, [b] rudresha_maths@sirmvit.edu, [c] vidyashreev@sjbit.edu.in

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This study investigates the influence of a time-periodic (modulation) magnetic field upon the development of ferroconvection in a densely packed medium saturated with couple stress ferromagnetic fluid. The Darcy model is used to describe the flow in porous medium. The research is important from practical and theoretical point of view. A time-periodic magnetic field is essential in circumscribing channels where the effect of gravity is less or nonexistent to generate circulation. There are numerous engineering uses for this in the manufacturing of magnetic field sensors, charged particle electrode materials, modulators, magnetic resonators, and optical devices. The resulting physical eigenvalue problem is dealt with by using isothermal boundary conditions and the regular perturbation technique with a small time-periodic amplitude. The onset criteria were defined on the supposition that the exchange of stability principle holds. The shift in the thermal Rayleigh number is dependent on the associated parameters: magnetic parameter, Vadasz number, couple stress parameter, porosity, and frequency of the time-periodic function. The results in this case indicate that the onset of ferroconvection can be enhanced or reduced by appropriate changes in the governing parameters.

Keywords: ferromagnetic fluid, Darcy-model, couple stresses, perturbation method, modulation

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1. Introduction

Space technology utilizes a unique substance known as a ferromagnetic fluid, also called a magnetic fluid or ferrofluid. This fluid plays a crucial role in maintaining stability and controlling fuel flow in spacecraft. The fluid is created by suspending microscopic magnetic particles, such as magnetite, maghemite, or cobalt ferrite, in nonconductive liquid bases. These bases typically include water, mineral oil, or kerosene, and can become highly magnetized when subjected to an external magnetic field. In the current landscape of scientific advancement, magnetic fluids have transformed the field of hydrodynamics. Colloidal magnetite has emerged as the most extensively studied magnetic fluid, capturing the interest of numerous researchers and engineers. The applications of magnetic fluids are diverse and promising, spanning areas such as biomedicine, aeronautical engineering, computer storage drive seals, magnetic resonance imaging contrast agents, and thermal engineering [Horng et al., 2001; Torres-Díaz, Rinaldi, 2014; Kole, Khandekar, 2021]. The phenomenon of magnetic fluid convection in relation to thermal expansion within a magnetic fluid layer resembles Chandrasekhar's classic Rayleigh – Bénard instability [Chandrasekhar, 1961]. Magnetic fluids have become a focal point in this scientific study because of their potential applications in heat-transfer mechanisms. Expanding on Chandrasekhar's foundational work, Finlayson conducted an in-depth analysis of the Rayleigh – Bénard problem for magnetic fluids using the normal-mode technique with infinite amplitude disturbance [Finlayson, 1970]. This research subsequently inspired numerous researchers to investigate various aspects of the ferroconvection models over time [Maruthamanikandan, 2003; Zakaria, Sirwah, 2012; Nisha Mary, Maruthamanikandan, 2021; Vidya Shree et al., 2024].

Modulation (time-periodic field) is a significant and contemporary research topic due to its diverse applications in engineering and medicine. Adjusting appropriate parameters such as temperature, concentration, or electric field can substantially influence the movement of various fields. Fluctuating these parameters over time can enhance system stability. The present research focuses on time-periodic magnetic fields, which play a crucial role in generating circulation within extremely narrow channels where gravitational effects are negligible. This phenomenon has numerous engineering applications, including magnetic field detection devices, charged particles in electrode materials, modulators, magnetic resonators, and optical devices. Prominent researchers have extensively documented the time-periodic magnetic field at the onset of ferroconvection and the interplay between harmonic and subharmonic modes using Floquet theory, the Chebyshev pseudo-procedure, and the QZ method [Aniss, Belhaq, Souhar, 2001; Matura, Lucke, 2009; Lange, Odenbach, 2011; Shliomis, Smorodin, 2002]. Broadening the research focus, theoretical investigations have examined the combined effects of magnetic field fluctuations, rotation, and porous media. The research indicates that when both magnetization and rotation are present, the modulation of the magnetic field has a destabilizing effect on the system, causing convection to occur more rapidly than in systems without modulation [Balaji et al., 2023]. Nevertheless, as the porous parameter values increase, the stabilizing influence of magnetic field modulation becomes more pronounced [Balaji et al., 2024].

The study of heat transfer through convection in fluid-saturated porous media has attracted considerable attention due to its diverse applications in science and technology. These applications include harnessing geothermal energy, disposing of nuclear waste, creating thermal insulation for buildings, cleaning up aquifers, and enhancing drying processes. Numerous scientists have investigated porous media in conjunction with Rayleigh – Bénard instability, taking various parameters into account. Vadasz provided a comprehensive explanation of the effects of Coriolis force on Darcy porous media using linear and weakly nonlinear stability theory. This research demonstrated that in porous media, similar to pure fluids subjected to rotation and heated from below, viscosity has a destabilizing effect on the onset of stationary convection at high rotation rates [Vadasz, 1998]. A theoretical study examined the combined influence of gravity modulation and rotation in porous media. The findings revealed that gravity modulation generally stabilizes viscous fluid layers, while it can both stabilize and destabilize Brinkman porous layers. In contrast, gravity modulation consistently destabilizes Darcy porous layers. The results for Darcy and viscous flow cases were derived as special cases of the Brinkman model [Malashetty, Swamy, 2011]. The Darcy – Brinkman – Rayleigh – Bénard model was employed to analyze the impact of thermal diffusion on the starting point of double-diffusive convection in composite systems [Sumithra, Komala, Manjunatha, 2022]. Recent research has shown that the onset of anisotropic porous medium ferroconvection can be accelerated or decelerated by fine-tuning various governing parameters using regular perturbation techniques [Chandrashekar et al., 2022].

Conventional continuum theory fails to adequately describe the flow behavior of non-Newtonian couple stress fluids. The microcontinuum theory proposed by Stokes enables the incorporation of couple stresses, and nonsymmetric tensors. Among polar flow models, the Stokes couple-stress flow model stands out by considering couple stresses along with the typical Cauchy stress. This model represents the simplest extension of classical fluid theory that accounts for polar phenomena in liquid flow, including the emergence of couple stresses and body couplings [Stokes, 1966]. An eminent researcher studied the effects of couple stress and a magnetic field in a porous medium described by the Darcy model. The result indicates that the impacts of the couple-stress and the magnetic field decelerate the onset of convection, whereas the permeability accelerates the onset of convection [Sharma, Thakur, 2000]. Numerous researchers have extensively investigated the convective instability of couple stress fluid, taking into account factors such as porous media, rotation, double diffusion, chemical reaction, vibration, magnetic field and internal heating [Sharma, Pal, 2000; Sharma, Shivani Sharama, 2001; Sunil, Sharma, Pal, 2002; Malashetty, Kollur, 2011; Saravanan, Premalatha, 2012; Malashetty et al., 2012; Taj, Maruthamanikandan, Akbar, 2013; Gaikwad, Kouser, 2014].

The available literature clearly shows that no one has previously examined the impact of a time-periodic magnetic field characteristics on the threshold of convection in a densely packed porous medium saturated with couple stress ferromagnetic fluid. As a result, the present study aims to analyze the ferroconvective instability in a densely packed porous medium saturated with couple stress ferromagnetic fluid in the presence of a time-periodic magnetic field. The results could be immensely beneficial in diagnostic systems, lubricants, in-line polarized fiber modulators, biological fluids, tumor cell treatment, polymer solutions, dynamic loudspeakers, climatology, clay coatings, and zero-gravity application situations that use magnetic fluid as a working medium. The direction of this work is to determine the base state and its stability through a linear analysis and the eigenvalue of the problem by means of a regular perturbation approach. Further, the study is based on the assumption that the convective currents are minimal and the frequency of the time-periodic magnetic field is extremely small. As a result, depending on the frequency of the time-periodic magnetic field, the onset of couple stress porous medium magnetic fluid convection can be either accelerated or decelerated. This paper is therefore structured as follows: The formulation of the problem and conservation equations of the considered system are presented in Sec. 2. In Sec. 3 a linear stability analysis, followed by method of solution, is carried out in Sec. 4. The results obtained are presented graphically and are discussed in Sec. 5. Section 6 contains concluding remarks.

2. Mathematical model

This model is examined with a Cartesian reference frame (x, y, z), where the horizontal directions are denoted by the x-axis and y-axis, and the z-axis denotes the vertical upward direction. The system is comprised of a thin surface of ferromagnetic couple stress fluid saturating densely packed porous medium in the horizontal directions (i. e., in the x-y plane), which is heated from the underside and cooled from the upper side. The region is supposed to have two surfaces, say $z \geqslant 0$ as the bottom surface of ferromagnetic fluid and $z \leqslant d$ as the top surface of ferromagnetic fluid. The flow in the densely packed porous medium is described by the Darcy model. The system is working under the impact of a vertically upward, external time-periodic magnetic field $\vec{H}_0^{\text{ext}}(t) = H_0^{\text{ext}}(t) = H_0\widehat{(1 + \varepsilon \cos \omega t)k}$ due to magnetic field modulation and acceleration due to vertically downward gravitational force $\vec{g} = -g\widehat{k}$. Here, H_0, ε, ω, t and g stand for uniform magnetic field, amplitude, frequency, time and gravitational acceleration, respectively. The bottom and top surfaces are kept at different uniform temperatures with a temperature difference ΔT. The Boussinesq approximation is applied to account for the influence of density variations. With these assumptions, the relevant model equations governing the flow of an incompressible ferromagnetic couple stress fluid in a densely packed porous medium with time-periodic magnetic field are as follows [Finlayson, 1970; Sharma, Thakur, 2000; Sharma, Pal, 2000; Sharma, Shivani Sharama, 2001; Sunil, Sharma, Pal, 2002; Aniss, Belhaq, Souhar, 2001; Vadasz, 1998; Stokes, 1966; Malashetty, Kollur, 2011; Saravanan, Premalatha, 2012; Malashetty et al., 2012; Gaikwad, Kouser, 2014]:

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\rho_0 \left[\frac{1}{\varepsilon_p}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon_p^2}(\vec{q}\cdot\nabla)\vec{q}\right] = -\nabla p + \rho\vec{g} - \frac{1}{K}\left(\mu_f - \mu_c\nabla^2\right)\vec{q} + \nabla\cdot(\vec{H}\vec{B}), \tag{2}$$

$$\varepsilon_p C_1\left[\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T\right] + (1-\varepsilon_p)(\rho_0 C)_s\frac{\partial T}{\partial t} + \mu_0 T\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\cdot\left[\frac{\partial \vec{H}}{\partial t} + (\vec{q}\cdot\nabla)\vec{H}\right] = K_1\nabla^2 T, \tag{3}$$

$$\rho = \rho_0[1 - \alpha(T - T_a)], \tag{4}$$

$$\vec{M} = \frac{\vec{H}}{H}M(H, T), \tag{5}$$

$$M = M_0 + \chi_m(H - H_0) - K_m(T - T_a), \tag{6}$$

$$\nabla\cdot\vec{B} = 0, \quad \nabla\times\vec{H} = 0, \quad \vec{B} = \mu_0(\vec{H} + \vec{M}), \tag{7}$$

where $\vec{q}$ is the velocity of fluid, ρ is the density, ρ_0 is the reference density, ε_p is the porosity, p is the pressure, μ_f is the dynamic viscosity, μ_0 is the magnetic permeability, μ_c is couple stress viscosity, T is the temperature, $\vec{H}$ is the total magnetic field, $\vec{M}$ is the magnetization, $\vec{B}$ is the magnetic induction, α is the coefficient of thermal expansion, T_a is the reference temperature, $C_1 = \rho_0 C_{V,H} - \mu_0\vec{H}\cdot\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}$, $C_{V,H}$ is the specific heat at constant volume and magnetic field, χ_m is the differential magnetic susceptibility, and K_m is the pyromagnetic coefficient. The lower and upper surface temperatures are $T = T_a + \frac{1}{2}\Delta T$ at $z = 0$ and $T = T_a - \frac{1}{2}\Delta T$ at $z = d$, respectively.

3. Linear stability analysis

Applying the method of an infinitesimal perturbation [Malashetty, Kollur, 2011; Chandrashekar et al., 2022] and introducing the magnetic potential ϕ, we obtain the following stability equations:

$$\left(\frac{1}{V_z}\frac{\partial}{\partial t} - C_S\nabla^2 + 1\right)\nabla^2 W = \left[R + RM_1(1+\varepsilon L)^2\right]\nabla_1^2 T - RM_1(1+\varepsilon L)^2\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right), \tag{8}$$

$$\begin{gathered}\lambda_p\frac{\partial T}{\partial t} - W + \frac{M_2}{\varepsilon_p}\left(\frac{(1+\varepsilon L)^2}{(1+\chi_0)^2}\right)W + M_2\left(\frac{(1+\varepsilon L)^2}{\chi_0(1+\chi_0)}\right)\left(\frac{\partial T}{\partial t} - W\right) - \\ -M_2\frac{1}{H_0}\frac{\partial}{\partial t}\frac{H_0(1+\varepsilon L)}{(1+\chi_0)}\left(\frac{\partial\phi}{\partial z}\right) - M_2\left(\frac{(1+\varepsilon L)^2}{(1+\chi_0)}\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right) - \\ -M_2\frac{1}{H_0}T\frac{\partial}{\partial t}H_0(1+\varepsilon L) = \nabla^2 T,\end{gathered} \tag{9}$$

$$\nabla^2\phi = \frac{\partial T}{\partial z}, \tag{10}$$

where $L = \mathrm{Re}\left\{e^{-i\omega t}\right\} = \cos\omega t$, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$ and various nondimensional parameters are $V_z = \frac{\varepsilon_p d^2 \mu_f C_1}{\rho_0 K K_1} = \frac{\varepsilon_p \mathrm{Pr}}{\mathrm{Da}}$; $R = \frac{\alpha\rho_0 g \Delta T d}{\mu_f \kappa}$, $\kappa = \frac{K_1}{C_1}$, $C_S = \frac{\mu_c}{\mu_f d^2}$, $M_2 = \frac{\mu_0 \chi_0^2 H_0^2}{C_1(1+\chi_0)T_a}$, $M_1 = \frac{\mu_0 \Delta T \chi_0^2 H_0^2}{T_a^2(1+\chi_0)^3 \alpha\rho_0 g d}$, $RM_1 = \frac{\mu_0\chi_0^2(\Delta T)^2 d H_0^2}{\mu\kappa(1+\chi_0)^3 T_a^2}$.

Here, ∇^2 is the Laplacian differential operator, V_z is the Vadasz number, Pr is the Prandtl number, Da is the Darcy number, R is the Darcy – Rayleigh number, κ is the effective thermal diffusivity, C_S is the Couple stress parameter, M_2 is the magnetic parameter, M_1 is the magnetic number, and RM_1 is the magnetic Rayleigh number.

Since the typical value of M_2 is equivalent to the order of 10^{-6} [Finlayson, 1970; Balaji et al., 2024] for magnetic fluid with different base liquids and hence its effect is neglected. Equations (8)–(10) are to be solved using suitable boundary conditions [Chandrasekhar, 1961]

$$W = \frac{\partial^2 W}{\partial z^2} = T = \frac{\partial \phi}{\partial z} = 0 \quad \text{at } z = 0,\ 1. \tag{11}$$

It is suitable to rewrite the whole problem in terms of the vertical component of the velocity W. Upon combining Eqs. (8)–(10), we obtain the following equations

$$L_1 L_2 L_3 \nabla^2 W = L_3 R \nabla_1^2 W + RM_1 (1 + \varepsilon L)^2 \nabla_1^4 W, \tag{12}$$

where

$$L_1 = \frac{1}{V_z}\frac{\partial}{\partial t} - C_S \nabla^2 + 1, \quad L_2 = \lambda_p \frac{\partial}{\partial t} - \nabla^2, \quad L_3 = \frac{\partial^2}{\partial z^2} + \nabla_1^2.$$

The boundary conditions in Eq. (11) can also be rearranged in terms of W in the form [Chandrasekhar, 1961]

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = 0 \quad \text{at } z = 0,\ 1. \tag{13}$$

4. Method of solution

The eigenfunctions, W, and the eigenvalues, R, are associated with the above eigenvalue problem for a time-periodic magnetic field that is different from the stationary magnetic field by a small quantity of order ε. Therefore, we assumed the solution of Eq. (12) of the form [Malashetty, Kollur, 2011; Balaji et al., 2024]

$$\begin{gathered} W = W_0 + \varepsilon W_1 + \varepsilon^2 W_2 + \cdots, \\ R = R_0 + \varepsilon R_1 + \varepsilon^2 R_2 + \cdots \end{gathered} \tag{14}$$

Substituting Eq. (14) into Eq. (12), we get the equation for the couple-stress Darcy – Rayleigh number R_0 for the densely packed porous medium saturated with couple stress ferromagnetic fluid in the absence of time-periodic (modulation) magnetic field

$$R_0 = \frac{\left(\pi^2 + \alpha^2\right)^3 + C_S\left(\pi^2 + \alpha^2\right)^4}{\alpha^2\left[\pi^2 + (1 + M_1)\alpha^2\right]}, \tag{15}$$

where $\alpha = \sqrt{\alpha_x^2 + \alpha_y^2}$ is the overall horizontal wavenumber with α_x and α_y being wavenumbers in x and y directions, respectively. Equation (15) is the expression for the couple-stress Darcy – Rayleigh number as a function of the wavenumber, couple stresses and the magnetic force in the absence of a time-periodic magnetic field for the Rayleigh – Bénard couple-stress ferroconvection problem in a densely packed porous medium. When the values of M_1 and C_S become zero, the expression for R_0 attenuates to that of Chandrasekhar's foundational work [Chandrasekhar, 1961]. On the other hand, the problem reduces to that of Finlayson's Rayleigh – Bénard study on ferromagnetic fluids [Finlayson, 1970], when $C_S = 0$. Further, the plot of Darcy – Rayleigh number R_0 versus wavenumber α is delineated in Section 5.

Following the analysis of [Malashetty, Kollur, 2011; Chandrashekar et al., 2022], we get the following expression for R_2:

$$R_2 = -\frac{R_0^2 M_1^2 \alpha^6}{\pi^2 + (1 + M_1)\alpha^2} \sum_{n=1}^{\infty} \frac{C_n}{D_n}, \tag{16}$$

where

$$C_n = -2\left(\omega^2 \frac{\lambda_p}{V_z}\left(n^2\pi^2 + \alpha^2\right)^2 - \left(n^2\pi^2 + \alpha^2\right)^3 - C_S\left(n^2\pi^2 + \alpha^2\right)^4 + R_0\alpha^2\left[n^2\pi^2 + (1 + M_1)\alpha^2\right]\right),$$

$$D_n = \left(\omega^2 \frac{\lambda_p}{V_z}\left(n^2\pi^2 + \alpha^2\right)^2 - \left(n^2\pi^2 + \alpha^2\right)^3 - C_S\left(n^2\pi^2 + \alpha^2\right)^4 + R_0\alpha^2\left[n^2\pi^2 + (1 + M_1)\alpha^2\right]\right)^2 + \\ +\omega^2\left(-\frac{1}{V_z}\left(n^2\pi^2 + \alpha^2\right)^3 - \lambda_p\left(n^2\pi^2 + \alpha^2\right)^2 - C_S\lambda_p\left(n^2\pi^2 + \alpha^2\right)^3\right)^2.$$

5. Results and discussion

Here, a discussion of the starting point of convection in a densely packed porous medium saturated with couple stress ferromagnetic fluid exposed to a magnetic field is meticulously carried out in detail. A collection of conservation equations considering temperature, pressure, magnetic field, viscosity, permeability, and couple stresses has been formulated and further set to infinitesimal perturbation incorporating pressure, magnetic field, temperature, magnetic induction, and magnetization of magnetic fluid. A related eigenvalue problem has been derived using the normal mode method and is further investigated by means of the regular perturbation technique. Significant insights into various parameters involved in the study are elucidated in detail for the following cases: the absence of magnetic field modulation and the presence of magnetic field modulation. These cases are discussed and analyzed with relevant graphs.

Absence of magnetic field modulation

This section deals with variations in the buoyancy-magnetization parameter M_1 and the couple-stress parameter C_S for the present study in the absence of time-periodic (modulation) magnetic field. In Fig. 1, *a*, the impact of M_1 on the stationary Darcy – Rayleigh number expressed by R_0 and the wave number expressed by α is shown. The parameter M_1 is the ratio of magnetic force to gravitational force, it bears the values as $M_1 = 5$, 25, 50 and the other parameter is supposed to be constant as $C_S = 0.1$. It can be seen that the parameter M_1 shows the subcritical motion on the configuration for ferroconvective instability in the absence of a time-periodic magnetic field. Because, the critical value of Darcy – Rayleigh number R_{0c} is diminished, as the values of M_1 are increased. It is clear from the definition of buoyancy-magnetization parameter that an increase in M_1 either amplifies the magnetic force or diminishes the gravitational force. Thus, the magnetic parameter M_1 helps accelerate the onset of ferroconvection in the absence of a time-periodic magnetic field. Further, the critical value of wave number α_c corresponding to Darcy – Rayleigh number R_0, increases with an increase in M_1.

Figure 1, *b* shows the influence of the couple-stress parameter with $C_S = 0.05$, 0.1, 0.2 while assuming the other parameter to be constant as $M_1 = 25$. The couple stress parameter C_S is the ratio of couple stress viscosity to that of dynamic viscosity. It is clear from Fig. 1, *b* that the curves for

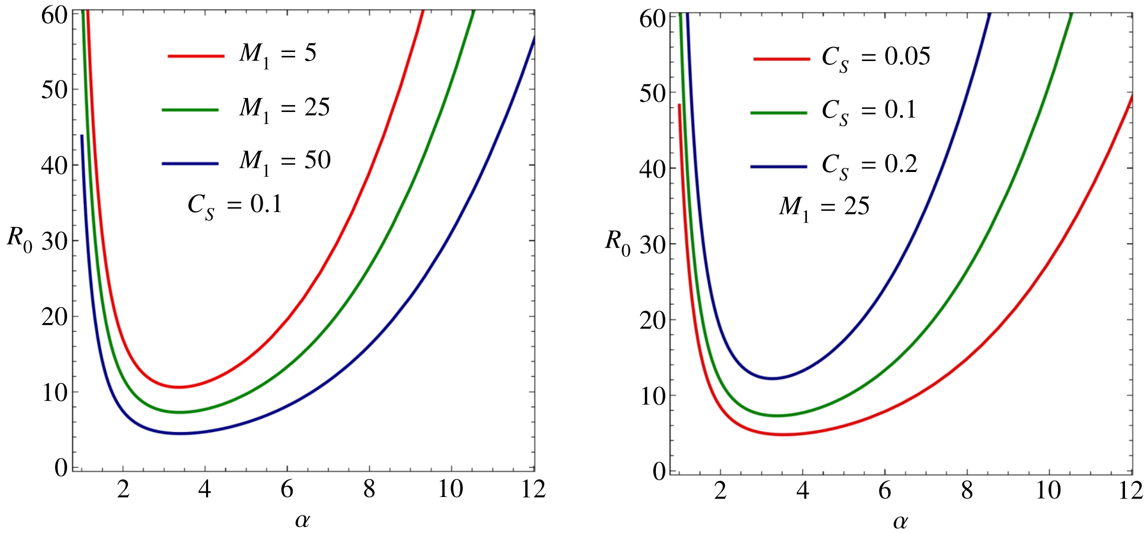

Figure 1. Variation in Darcy–Rayleigh number R_0 versus wave number α on varying (a) the buoyancy-magnetization parameter M_1 and (b) the couple stress parameter C_S

increasing values of C_S show the supercritical motion. This is due to the fact that increasing the values of C_S escalates the viscosity in the fluid and makes the system more stable. Thus, the couple stress parameter C_S helps delay the onset of ferroconvection in the absence of a time-periodic magnetic field. On the other hand, the critical value of wave number α_c corresponding to Rayleigh number R_0, decreases with an increase in C_S.

Presence of magnetic field modulation

This section analyzes the impacts of the buoyancy-magnetization parameter M_1, the Vadasz number V_z, normalized porosity λ_p, and the couple stress parameter C_S in the presence of a time-periodic magnetic field. The principle of exchange of stability analysis is based on the assumption of minimal amplitude of the time-periodic magnetic field. Figures 2, *a*, 2, *b*, 3, *a* and 3, *b* are used to elucidate the findings of the present study. The supercritical and subcritical effects of the time-periodic magnetic field depend on the sign of correction to the critical thermal Rayleigh number R_{2c}. The subcritical effect of the time-periodic magnetic field occurs when the values of R_{2c} is greater than zero. On the other hand, a supercritical effect of the time-periodic magnetic field takes place when the value of R_{2c} is less than zero.

Figure 2, *a* corresponds to the fluctuations in the critical values of correction Rayleigh number R_{2c} with the variation in frequency ω for fixed values of parameters $V_z = 3$, $\lambda_p = 0.5$, and $C_S = 0.1$ with the fluctuation in the buoyancy-magnetization parameter by taking its different values as $M_1 = 5, 25, 50$. It is clear from the figure that the impact of increasing magnetic parameter M_1 leads to attenuation in the critical values of the correction Rayleigh number R_{2c} on the configuration for the ferroconvective instability over a moderate and large range of frequency (i. e., $0 < \omega \leqslant 50$). This is due to the fact that under the impact of a time-periodic magnetic field, the viscosity of magnetic fluid depends on the time-periodic frequency. If the time-periodic frequency is less, then the applied time-periodic magnetic field amplifies the viscosity of magnetic fluid, and for moderate and large ranges of frequency, it diminishes the viscosity of magnetic fluid. Thus, the subcritical motion becomes apparent by the action of M_1, which is more noticeable at moderate and large time-periodic frequency values (i. e., $0 < \omega \leqslant 50$). As a result, M_1 plays a prominent role in accelerating the onset of ferroconvection. In contrast, a supercritical motion turned up for increasing values of M_1 at very small ranges of frequency

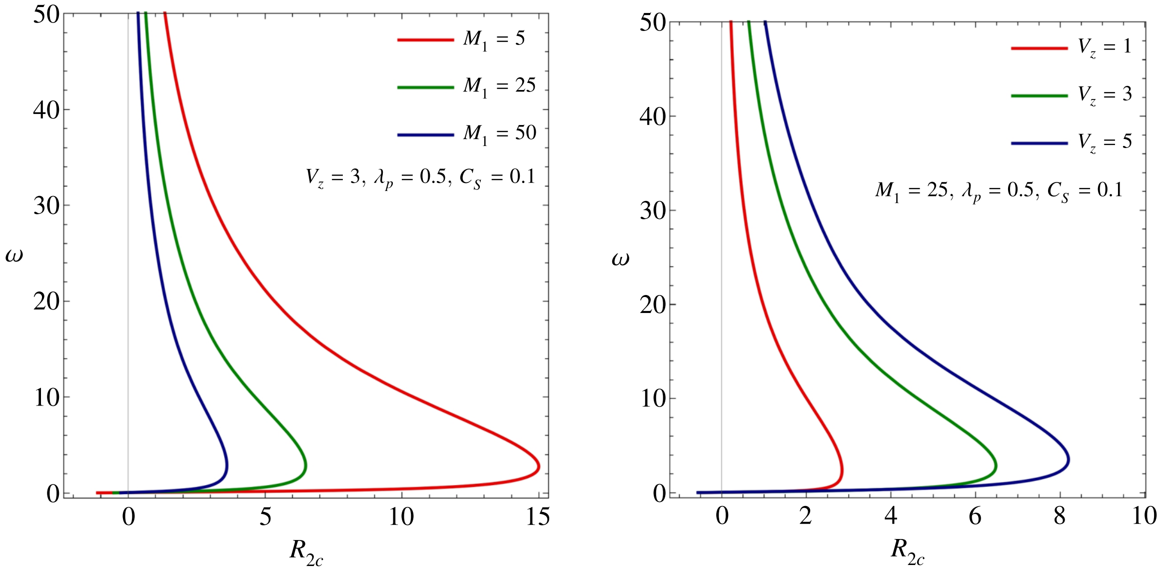

Figure 2. Variation in the correction to the critical Darcy – Rayleigh number R_{2c} versus frequency ω on varying (a) the buoyancy-magnetization parameter M_1 and (b) the Vadasz number V_z

(i. e., $0 < \omega \leqslant 50$). This is observed because, for a very small value of time-periodic frequency ω, the viscosity is high in magnetic fluids due to the suspended microscopic magnetic particles in the liquid carrier wrapped with surfactant. Eventually, for a very small value of ω, the suspended magnetic particles in the magnetic fluid take time to expand in the fluid and hence postpone the onset of couple-stress porous medium ferroconvection. Thus, the system exhibits stability in dual modes.

In Fig. 2, *b*, variation in R_{2c} is plotted with variation in the time-periodic frequency ω when the Vadasz number V_z is present in the system. The impact of Vadasz number is explored by keeping all other parameters fixed as $M_1 = 25$, $\lambda_p = 0.5$, $C_S = 0.1$ and varying Vadasz number as $V_z = 1, 3, 5$. It is interpreted from the graph that if the Vadasz number is increasing, then the value of correction to the critical Rayleigh number R_{2c} is intensified significantly over the entire range of values of time-periodic frequency (i. e., $0 < \omega \leqslant 50$), which leads to a rapid supercritical motion within the system. The parameter V_z depends on several factors such as the viscosity of the fluid, thermal diffusivity, porosity and permeability of the porous medium. Here, the Vadasz number inflates the supercritical motion because the porous medium causes the magnetic fluid to become hampered, leading to fluctuations in the viscous effect, thermal diffusivity, porosity and permeability. Thus, the Vadasz number plays a prominent role in suppressing the onset of couple stress porous medium ferroconvection in the presence of time-periodic magnetic field. Furthermore, the influence of the time-periodic magnetic field disappeard altogether for a very low range of values of freqeuncy ω.

Figure 3, *a* shows the impact of normalized porosity λ_p. The variation in λ_p is taken as $\lambda_p = 0.3$, 0.5, 0.8 while keeping additional parameters fixed as $M_1 = 25$, $V_z = 3$, $C_S = 0.1$ and a graph is plotted between the critical correction Rayleigh number R_{2c} and time-periodic frequency ω. The normalized porosity λ_p is proportional to the ratio of porosity to that of specific heat. On varying normalized porosity λ_p, it is delineated through Fig. 3, *a* that the action of normalized porosity results in subcritical motion within the system for small and intermediate time-periodic frequency values (i. e., $0 < \omega \leqslant 15$). On the other hand, supercritical motion occurred up for higher values of time-periodic frequency (i. e., $15 < \omega \leqslant 50$), when the normalized porosity is increased. Therefore, normalized porosity exhibits dual impacts on the stability of couple stress magnetic fluid saturated porous medium. This is observed because, from the description of normalized porosity on increasing values of λ_p, leading to changes in the pore space and temperature fields. Hence, it creates the delaying process of ferroconvective

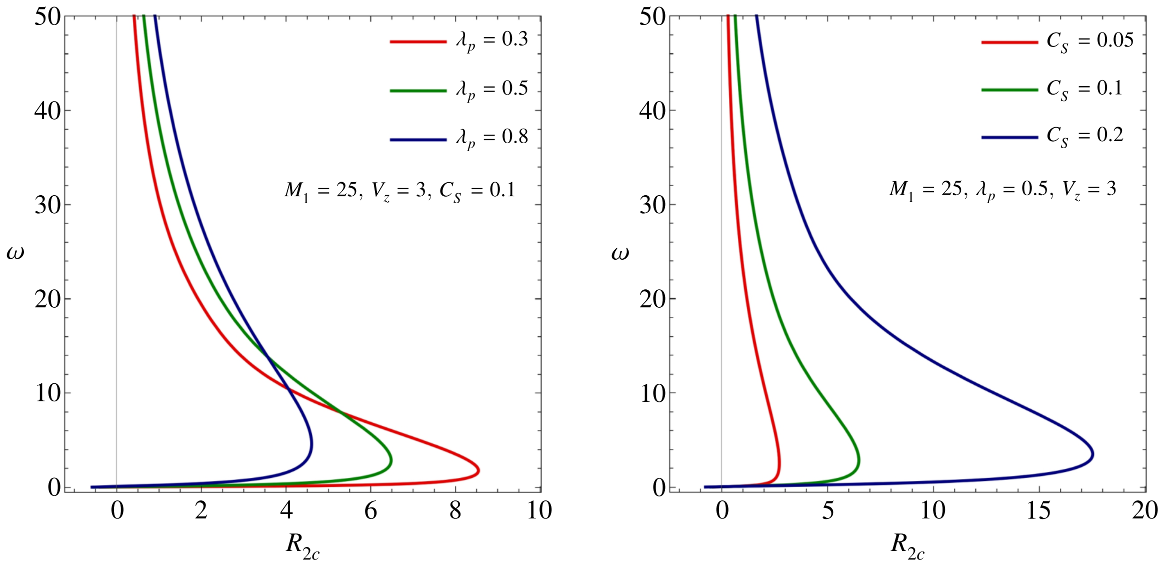

Figure 3. Variation in the correction to the critical Darcy – Rayleigh number R_{2c} versus frequency ω on varying (a) the normalized porosity λ_p and (b) the Couple stress parameter C_S

instability for a large values of frequency (i. e., $15 < \omega \leqslant 50$), whereas the normalized porosity under the small and intermediate values of frequency (i. e., $0 < \omega \leqslant 15$) expresses the hastening process of ferroconvection.

Figure 3, *b* depicts the correction to the critical Rayleigh number R_{2c} as a function of time-periodic frequency ω for different values of couple-stress parameter represented as C_S. The results presented here are for $C_S = 0.05, 0.1, 0.2$ when $M_1 = 25$, $V_z = 3$, $\lambda_p = 0.5$. The couple-stress parameter C_S is the ratio of couple-stress viscosity to that of dynamic viscosity. It is clear from Fig. 3, *b* that the effect of increasing couple-stress parameter C_S leads to amplification in the critical values of R_{2c} on the ferroconvective instability for the entire range of frequency (i. e., $0 < \omega \leqslant 50$). This is due to the fact that increasing the values of C_S escalates the viscosity in the fluid and makes the system more stable. Thus, a rapid supercritical motion becomes apparent by the action of C_S, which is more noticeable over the entire range of time-periodic frequency (i. e., $0 < \omega \leqslant 50$). As a result, C_S plays a prominent role in decelerating the onset of magnetic fluid convection in couple-stress fluid saturated porous medium.

6. Conclusions

The Rayleigh – Bènard ferroconvection problem of couple-stress fluid saturating porous medium is investigated for the following cases: the absence of magnetic field modulation (time-periodic) and the presence of magnetic field modulation. Small linear perturbations are introduced to the system, and the resultant equations are solved using the normal mode approach. Analytical solutions are obtained by the regular perturbation method. The impacts of the buoyancy-magnetization parameter, Vadasz number, porous parameter, and couple-stress parameter are explained in graphs, and the following observations are analyzed: The buoyancy-magnetization parameter M_1 shows subcritical motion in the system for both cases (absence and presence) of a time-periodic magnetic field. The impact of Vadasz number is exactly opposite to that of buoyancy-magnetization parameter M_1, increasing V_z leads to supercritical motion in the system in the presence of time-periodic frequency. However, normalized porosity λ_p exhibits dual impacts on the stability of the system. That is subcritical instability on increasing λ_p when ω is small and moderate and supercritical instability on increasing λ_p when ω is moderate and large. In addition, couple-stress parameter C_S plays a significant role in slowing down the process of magnetic fluid convection in a couple-stress fluid saturating porous medium for both cases (absence and presence) of a time-periodic magnetic field.

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