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CONSISTENCY AND CONTROL OVER EIGENVALUE SPECTRUM ¹

We consider a linear control system with an incomplete feedback

$$\dot{x} = A(t)x + B(t)u, \quad y = C(t)x, \quad u = U(t)y.$$

We study the problem of control over asymptotic behaviour of the closed-loop system

$$\dot{x} = (A(t) + B(t)U(t)C(t))x, \quad x \in \mathbb{R}^n. \quad (1)$$

For the above system, we introduce the concept of consistency, which is a generalization of the concept of complete controllability onto systems with incomplete feedback. The focus is on the consistency property of system (1). We have obtained new necessary conditions and sufficient conditions for the consistency of the above system including the case when the system is time-invariant.

For time-invariant system (1), we study the problem of global control over eigenvalue spectrum. The objective is to reduce a characteristic polynomial of a matrix of stationary system (1) to any prescribed polynomial by means of time-invariant control U . By methods of linear algebra, we obtain necessary and sufficient conditions for global controllability over the spectrum in the case where the system coefficients have a special form. In that case, we establish that the property of consistency is sufficient for the global controllability over the spectrum, and under certain assumptions it is necessary too.

Keywords: linear systems with delay, reducibility, Lyapunov exponents, Lyapunov invariants.

Introduction

We consider a differential inclusion

$$\dot{x} \in F(f^t \sigma, x), \quad \sigma \in \Sigma, \quad x \in \mathbb{R}^n, \quad (0.1)$$

where $F(\sigma, x)$ is a compact set in \mathbb{R}^n and Σ is a compact metric space. System (0.1) generates the topological flow ... as it was noted by N. N. Krasovskii [1, Chapter 3]. The works [1, 3–5, 8–10] are ...

§ 1. Notations and definitions

Let \mathbb{R}^n be an n -dimensional Euclidean space and

Consider the system

$$\dot{x}(t) = \int_{-r}^0 dA(t, s)x(t+s), \quad t \in \mathbb{R} = (-\infty, \infty). \quad (1.1)$$

We identify system (1.1) with

$$\dot{y}(t) = \int_{-r}^0 dB(t, s)y(t+s), \quad t \in \mathbb{R} = (-\infty, \infty). \quad (1.2)$$

Remark 1. We note that system $\dot{y}(t) = \int_{-r}^0 dB(t, s)y(t+s)$ is

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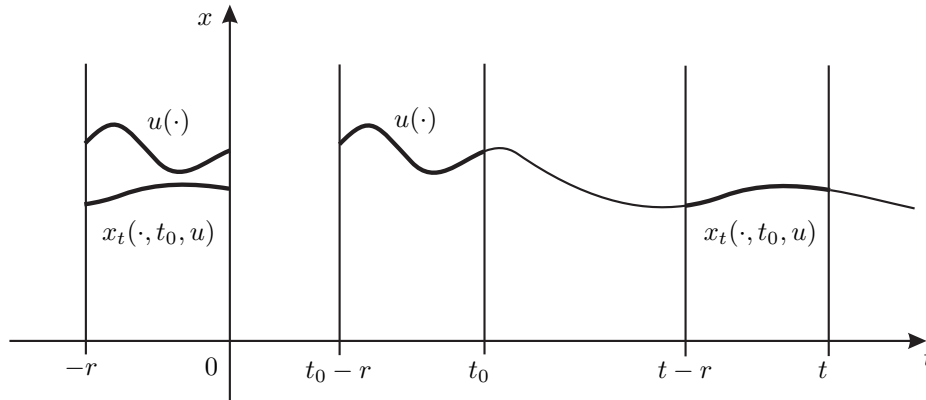


Fig 1. Motion generated by solution of equation (1.1)

§ 2. Invariants sets

Definition 1 (see [8], [9, p.110]). We say that \mathfrak{X}_0 is *regular* if ...

Lemma 1 (see [13, p.123]). Let \mathfrak{X}_0 be the

P r o o f. We show that see section 1

□

§ 3. Reducibility theorem

Theorem 1 (about triangulation). If \mathbb{S}^p is completely regular then:

- a) there exists a system B and Lyapunov transformation $x = L(t)y$ such that ...;
- b) in the set $\{B\}$ of all systems kinematically similar to the system (A, \mathbb{S}^p) there exists a system $\dot{y} = C(t)y$ such that ...

§ 4. Proof of theorem 1

1. We fix some basis in the space
2. Take a continuous function
3. We construct the function $t \rightarrow \tilde{B}(t)$ so that

Then we have

$$|\hat{Y}(t, 0)| \leq \alpha |V(t)Z(t)| = \dots = \alpha |Z(t)| \leq \alpha \sqrt{r} \|U_t\|_{\mathbb{R}^p \rightarrow \mathfrak{S}}, \quad (4.1)$$

... Thus, the formula is true. Q.E.D.

□

Theorem 2. Let X be Banach space. Then ...

Lemma 2. Suppose that.....

Proposition 1. Let

Corollary 1. For any continuous map ... there exist a

Hypothesis 1. Theorem 2 is true.

Definition 2. A group is called *abelian* if

Remark 2. Note that

Example 1. Consider the set of all points ... such that

Assumption 1. Suppose the function $\xi_i(t)$ is periodic.

Condition 1. The function $f(x)$ is nonnegative

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Согласованность и управление спектром собственных значений

Ключевые слова: линейные системы с последствием, приводимость, показатели Ляпунова, ляпуновские инварианты.

УДК 517.917

Рассматривается линейная управляемая система с неполной обратной связью

$$\dot{x} = A(t)x + B(t)u, \quad y = C(t)x, \quad u = U(t)y.$$

Исследуется задача управления асимптотическим поведением замкнутой системы

$$\dot{x} = (A(t) + B(t)U(t)C(t))x, \quad x \in \mathbb{R}^n. \quad (1)$$

Для такой системы вводится понятие согласованности. Это понятие является обобщением понятия полной управляемости на системы с неполной обратной связью. Исследовано свойство согласованности системы (1), получены новые необходимые условия и достаточные условия согласованности системы (1), в том числе в стационарном случае.

Для стационарной системы вида (1) исследуется задача о глобальном управлении спектром собственных значений, которая заключается в приведении характеристического многочлена матрицы стационарной системы (1) с помощью стационарного управления U к произвольному наперед заданному полиному. С помощью методов линейной алгебры получены необходимые и достаточные условия глобальной управляемости спектра в случае, когда коэффициенты системы имеют специальный вид. Установлено, что в этом случае свойство согласованности является достаточным, а при определенных предположениях и необходимым условием глобальной управляемости спектра.

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