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PRODUCTS OF SPACES AND THE CONVERGENCE OF SEQUENCES

By the Hewitt–Marczewski–Pondiczery theorem, the Tychonoff product of 2^ω separable spaces is separable. We continue to explore the problem of the existence in the Tychonoff product $\prod\limits_{\alpha\in 2^\omega} Z_\alpha$ of 2^ω separable spaces a dense countable subset, which does not contain non-trivial convergent sequences. We say that a sequence $\lambda = \{x_n : n \in \omega\}$ is simple, if, for every $x_n \in \lambda$, a set $\{n' \in \omega : x_{n'} = x_n\}$ is finite. We prove that in the product of separable spaces $\prod\limits_{\alpha\in 2^\omega} Z_\alpha$, such that Z_α $(\alpha \in 2^\omega)$ contains a simple nonconvergent sequence, there is a countable dense set $Q \subseteq \prod\limits_{\alpha\in 2^\omega} Z_\alpha$, which does not contain non-trivial convergent in $\prod\limits_{\alpha\in 2^\omega} Z_\alpha$ sequences.

Keywords: Tychonoff product, dense set, convergent sequence, independent matrix.

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Introduction

The Hewitt–Marczewski–Pondiczery theorem (see [3]) states that if $\prod\limits_{\alpha\in A} X_\alpha$ is the Tychonoff product of topological spaces, $d(X_\alpha) \leqslant \tau \geqslant \omega$ for all $\alpha \in A$ and $|A| \leqslant 2^\tau$, then $d(\prod\limits_{\alpha\in A} X_\alpha) \leqslant \tau$.

We consider the problem of the existence in the product of topological spaces a subspace without non-trivial convergent sequences. This problem has been studied extensively in recent decades. In [7], M. Hrušak, U. A. Ramos-Garcia, S. Shelah and J. van Mill constructed, in ZFC, the subgroup of 2^c without non-trivial convergent sequences. In [8], W. H. Priestley proved that the Tychonoff cube I^c contains a countable dense set without non-trivial convergent sequences. In [9], P. Simon proved the existence of such countable set in 2^c. In [5], it was proved the existence of such countable dense set in the product of c many spaces, which contain two disjoint non-empty closed sets. In [6], it was constructed two countable dense sets in I^c with properties which ensure, among other properties, that these sets contain no non-trivial convergent sequences.

We say that a sequence $\lambda = \{x_n : n \in \omega\}$ is simple if the set $\{n' \in \omega : x_n = x_{n'}\}$ is finite for every $x_n \in \lambda$. We prove (Theorem 2.1) that in the product of separable spaces $\prod\limits_{\alpha\in 2^\omega} Z_\alpha$ of 2^ω, such that Z_α $(\alpha \in 2^\omega)$ contains a simple nonconvergent sequence, there is a countable dense set $Q \subseteq \prod\limits_{\alpha\in 2^\omega} Z_\alpha$, which does not contain non-trivial convergent in $\prod\limits_{\alpha\in 2^\omega} Z_\alpha$ sequences.

§ 1. Preliminaries

Definitions and the notions used in the paper can be found in [1–3]. $I^c = \prod\limits_{\alpha\in 2^\omega} I_\alpha$ is the Tychonoff cube of the weight c; $2 = \{0, 1\}$ is the two point discrete space. $d(X)$ stands for the density of a space X; by $[A]$ we denote the closure of A; $\exp A$ denotes the set of all subsets of A; by $\operatorname{Exp} A$ we denote the set of all non-empty subsets of A.

We say that X is a countable set if $|X| = \omega$. A sequence $\{x_n\}_{n=1}^{\omega}$ is called *trivial*, if there is $n_0 \in \omega$ such that $x_n = x_{n_0}$ for all $n > n_0$. A sequence $\lambda = \{x_n\}_{n=1}^{\omega}$ is called *simple*, if for every $x_n \in \lambda$ the set $\{n' \in \omega : x_n = x_{n'}\}$ is finite.

We use the notion of an independent matrix. It was defined by J. van Mill [10] as a subfamily of an independent linked family defined by K. Kunen [11].

Definition 1.1 (see [10]). For a countable set X, an indexed family $\{A_{i,j}\colon i \in I,\ j \in J\}$ of subsets of X is called a J by I independent matrix, if:

- whenever $j_0,\ j_1 \in J$ are distinct and $i \in I$ then $|A_{ij_0} \cap A_{ij_1}| < \omega$;
- if $i_1, \dots, i_n \in I$ are distinct and $j_1, \dots, j_n \in J$ then $|\bigcap\{A_{i_k j_k}\colon k = 1, \dots, n\}| = \omega$.

The space X is decomposable, if X contains two disjoint non empty closed sets.

Let us present the constructions developed in [4]. These constructions are not widely known but quite complex. We will use them further in the proof of the main result.

Consider $(\operatorname{Exp} k)^{\exp k}$ for $k \in \omega$. Elements of this set we will denote by v, u, etc. For $k \in \omega$, denote

$$H_k = \{u \in (\operatorname{Exp} k)^{\exp k}\colon \{n\} \in u(\exp k) \ \text{ for all } \ n < k\},$$

$$H = \bigcup\{H_k\colon k \in \omega\}.$$

For $X \in \exp\omega$, $Y \in \operatorname{Exp}\omega$ and $k \in \omega$, denote

$$A_k(X, Y) = \{u \in H_k\colon u(X \cap k) = Y \cap k\},$$

$$A(X, Y) = \bigcup\{A_k(X, Y)\colon k \in \omega\}.$$

The family

$$\mathcal{M}_1 = \{A(X, Y)\colon X \in \exp\omega, Y \in \operatorname{Exp}\omega\}$$

is the independent matrix.

Lemma 1.1. *Let $u \in H_{k_0} \subseteq H$ for some $k_0 \in \omega$ and a set $F \subseteq H$ such that*

- *$|F \cap H_k| \leqslant 1$ for all $k \in \omega$;*
- *$|F \cap H_k| = \varnothing$ for all $k < k_0$.*

Then for every set $X \in \exp\omega$ there exists a set $Y \in \operatorname{Exp}\omega$ such that $u \in A(X, Y)$ and $A(X, Y) \cap F = \varnothing$.

Lemma 1.2. *Let $u, v \in H$, $u \neq v$. For every $B \subseteq \exp\omega$, $|B| < 2^\omega$, there is $X \in \exp\omega \setminus B$ and $Y \in \operatorname{Exp}\omega$ such that $u \in A(X, Y)$ and $v \notin A(X, Y)$.*

Definition 1.2. Let $\mathcal{F}$ be the family of countable sets of H such that for every $F \in \mathcal{F}$ the following holds:

1) $|F \cap H_k| \leqslant 1$ for all $k \in \omega$;

2) $|\{Y \in \operatorname{Exp}\omega\colon A(X, Y) \cap F = \varnothing| \geqslant \omega$ for every $X \in \exp\omega$.

The family $\mathcal{F}$ has the following property: $|\mathcal{F}| = c$.

Lemma 1.3. *For every countable set $E \subseteq H$ there are 2^ω many $F \in \mathcal{F}$ such that $F \subseteq E$.*

Lemma 1.4. *Let $F \in \mathcal{F}$ and $X \in \exp\omega$. Then there is a family $T^1(X, F) \subseteq \operatorname{Exp}\omega$ such that:*

- *$|T^1(X, F)| = \omega$;*

– $|A(X,Y) \cap F| < \omega$ *for all* $Y \in T^1(X,F)$*;*

– $\bigcup\{A(X,Y)\colon Y \in T^1(X,F)\} = H$.

For $F \in \mathcal{F}$ there is $T^2(X,F) \subseteq \operatorname{Exp}\omega$, $|T^2(X,F)| = \omega$ such that $A(X,Y) \cap F = \varnothing$ for all $Y \in T^2(X,F)$. Denote $T(X,F) = T^1(X,F) \cup T^2(X,F)$.

Let P be the set of all ordered pairs (u,v) of elements $u,v \in H$. By Lemma 1.2, there is a countable family

$$\mathcal{L} \subseteq \exp\omega,\ \mathcal{L} = \{X_{(u,v)}\colon (u,v) \in P\}$$

such that for every $X_{(u,v)} \in \mathcal{L}$ there is $Y_{(u,v)} \in \operatorname{Exp}\omega$ such that $u \in A(X_{(u,v)}, Y_{(u,v)})$, $v \notin A(X_{(u,v)}, Y_{(u,v)})$ and $X_{(u,v)} \neq X_{(u',v')}$ if $(u,v) \neq (u',v')$.

Denote $\mathcal{R} = \exp\omega \setminus \mathcal{L}$. Let $\mathcal{X}\colon \mathcal{F} \to \mathcal{R}$ be one-to-one mapping. For $F \in \mathcal{F}$ denote $X_F = \mathcal{X}(F)$.

Consider $(u,v) \in P$. Let $T_{(u,v)} \subseteq \operatorname{Exp}\omega$ be a countable family such that $Y_{(u,v)} \in T_{(u,v)}$ and $\bigcup\{A(X_{(u,v)}, Y)\colon Y \in T_{(u,v)}\} = H$.

Define for every $X \in \exp\omega$ the family $T_X \subseteq \operatorname{Exp}\omega$ by the following rule:

$$T_X = \begin{cases} T_{(u,v)}, & \text{for } X = X_{(u,v)} \in \mathcal{L}; \\ T(X_F, F), & \text{for } F \in \mathcal{F} \text{ and } X_F = \mathcal{X}(F). \end{cases}$$

By the similar way as in [4], by using the matrix $\mathcal{M}_1$, we define the matrix

$$\mathcal{M}_2 = \{\widetilde{A}(X,Y)\colon X \in \exp\omega,\ Y \in T_X\}$$

which satisfies the following conditions:

1) for every $(u,v) \in P$, there exist $X = X_{(u,v)} \in \mathcal{L}$ and $Y_{(u,v)} \in T_{(u,v)}$ such that $u \in \widetilde{A}(X_{(u,v)}, Y_{(u,v)})$, $v \notin \widetilde{A}(X_{(u,v)}, Y_{(u,v)})$;

2) for $F \in \mathcal{F}$ and $X = X_F = \mathcal{X}(F)$, $|\widetilde{A}(X_F, Y) \cap F| < \omega$ for all $Y \in T(X_F, F)$;

3) for $F \in \mathcal{F}$ and $X = X_F = \mathcal{X}(F)$, $|\{Y \in T(X,F)\colon \widetilde{A}(X_F, Y) \cap F \neq \varnothing\}| = \omega$;

4) $\widetilde{A}(X,Y) \cap \widetilde{A}(X,Y') = \varnothing$ if $Y, Y' \in T_X$, $Y \neq Y'$, $X \in \exp\omega$;

5) $\bigcup\{\widetilde{A}(X,Y)\colon Y \in T_X\} = H$ for all $X \in \exp\omega$;

6) if $X_1 \ldots, X_n \in \exp\omega$ are distinct and $Y_i \in T_{X_i}$ $(i = 1, \ldots, n)$ then there is $k_0 \in \omega$ such that $\big(\bigcap\{\widetilde{A}(X_i, Y_i)\colon i = 1, \ldots, n\}\big) \cap H_k \neq \varnothing$ for all $k > k_0$.

The matrix $\mathcal{M}_2$ generates a space

$$\Sigma = \prod_{X \in \exp\omega} T_X.$$

For $\xi \in \Sigma$ and $X \in \exp\omega$, denote

$$\xi_X = \pi_X(\xi) \in T_X$$

for the X projection $\pi_X\colon \Sigma \to T_X$. The ξ_X is a X-coordinate of ξ.

Lemma 1.5. *The set* Σ *satisfies the following conditions:*

1) *if* $\xi_1, \xi_2 \in \Sigma$, $\xi_1 \neq \xi_2$, *then there is* $X \in \exp\omega$ *such that*

$$\widetilde{A}(X, \pi_X(\xi_1)) \cap \widetilde{A}(X, \pi_X(\xi_2)) = \varnothing;$$

2) $\left|\bigcap\{\widetilde{A}(X, \xi_X)\colon X \in \exp\omega\}\right| \leqslant 1$ *for all* $\xi \in \Sigma$;

3) *for every* $u \in H$ *there is the only* $\xi^u \in \Sigma$ *such that* $\bigcap\{\widetilde{A}(X, \xi^u)\colon X \in \exp\omega\} = \{u\}$;

4) *if* $u_1, u_2 \in H$, $u_1 \neq u_2$, *then* $\xi^{u_1} \neq \xi^{u_2}$.

Denote by $\mu\colon H \to \Sigma$ a mapping from H into Σ defined by the rule $\mu(u) = \xi^u$ (see Lemma 1.5) for every $u \in H$.

Lemma 1.6. *The set* $\mu(H)$ *is a dense subset of the space* Σ.

§ 2. Main result

We consider the problem of the existence of a countable dense set in the product $\prod_{\alpha\in 2^\omega} Z_\alpha$ of 2^ω separable spaces, which does not contain non-trivial convergent sequences.

It has been proved in [4] that such a countable dense set exists in the product of separable T_1-spaces. In general case, the existence of such a countable dense set has been proved in the product $\prod_{\alpha\in 2^\omega} Z_\alpha$ of separable decomposable spaces Z_α $(\alpha \in 2^\omega)$.

Now we prove (Theorem 2.1) that there is a space Z, which is not T_1-space and is not a decomposable space, but $\prod_{\alpha\in 2^\omega} Z_\alpha$ $(Z_\alpha = Z$ for $\alpha \in 2^\omega)$ contains a countable dense set, which does not contain non-trivial convergent sequences.

Theorem 2.1. *Let* $\prod_{\alpha\in 2^\omega} Z_\alpha$ *be the product of separable spaces such that* Z_α $(\alpha \in 2^\omega)$ *contains a simple nonconvergent sequence. Then there is a countable dense set* $Q \subseteq \prod_{\alpha\in 2^\omega} Z_\alpha$*,which does not contain non-trivial convergent in* $\prod_{\alpha\in 2^\omega} Z_\alpha$ *sequences.*

P r o o f. Let $\prod_{X\in\exp\omega} Z_X$ be the product of separable spaces such that Z_X $(X \in \exp\omega)$ contains a simple non-convergent sequence.

In every Z_X $(X \in \exp\omega)$ there is a non-convergent sequence $q_X = \{a_n^X\colon n \in \omega\}$ such that $a_n^X \neq a_{n'}^X$ if $n \neq n'$, and a dense countable set $D_X \subseteq Z_X$ such that $|D_X \setminus q_X| = \omega$. For $\Sigma = \prod_{X\in\exp\omega} T_X$ and $\prod_{X\in\exp\omega} Z_X$, let us define the mapping from Σ onto $\prod_{X\in\exp\omega} D_X \subseteq \prod_{X\in\exp\omega} Z_X$:

$$\Psi\colon \Sigma \to \prod_{X\in\exp\omega} Z_X$$

by the following way.

Let $X \in \mathcal{L}$, i.e., $X = X_{(u,v)}$ for $(u, v) \in P$. In this case $T_X = T_{(u,v)}$.

Let $\phi_X\colon T_X \to D_X$ be a one-to-one mapping from $T_X = T_{(u,v)}$ onto D_X.

Let $X = X_F = \mathcal{X}(F)$ for some $F \in \mathcal{F}$. Consider $T_X = T(X_F, F) = T^1(X, F) \cup T^2(X, F)$.

By Lemma 1.4, we have $|T_X^1| = \omega$. Since $T_X^2 = T_X \setminus T_X^1$, we have $|T_X^2| = \omega$.

Define a one-to-one mapping $\phi_X\colon T_X \to D_X$ from T_X onto D_X such that

$$\phi_X(T_X^1) = q_X.$$

Define the mapping

$$\Psi\colon \Sigma \to \prod_{X\in\exp\omega} D_X \subseteq \prod_{X\in\exp\omega} Z_X$$

as follows: for $\xi = \{\xi_X\}_{X \in \exp\omega} \in \Sigma$ define $\Psi(\xi) = z = \{z_X\}_{X \in \exp\omega}$ such that $z_X = \phi_X(\xi_X)$ for $X \in \exp\omega$.

The mapping Ψ is one-to-one and continuous. We have $z_X = \pi_X(z) = \pi_X(\Psi(\xi)) = \phi_X(\xi_X)$ for the projection $\pi_X \colon \prod\limits_{X \in \exp\omega} Z_X \to Z_X$.

For $u \in H$, $\pi_X(\Psi(\mu(u))) = \phi_X(\xi^u_X)$.

Let us prove that, for $F \in \mathcal{F}$ and $X = X_F = \mathcal{X}(F)$,

$$\pi_X(\Psi \circ \mu(F)) = q_X.$$

By the definition of ξ_u, we have $\xi_u \in T^1_X$. Therefore $\{\xi_u \colon u \in F\} \subseteq T^1_X$.

By the property 4) of the matrix $\mathcal{M}_2$, we have

$$\{\xi_u \colon u \in F\} = T^1_X.$$

Therefore, $\phi_X(\{\xi^u_X \colon u \in F\}) = \phi_X(T^1_X) = q_X$. So we have

$$q_X = \phi_X(T^1_X) = \phi_X(\{\xi^u_X \colon u \in F\}) = \pi_X(\Psi(\mu(F))).$$

Let us prove that the set

$$Q = \Psi(\mu(H))$$

does not contain any convergent non-trivial sequence.

Suppose $\gamma = \{z_n\}_{n=1}^{\infty} \subseteq Q$ is a convergent sequence, $\lim\limits_{n\to\infty} z_n = \widetilde{z}$. Without loss of generality we can assume that $z_n \neq z_{n'}$ if $n \neq n'$.

Since Ψ is a one-to-one mapping from Σ onto $\prod\limits_{X \in \exp\omega} D_X \subseteq \prod\limits_{X \in \exp\omega} Z_X$ and $\mu \colon H \to \Sigma$ is a one-to-one mapping from H into Σ, for every $z_n \in \gamma$ there is the only $u_n \in H$ such that $\Psi(\mu(u_n)) = z_n$.

Consider the set $E = \{u_n \colon n \in \omega\}$.

By Lemma 1.3, there is $F \in \mathcal{F}$ such that $F \subseteq E$.

Let $F = \{u_{n_k} \colon k \in \omega\}$. Consider a sequence $\gamma' = \{z_{n_k} \colon k \in \omega\}$ where $z_{n_k} = \Psi(\mu(u_{n_k}))$.

The γ' is a convergent sequence too and

$$\pi_X(\gamma') = \pi_X(\Psi(\mu(F))) = q_X.$$

The projection $\pi_X \colon \prod\limits_{X \in \exp\omega} Z_X \to Z_X$ is a continuous mapping, then $\pi(\gamma') = q_X$ must be convergent, but q_X is not a convergent sequence. Contradiction.

So, the set Q does not contain a convergent non-trivial sequence.

Since Ψ is a continuous mapping, $\mu(H)$ is dense in Σ (Lemma 1.6) and $\prod\limits_{X \in \exp\omega} D_X$ is dense in $\prod\limits_{X \in \exp\omega} Z_X$, the set Q is a countable dense subset of $\prod\limits_{X \in \exp\omega} Z_X$. □

Let us present an example that illustrates Theorem 2.1.

Example 2.1. Let us consider the set $Z = \{0, 1, 2, \ldots\}$. We consider the following topology τ on Z: $\tau = \{\varnothing, Z\} \cup \{U_n \colon n \in \omega\}$, where $U_n = \{0, 1, \ldots, n\}$.

The space Z with this topology τ is not a T_1-space, but for every $n, m \in Z$ such that $n < m$, for the neighborhood $On = \{0, \ldots, n\}$, we have $m \notin On$, and for the neighborhood $Om = \{0, \ldots, m\}$ we have $n \in Om$. For every $n \in Z$ we have $[\{n\}] = \{n, n+1, \ldots\}$. Therefore the space Z does not contain disjoint closed sets and Z is not a decomposable space.

But every non-trivial sequence $q \subseteq Z$ is not a convergent sequence. In fact, let $n \in Z$. The neighborhood $On = \{0, \ldots, n\}$ of the point n is finite and therefore n is not a limit of q. So, the space Z is not a T_1-space and is not a decomposable space but satisfies the conditions of Theorem 2.1 and therefore the product $\prod\limits_{\alpha \in 2^\omega} Z_\alpha$, where $Z_\alpha = Z$ $(\alpha \in 2^\omega)$, contains a countable dense set, which does not contain non-trivial convergent sequences.

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Произведения пространств и сходимость последовательностей

Ключевые слова: тихоновское произведение, плотное множество, сходящаяся последовательность, независимая матрица.

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По теореме Хьюитта–Марчевского–Пондишери тихоновское произведение 2^ω сепарабельных пространств сепарабельно. Мы продолжаем исследовать проблему существования в тихоновском произведении $\prod\limits_{\alpha\in 2^\omega} X_\alpha$ сепарабельных пространств плотного счетного подмножества, не содержащего нетривиальных сходящихся последовательностей. Мы говорим, что последовательность $\lambda = \{x_n \colon n \in \omega\}$ является простой, если для каждого $x_n \in \lambda$ множество $\{n' \in \omega \colon x_{n'} = x_n\}$ конечно. Мы доказываем, что в произведении $\{Z_\alpha \colon \alpha \in 2^\omega\}$ сепарабельных пространств, где всякое Z_α $(\alpha \in \omega)$ содержит простую несходящуюся последовательность, есть счетное плотное множество $Q \subseteq \prod\limits_{\alpha\in 2^\omega} Z_\alpha$, которое не содержит нетривиальных сходящихся в $\prod\limits_{\alpha\in 2^\omega} Z_\alpha$ последовательностей.

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