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THE NUMERICAL SOLUTION OF A NONLOCAL BOUNDARY VALUE PROBLEM FOR AN ORDINARY SECOND-ORDER DIFFERENTIAL EQUATION BY THE FINITE DIFFERENCE METHOD

In the article a numerical technique based on the finite difference method is proposed for the approximate solution of a second order nonlocal boundary value problem for ordinary differential equations. It is clear that a bridge designed with two support points at each end point leads to a standard two-point local boundary value condition, and a bridge contrived with multi-point supports corresponds to a multi-point boundary value condition. At the same time if non-local boundary conditions can be set up near each endpoint of a multi-point support bridge, a two-point nonlocal boundary condition arises. The computational results for the nonlinear model problem are presented to validate the proposed idea. The effect of parameters variation on the convergence of the proposed method is analyzed.

Keywords: second-order boundary value problem, finite difference method, integral boundary conditions, parameters and convergence.

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Introduction

A second-order differential equation which models different physical situations arises in applied mathematics and physics subject to various boundary conditions. The standard boundary conditions that are in general imposed on two points are Dirichlet, Neumann and Robin type. But several phenomena in applied mathematics, and physics that modeled by the differential equation cannot be described by standard boundary conditions. For example, consider a bridge which is simply supported at many points other than end points. The modelling of this example will yield second-order multi point boundary value problem [1]. In modelling of thermal conduction phenomena, we get a second-order differential equation with integral boundary conditions [2–4].

In this article we consider the following second-order differential equation

$$u''(x) = f(x, u), \quad a < x < b, \tag{0.1}$$

subject to the nonlocal boundary conditions

$$u(a) = \int_{x_1}^{x_{N-2}} g(x)u(x)\,dx + \lambda_1, \;\; u(b) = \int_{x_1}^{x_{N-2}} r(x)u(x)\,dx + \lambda_2, \;\; a < x_1 < x < x_{N-2} < b.$$

where λ_1, λ_2 are parameters and $f(x, u)$ is real and uniformly continuous with respect to (x, u). We will define the nodal points x_1 and x_{N-2} in the domain (a, b) in next section.

Recently, many researchers have shown interest in the area of boundary value problems with nonlocal boundary conditions. For the solution and positive solutions of the nonlocal problems similar to considered boundary value problems with integral boundary conditions (0.1) in ordinary differential equations, we refer the reader to [5–8] and references therein. But a particular case of boundary value problem with integral boundary condition/s is a multi point boundary value problem. So multi point boundary value problem is also an interesting and important class of problems and the theoretical concepts of existence / nonexistence and uniqueness / multiplicity solution of these problems can be found in [9–12] and references therein.

In this article, we have assumed the existence and uniqueness of the solution of the problem (0.1). To the best of our knowledge, there is a rare work on the numerical solution of the problem (0.1) reported in the literature. However, we found a numerical method based on the shooting method for approximate solutions of nonlocal boundary value problems similar to the problem (0.1) in [1]. Inspired by the shooting method, in this article we consider the finite difference method for the numerical solution of the considered boundary value problems (0.1). Also, we apply numerical computation methods to understand and study the effect of changes in parameters in the approximate solution.

We have presented our work in this article as follows. In the next section we proposed a finite difference method. We have discussed derivation and convergence of the proposed method under the appropriate condition in Section 2 and Section 3 respectively. In Section 4, we give the application of the proposed method on the model problems and numerical results so produced to show the efficiency. Discussion and conclusion on the performance of the proposed method are presented in Section 5.

§ 1. The Difference Method

Let us define the set of nodal points $\Omega = \{x_0, x_1, \ldots, x_N\}$ such that $x_i = a + i \cdot h$, $i = 0, 1, \ldots, N$, in the interval $[a, b]$. The term h in the definition of nodal points is known as uniform step size. We wish to determine the numerical solution of the problem at these nodal points. We denote the numerical approximation of $u(x)$ at the node $x = x_i$ as u_i and the numerical approximation of the forcing function $f(x, u(x))$ at the node $x = x_i$ as f_i, $i = 0, 1, 2, \ldots, N$. Thus, we write problem (0.1) at these nodal points $x = x_i$ and the continuous problem transformed into a discrete problem by the application of the finite difference. Thus,

$$u_i'' = f_i, \quad a \le x_i \le b \quad \text{and} \quad i = 0, 1, \ldots, N, \tag{1.1}$$

and the boundary conditions are

$$u_0 = \int_{x_1}^{x_{N-2}} g(x)u(x)\,dx + \lambda_1 \quad \text{and} \quad u_N = \int_{x_1}^{x_{N-2}} r(x)u(x)\,dx + \lambda_2.$$

Following the idea in [13], let us approximate the integrals that appeared in the boundary conditions of the problem (1.1) by the composite / repeated trapezoidal quadrature method [15] which will yield the following:

$$u_0 = \int_{x_1}^{x_{N-2}} g(t)u(t)\,dt + \lambda_1 = \sum_{j=1}^{N-2} [g_j \xi_j u_j + E_{t_j}] + \lambda_1 \tag{1.2}$$

and

$$u_N = \int_{x_1}^{x_{N-2}} r(t)u(t)\,dt + \lambda_2 = \sum_{j=1}^{N-2} [r_j \xi_j u_j + \overline{E}_{t_j}] + \lambda_2, \tag{1.3}$$

where $x_1 = t_1 < t_2 < \ldots < t_M = x_{N-2}$, $j = 1, 2, \ldots, M$, using uniform step length h such that $t_j = x_1 + (j-1)h$, $j = 1, 2, \ldots, M$, $N = M + 2$, and E_{t_j}, $\overline{E}_{t_j}$ are the truncation error in the j^{th} interval. The numerical coefficients ξ_j, $j = 1, 2, \ldots, M$ are quadrature nodes and these coefficients are

$$\xi_j = \begin{cases} h/2 & \text{if } j = 1, M \\ h & \text{otherwise } j = 2, \ldots, M-1\,. \end{cases}$$

Truncating the error terms in (1.2) and (1.3), we propose the following finite difference method for the numerical solution of the problem (1.1) at nodes x_i,

$$\begin{aligned} \sum_{j=1}^{M} g_j\xi_j u_j - 2u_i + u_{i+1} &= -\lambda_1 + \frac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), \quad i = 1, \\ u_{i-1} - 2u_i + u_{i+1} &= \frac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), \quad 2 \le i \le N-2 \\ u_{i-1} - 2u_i + \sum_{j=1}^{M} r_j\xi_j u_j &= -\lambda_2 + \frac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), \quad i = N-1. \end{aligned} \tag{1.4}$$

However the quadrature nodes and truncation errors depend on the number of nodal points in $[a, b]$ and for a large number of nodal points reduces truncation errors E_{t_j} and $\overline{E}_{t_j}$ considerably. Thus we have obtained a system of equations in the variable u_i, $i = 1, 2, \ldots, N-1$. The solution of a system of equations is the approximate solution of the problem considered. We solved the system of equations (1.4) using an appropriate iterative method.

§ 2. Development of the Finite Difference Method

In this section we will discuss the development of the proposed finite difference method (1.4). Let us consider the following second-order differential equation:

$$u''(x) = f(x, u).$$

We consider the following linear combination of the solution $u(x)$ of the problem and forcing function $f(x, u(x))$ at nodes x_i,

$$a_0u_{i-1} + a_1u_i + a_2u_{i+1} + h^2(b_0f_{i-1} + b_1f_i + b_2f_{i+1}) = 0, \qquad i = 1, \tag{2.1}$$

where a_0, a_1, a_2, b_0, b_1, and b_2 are constant to be determined under appropriate conditions. Let us write each term of (2.1) in a Taylor series about point x_i. In Taylor series we apply $u_i'' = f_i$ and compare the coefficients of h^p, $p = 0, 1, \ldots, 5$. Thus, we have the following system of equations:

$$\begin{aligned} a_0 + a_1 + a_2 &= 0, \\ a_0 - a_2 &= 0, \\ a_0 + a_2 + 2(b_0 + b_1 + b_2) &= 0, \\ a_0 - a_2 + 6(b_0 - b_2) &= 0, \\ a_0 + a_2 + 12(b_0 + b_2) &= 0. \end{aligned} \tag{2.2}$$

Solving the system of equations (2.2), we have

$$(a_0, a_1, a_2, b_0, b_1, b_2) = \left(1, -2, 1, \frac{-1}{12}, \frac{-10}{12}, \frac{-1}{12}\right).$$

Substituting these constants in (2.1), we have

$$u_{i-1} - 2u_i + u_{i+1} = \frac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), \qquad i = 1. \tag{2.3}$$

Which is in fact the fourth-order Numerov method for the solution of two point second-order BVPs in ODEs. Using the nonlocal boundary condition (1.2) in (2.3), we have

$$\sum_{j=1}^{M} g_j\xi_j u_j - 2u_i + u_{i+1} = -\lambda_1 + \frac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), \quad i = 1.$$

In a similar manner, we may obtain a difference approximation for the boundary condition (1.3) in the form

$$u_{i-1} - 2u_i + \sum_{j=1}^{M} r_j \xi_j u_j = -\lambda_2 + \frac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), \quad i = N-1.$$

§ 3. Convergence analysis

For the discussion of the convergence of the proposed finite difference method (1.4), let us consider the following linear test problem:

$$-u''(x) + f(x) = 0, \qquad a < x < b,$$

subject to the boundary conditions $u_0 = \int_{x_1}^{x_{N-2}} g(x)u(x)\,dx + \lambda_1$, $u_N = \int_{x_1}^{x_{N-2}} r(x)u(x)\,dx + \lambda_2$.

The exact solution $\mathbf{U}$ of the (1.4) satisfies the matrix equation

$$\mathbf{JU} + \mathbf{F} + \mathbf{T} = \mathbf{0}, \tag{3.1}$$

where the matrix $\mathbf{T}$ is truncation error terms in the method and

$$\mathbf{J} = \begin{pmatrix} 2 - g_1\xi_1 & -1 - g_2\xi_2 & & & -g_{N-2}\xi_{N-2} & 0 \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & \ddots & & \ddots & \\ & & & -1 & 2 & -1 \\ -r_1\xi_1 & -r_2\xi_2 & & & -1 - r_{N-2}\xi_{N-2} & 2 \end{pmatrix}_{(N-1)\times(N-1)},$$

$\mathbf{U} = (U_i)_{(N-1)\times 1}$, $\quad \mathbf{F} = (F_i)_{(N-1)\times 1}$, $\quad$ where

$$F_i = \begin{cases} \lambda_1 + \dfrac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), & i = 1, \\ \dfrac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), & 1 < i < N-1, \\ \lambda_2 + \dfrac{h^2}{12}(f_{i-1} + 10f_i + f_{i+1}), & i = N-1, \end{cases}$$

and $\mathbf{T} = (T_i)_{(N-1)\times 1}$, where

$$T_i = \frac{h^6}{240} u_i^{(6)}, \quad i = 1, \ldots, N-1.$$

Let matrix $\mathbf{u} = (u_i)_{(N-1)\times 1}$ be the approximate solution of the (1.4) satisfying the matrix equation

$$\mathbf{Ju} + \mathbf{F} = \mathbf{0}. \tag{3.2}$$

Subtracting (3.1) from (3.2) and defining an error $\epsilon_i = u_i - U(x_i)$ at each node x_i, $i = 1, 2, \ldots, N-1$, we have

$$\mathbf{J}\boldsymbol{\epsilon} = \mathbf{T}, \tag{3.3}$$

where matrix $\boldsymbol{\epsilon} = (\epsilon_i)_{(N-1)\times 1}$.

Let

$$\mathbf{J} = \mathbf{A} - \mathbf{B},$$

where

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & & & 0 \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ 0 & & & -1 & 2 \end{pmatrix}_{(N-1)\times(N-1)},$$

$$\mathbf{B} = h \begin{pmatrix} \frac{1}{2}g_1 & g_2 & & \frac{1}{2}g_{N-2} & 0 \\ & & 0 & & \\ \frac{1}{2}r_1 & r_2 & & \frac{1}{2}r_{N-2} & 0 \end{pmatrix}_{(N-1)\times(N-1)}.$$

Let us assume that

$$M_{gr} = \left| \max_{x\in[a,b]} \{g(x), r(x)\} \right|, \qquad M = \max_{x\in[a,b]} \left| u^{(6)}(x) \right|,$$

and

$$\left(\frac{b-a}{h} - 3 \right) \frac{(b-a)^2 M_{gr}}{8h} < 1.$$

Then matrix $\mathbf{J}$ is invertible [14] and hence from (3.3) we have,

$$\|\boldsymbol{\epsilon}\| \leq \frac{1}{1 - \|\mathbf{A}^{-1}\|\|\mathbf{B}\|} \|\mathbf{A}^{-1}\|\|\mathbf{T}\|. \tag{3.4}$$

Estimating $\|\mathbf{A}^{-1}\|$ in (3.4) by [15] and simplifying, we obtain

$$\|\boldsymbol{\epsilon}\| \leq \frac{h^4(b-a)^2}{1920} \left(1 + \frac{(b-a)^3 M_{gr}}{8h^2} \right) M. \tag{3.5}$$

It follows from (3.5) that the error in the proposed finite difference method is bounded and $\|\boldsymbol{\epsilon}\| \to 0$ as $h \to 0$. Thus, we have established the convergence of the proposed method (1.4) and the order of convergence of the proposed method (1.4) is at least $O(h^2)$.

§ 4. Numerical results

To verify the theoretical development and computational efficiency of the proposed method, we have considered three linear and nonlinear model problems. We have presented numerical results in Tables. In each tabulated numerical results, we have shown χ^* and $_*\chi$ respectively for the maximum and minimum absolute error in the approximate solution $u(x)$ of the problems (0.1) for different values of N and uniform step size h. We have used the following formulas in computation of χ^* and $_*\chi$:

$$\chi^* = \max_{1\leq i\leq N} |U(x_i) - u_i|, \quad {}_*\chi = \min_{1\leq i\leq N} |U(x_i) - u_i|,$$

where $U(x_i)$ and u_i are respectively exact and computed value of the solution $u(x)$. In the tables we have used the computer notation, i.e., $.19714534e - 2$ for $.19714534 \times 10^{-2}$. For the solution of the system of equations (1.4), we have used Gauss Seidel and Newton–Raphson methods respectively for linear and nonlinear systems of equations. All computations were performed on a Windows 2007 Home Basic operating system in the GNU FORTRAN environment version 99 compiler (2.95 of gcc) on Intel Core i3-2330M, 2.20 Ghz PC. The solutions are computed on N nodes and iteration is continued until either the maximum difference between two successive iterates is less than 10^{-10} or the number of iterations reached 10^4.

Problem 1. The nonlinear model problem in [1] with different boundary conditions is given by

$$u''(x) = (x - 0.5^x)u^2(x) + f(x), \quad 0 < x < 1,$$

subject to the boundary conditions

$$u(0) = \int_{x_1}^{x_{N-2}} g(x)u(x)\,dx + \lambda_1 \quad \text{and} \quad u(1) = \int_{x_1}^{x_{N-2}} r(x)u(x)\,dx + \lambda_2,$$

where $g(x) = \dfrac{x}{1.33 - x}$, $r(x) = \dfrac{x^2 - 1}{2 - \sqrt{x}}$ and $f(x)$ is calculated so that the analytical solution of the problem is $u(x) = (x - 1)\sin(x)$. In the boundary conditions, λ_1 and λ_2 are parameters. The MAE and MIE computed by method (1.4) for different values of N, λ_1 and λ_2 are presented in Table 1.

Problem 2. The nonlinear model problem in [1] with different boundary conditions is given by

$$u''(x) = (x^3 + x + 1)u^2(x) + f(x), \quad 0 < x < 1,$$

subject to the boundary conditions

$$u(0) = \int_{x_1}^{x_{N-2}} g(x)u(x)\,dx + \lambda_1 \quad \text{and} \quad u(1) = \int_{x_1}^{x_{N-2}} r(x)u(x)\,dx + \lambda_2,$$

where $g(x) = \dfrac{\exp(x - .8659)}{x - 2}$, $r(x) = \exp(-x)$, and $f(x)$ is calculated so that the analytical solution of the problem is $u(x) = x(1 - x)\exp(x)$. In the boundary conditions, λ_1 and λ_2 are parameters. The MAE and MIE computed by method (1.4) for different values of N, λ_1 and λ_2 are presented in Table 2.

Problem 3. The nonlinear model problem is given by

$$u''(x) = \exp(x)u^2(x) + f(x), \quad 0 < x < 1,$$

subject to the boundary conditions

$$u(0) = \int_{x_1}^{x_{N-2}} g(x)u(x)\,dx + \lambda_1 \quad \text{and} \quad u(1) = \int_{x_1}^{x_{N-2}} r(x)u(x)\,dx + \lambda_2,$$

where $g(x) = \dfrac{x}{2 - x}$, $r(x) = \dfrac{x - 1}{1.1 - \sqrt{x}}$ and $f(x)$ is calculated so that the analytical solution of the problem is $u(x) = x^2\exp(-x)$. In the boundary conditions, λ_1 and λ_2 are parameters. The MAE and MIE computed by method (1.4) for different values of N, λ_1 and λ_2 are presented in Table 3 and Table 4.

We have considered second-order nonlocal boundary value problems in ordinary differential equations to test the computational efficiency of the proposed finite difference method (1.4). We observed in numerical experiments for different values of N presented in tables that maximum and minimum absolute errors in the solution decrease as h decreases. Since composite / repeated

Table 1. Maximum and minimum absolute error (Problem 1)

λ_2	λ_1		N 8	16	32	64
-0.0765	.113	χ^*	.35345e-1	.12714e-1	.27907e-2	.48769e-3
		$Order$	-	1.4750	2.1878	2.5166
		$_*\chi$	.19714e-2	.30860e-3	.27254e-4	.42914e-3
	.1135	χ^*	.35829e-1	.13243e-1	.33306e-2	.59750e-3
		$Order$	-	1.4359	1.9913	2.4786
		$_*\chi$	.19026e-2	.25209e-3	.17702e-4	.50663e-6

Table 2. Maximum and minimum absolute error (Problem 2)

λ_2	λ_1		N 8	16	32	64
-0.1671	.1513	χ^*	.38335e-1	.13717e-1	.31396e-2	.14375e-2
		$Order$	-	1.4827	2.1274	1.1270
		$_*\chi$	.93517e-4	.24139e-3	.66737e-4	.22988e-3
	.1514	χ^*	.34112e-1	.13794e-1	.32165e-2	.14048e-2
		$Order$	-	1.4774	2.1005	1.1951
		$_*\chi$	.13263e-3	.20229e-3	.22898e-4	.15278e-3

Table 3. Maximum and minimum absolute error (Problem 3)

λ_2	λ_1		N 8	16	32	64
.5181	-0.0845	χ^*	.41119e-1	.23717e-1	.98905e-2	.86497e-3
		$Order$	-	.7939	1.2618	3.5153
		$_*\chi$	.11667e-2	.54356e-3	.19939e-4	.86685e-5
	-0.0847	χ^*	.41150e-1	.23905e-1	.10085e-1	.10630e-2
		$Order$	-	.7836	1.2450	3.2461
		$_*\chi$	.11032e-2	.50055e-3	.12753e-4	.42046e-5
0.5179	-0.0845	χ^*	.40993e-1	.23744e-1	.99205e-2	.89694e-3
		$Order$	-	.7878	1.2591	3.4673
		$_*\chi$	.10990e-2	.47101e-3	.55043e-4	.10555e-4

Table 4. Maximum and minimum absolute error (Problem 3)

λ_1	λ_2		N			
			8	16	32	64
-.0798	0.5146	χ^*	.38199e-1	.19783e-1	.58304e-2	.32314e-2
		$Order$	-	.9493	1.7626	.8514
		${}_*\chi$	.14733e-2	.28487e-3	.15447e-4	.20409e-4
	0.5148	χ^*	.38324e-1	.19756e-1	.58004e-2	.32635e-2
		$Order$	-	.9560	1.7681	.8297
		${}_*\chi$	.15411e-2	.35748e-3	.97209e-4	.47137e-4
-.0799	.5147	χ^*	.38277e-1	.19863e-1	.59129e-2	.31482e-2
		$Order$	-	.9464	1.7482	.9093
		${}_*\chi$	.14755e-2	.29963e-3	.48006e-4	.13286e-4

trapezoidal quadrature method has better computational performance for the larger no. of subintervals N, i.e., E_{t_j} and $\overline{E}_{t_j}$ decreases as N increases. It is evident from the tabulated results that method (1.4) is convergent. Since we discretized boundary conditions using composite / repeated quadrature method, this may be the possible reason for variation in order of the convergence of the proposed method for the finer mesh.

§ 5. Conclusion

A second-order boundary value problem with nonlocal boundary conditions in ODEs was considered for the numerical solution in this article. A finite difference method has been developed and discussed for the problem considered. The proposed finite difference method is a system of equations in solution $u(x)$ at discrete nodes x_i, $i = 1, \ldots, N-1$. The numerical results those we obtained by the application of the proposed method (1.4) approve the theoretical development of the proposed method. Improvement in proposed finite difference method is possible. Work in this direction is in progress.

REFERENCES

1. Zou Y., Hu Q., Zhang R. On numerical studies of multi-point boundary value problem and its fold bifurcation, *Applied Mathematics and Computation*, 2007, vol. 185, issue 1, pp. 527–537. https://doi.org/10.1016/j.amc.2006.07.064
2. Cannon J.R. The solution of the heat equation subject to the specification of energy, *Quarterly of Applied Mathematics*, 1963, vol. 21, no. 2, pp. 155–160. https://www.jstor.org/stable/43635292
3. Ionkin N.I. The solution of a certain boundary value problem of the theory of heat conduction with a nonclassical boundary condition, *Differentsial'nye Uravneniya*, 1977, vol. 13, no. 2, pp. 294–304 (in Russian). http://mi.mathnet.ru/eng/de2993
4. Chegis R.Y. Numerical solution of the heat conduction problem with an integral condition, *Litovskii Matematicheskii Sbornik*, 1984, vol. 24, no. 4, pp. 209–215 (in Russian).
5. Khan R.A. The generalized method of quasilinearization and nonlinear boundary value problems with integral boundary conditions, *Electronic Journal of Qualitative Theory of Differential Equations*, 2003, no. 19, pp. 1–15. https://doi.org/10.14232/ejqtde.2003.1.19

6. Feng M., Ji D., Ge W. Positive solutions for a class of boundary value problem with integral boundary conditions in Banach spaces, *Journal of Computational and Applied Mathematics*, 2008, vol. 222, issue 2, pp. 351–363. https://doi.org/10.1016/j.cam.2007.11.003
7. Boucherif A. Second-order boundary value problems with integral boundary conditions, *Nonlinear Analysis: Theory, Methods & Applications*, 2009, vol. 70, issue 1, pp. 364–371. https://doi.org/10.1016/j.na.2007.12.007
8. Liu L., Hao X., Wu Y. Positive solutions for singular second order differential equations with integral boundary conditions, *Mathematical and Computer Modelling*, 2013, vol. 57, issues 3–4, pp. 836–847. https://doi.org/10.1016/j.mcm.2012.09.012
9. Gupta C.P. A generalized multi-point boundary value problem for second order ordinary differential equations, *Applied Mathematics and Computation*, 1998, vol. 89, issues 1–3, pp. 133–146. https://doi.org/10.1016/S0096-3003(97)81653-0
10. Guo Y., Shan W., Ge W. Positive solutions for second-order m-point boundary value problems, *Journal of Computational and Applied Mathematics*, 2003, vol. 151, issue 2, pp. 415–424. https://doi.org/10.1016/S0377-0427(02)00739-2
11. Kong L., Kong Q. Multi-point boundary value problems of second-order differential equations (I), *Nonlinear Analysis: Theory, Methods & Applications*, 2004, vol. 58, issues 7–8, pp. 909–931. https://doi.org/10.1016/j.na.2004.03.033
12. Ma R. Positive solutions for nonhomogeneous m-point boundary value problems, *Computers & Mathematics with Applications*, 2004, vol. 47, issues 4–5, pp. 689–698. https://doi.org/10.1016/S0898-1221(04)90056-9
13. Pandey P.K. A finite difference method for the numerical solving general third order boundary-value problem with an internal boundary condition, *Russian Mathematics*, 2017, vol. 61, issue 12, pp. 29–38. https://doi.org/10.3103/S1066369X17120040
14. Froberg C.E. *Introduction to numerical analysis, 2nd ed.*, New York: Addison-Wesley, 1969.
15. Jain M.K., Iyenger S.R.K., Jain R.K. *Numerical methods for scientific and engineering computation (2/e)*, New Delhi: Wiley Eastern Limited, 1987.

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Численное решение нелокальной краевой задачи для обыкновенного дифференциального уравнения второго порядка методом конечных разностей

Ключевые слова: краевая задача второго порядка, метод конечных разностей, интегральные граничные условия, параметры и сходимость.

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В статье предложена численная методика, основанная на методе конечных разностей, для приближенного решения нелокальной краевой задачи второго порядка для обыкновенных дифференциальных уравнений. Ясно, что мост, построенный с двумя опорными точками в каждой конечной точке, приводит к стандартному двухточечному локальному граничному условию, а мост, созданный с помощью многоточечных опор, соответствует многоточечному граничному условию. В то же время, если нелокальные граничные условия могут быть установлены вблизи каждой конечной точки многоточечного опорного моста, возникает двухточечное нелокальное граничное условие. Результаты расчетов для нелинейной модельной задачи представлены для проверки предложенной идеи. Проанализировано влияние изменения параметров на сходимость предложенного метода.

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