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On Universality of Critical Behavior in the Focusing Nonlinear Schrödinger Equation, Elliptic Umbilic Catastrophe and the *Tritronquée* Solution to the Painlevé-I Equation

B. Dubrovin · T. Grava · C. Klein

Abstract

We argue that the critical behavior near the point of "gradient catastrophe" of the solution to the Cauchy problem for the focusing nonlinear Schrödinger equation $i\epsilon\Psi_t + \frac{\epsilon^2}{2}\Psi_{xx} + |\Psi|^2\Psi = 0$, $\epsilon \ll 1$, with analytic initial data of the form $\Psi(x, 0; \epsilon) = A(x)e^{\frac{i}{\epsilon}S(x)}$ is approximately described by a particular solution to the Painlevé-I equation.

B. Dubrovin (✉) · T. Grava
SISSA, Trieste, Italy
e-mail: dubrovin@sissa.it

C. Klein
Max-Planck Institute, Leipzig, Germany

Present address:
C. Klein
Institut de Mathématiques de Bourgogne, Université de Bourgogne, 9 avenue Alain Savary, BP 47970, 21078 Dijon Cedex, France