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NUMERICAL METHODS AND THE BASIS FOR THEIR APPLICATION

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Noise removal from images using the proposed three-term conjugate gradient algorithm

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Conjugate gradient algorithms represent an important class of unconstrained optimization algorithms with strong local and global convergence properties and simple memory requirements. These algorithms have advantages that place them between the steep regression method and Newton's algorithm because they require calculating the first derivatives only and do not require calculating and storing the second derivatives that Newton's algorithm needs. They are also faster than the steep descent algorithm, meaning that they have overcome the slow convergence of this algorithm, and it does not need to calculate the Hessian matrix or any of its approximations, so it is widely used in optimization applications. This study proposes a novel method for image restoration by fusing the convex combination method with the hybrid (CG) method to create a hybrid three-term (CG) algorithm. Combining the features of both the Fletcher and Revees (FR) conjugate parameter and the hybrid Fletcher and Revees (FR), we get the search direction conjugate parameter. The search direction is the result of concatenating the gradient direction, the previous search direction, and the gradient from the previous iteration. We have shown that the new algorithm possesses the properties of global convergence and descent when using an inexact search line, relying on the standard Wolfe conditions, and using some assumptions. To guarantee the effectiveness of the suggested algorithm and processing image restoration problems. The numerical results of the new algorithm show high efficiency and accuracy in image restoration and speed of convergence when used in image restoration problems compared to Fletcher and Revees (FR) and three-term Fletcher and Revees (TTFR).

Keywords: nonsmooth, restoration, globally, descent, numerical, optimization

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Удаление шума из изображений с использованием предлагаемого алгоритма трехчленного сопряженного градиента

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Алгоритмы сопряженных градиентов представляют собой важный класс алгоритмов безусловной оптимизации с хорошей локальной и глобальной сходимостью и скромными требованиями к памяти. Они занимают промежуточное место между методом наискорейшего спуска и методом Ньютона, поскольку требуют вычисления и хранения только первых производных и как правило быстрее методов наискорейшего спуска. В данном исследовании рассмотрен новый подход в задаче восстановления изображений. Он наследует одновременно методу сопряженных градиентов Флетчера – Ривза (FR) и трехкомпонентному методу сопряженных градиентов (TTCG), и поэтому назван авторами гибридным трехкомпонентным методом сопряженных градиентов (HYCGM). Новое направление спуска в нем учитывает текущее направления градиента, предыдущее направления спуска и градиент из предыдущей итерации. Показано, что новый алгоритм обладает свойствами глобальной сходимости и монотонности при использовании неточного линейного поиска типа Вулфа при некоторых стандартных предположениях. Для подтверждения эффективности предложенного алгоритма приводятся результаты численных экспериментов предложенного метода в сравнении с классическим методом Флетчера – Ривза (FR) и трехкомпонентным методом Флетчера – Ривза (TTFR).

Ключевые слова: негладкий, восстановление, глобально, спуск, числовой, оптимизация

1. Introduction

Optimization problems are notoriously difficult because of their nonsmooth nature. Restoring images that were damaged by background noise while being sent or recorded is one such issue. Most gradient-based methods are not suitable for solving these problems directly because of their structure. Recent developments in gradient-based techniques have made it possible to employ more effective and trustworthy noise suppression processes, leading to more precise outcomes. Impulse noise has been proposed as a classical noise model by a number of academics. How well different gradient-based techniques perform when used to image restoration problems has been the subject of recent research (for examples, see [Aminifard, Babaie-Kafaki, 2022; Babaie-Kafaki, Mirhoseini, Aminifard, 2023; Khudhur, Fawze, 2023; Laylani et al., 2023]).

To do so, let's first look at the noise candidate's index set, which is as follows:

$$K = \{(i,\ j) \in W \mid \overline{\xi}_{ij} \neq \xi_{ij},\ \xi_{ij} = s_{\min} \text{ or } s_{\max}\}, \tag{1}$$

where $W = \{1, 2, \ldots, M\}\times\{1, 2, \ldots, N\}$ and is an adaptive median filter of the observed noisy picture of x corrupted by salt and pepper impulse noise, $x_{i,j}$ represents the grey level of the real image x at the pixel location $(i,\ j)$. Additionally, a noisy pixel's minimum and maximum values are denoted by $s_{\min}$ and $s_{\max}$. In light of the foregoing, we give the following definition of the picture restoration problem:

$$\min \mathcal{G}(u), \tag{2}$$

where

$$\mathcal{G}(u) = \sum_{(i,j)\in K} \left\{ \sum_{(m,n)\in V_{i,j}/K} \phi_\alpha(u_{i,j} - \xi_{m,n}) + \frac{1}{2} \sum_{(m,n)\in V_{i,j},\, K} \phi_\alpha(u_{i,j} - u_{m,n}) \right\}, \tag{3}$$

where $V_{i,j} = \{(i,\ j-1), (i,\ j+1), (i-1,\ j), (i+1,\ j)\}$ is the neighborhood of $(i,\ j)$. In the following equation, $\phi_\alpha(t) = \sqrt{t^2 + \alpha}$ with $\alpha = 1$, represents the Huber function, and it is clear that this function is responsible for the regularity of $\mathcal{G}$. The Huber function is chosen because it preserves edges.

The following unconstrained optimization problem is addressed in this work using conjugate gradient methods:

$$\min(\mathcal{G}(u)), \quad x \in \mathbb{R}^n \tag{4}$$

is frequently taken into account, when the smooth function $\mathcal{G}\colon R^n \to R$ has gradient $g(x) = \nabla\mathcal{G}(x)$ which is attainable. The optimization community paid close attention to problems in the form of (4) due to their numerous applications in a number of domains [Dai, Zhu, Zhang, 2022; Hassan, Sadiq, 2022; Laylani, Hassan, Khudhur, 2022], including picture restoration [Hassan, Alashoor, 2023; Ismail Ibrahim, Mohammed Khudhur, 2022; Jiang et al., 2023; Wang, Tian, Pang, 2023]. Iterative techniques that utilize the gradient of $\mathcal{G}$ are typically the most efficient way to deal with such problems.

Due to its nice theoretical property and low memory requirements, the Conjugate Gradient (CG) method is an example of such a technique. By utilizing $x_0 \in R^n$ as a starting point, the approach constructs the sequence $\{x_k\}$ according to the scheme

$$x_{k+1} = x_k + \alpha_k d_k, \quad k = 0, 1, 2, \ldots \tag{5}$$

Step size, often known as line step, is denoted by the scalar $\alpha_k > 0$.

The Wolfe rule [Powell, 1970] is a common method for calculating the step size. In the kth iteration method, α_k is calculated to adhere to a set of predetermined constraints [Hager, Zhang, 2006; Elhamid, Khudhur, 2024; Souli et al., 2024]. The following inequalities characterize the widely employed method known as the standard line search approach:

$$\mathcal{G}(x_k + \alpha_k d_k) \leqslant \mathcal{G}(x_k) + \delta \alpha_k g_k^T d_k \tag{6}$$

$$d_k^T g(x_k + \alpha_k d_k) \geqslant \sigma d_k^T g_k. \tag{7}$$

Replacing (7) with the formula

$$\left| d_k^T g(x_k + \alpha_k d_k) \right| \leqslant -\sigma d_k^T g_k \tag{8}$$

provides the rule where $0 < \delta < \sigma < 1$ known as strong Wolfe condition. The spectral search direction, d_{k+1}, is used to determine this step size, which is then expressed as

$$d_{k+1} = \begin{cases} -g_{k+1} & \text{if } k = 0, \\ -g_{k+1} + \beta_k d_k & \text{if } k > 0, \end{cases} \tag{9}$$

where β_k stands for the CG updating parameters. The parameter β_k plays a significant role in developing and selecting a CG approach. Fletcher and Revees (FR) [Fletcher, 1964] and Polak, Ribière, and Polyak (PRP) [Polak, Ribière, 1969] suggest some antecedent formulations for the parameters with the following formulas:

$$\beta_k^{\mathrm{FR}} = \frac{\|g_{k+1}\|^2}{\|g_k\|^2}, \quad \beta_k^{\mathrm{PRP}} = \frac{g_{k+1}^T y_k}{\|g_k\|^2}$$

with $y_k = g_{k+1} - g_k$ and $\|\cdot\|$ stands for the Euclidean norm of vectors.

In addition to these, there are also the methods of Conjugate Descent (CD) [Fletcher, 1980], Hestenes and Steifel (HS) [Hestenes, Stiefel, 1952], Liu and Storey (LS) [Liu, Storey, 1991], Dai and Yuan (DY), and so on. Recently, three-term conjugate gradient methods were introduced by Zhang, Zhou, and Li [Zhang, Zhou, Li, 2006a; Zhang, Zhou, Li, 2006b; Zhang, Zhou, Li, 2007]. These methods always fulfill the sufficient descent criterion:

$$g_k^T d_k \leqslant -k\|g_k\|^2 \ \forall k \tag{10}$$

for some positive constant k, apart from line-of-sight considerations. They presented the modified FR approach, which is outlined in [Zhang, Zhou, Li, 2006a]:

$$d_k = -g_k + \beta^{\mathrm{FR}} d_{k-1} - \theta_{k-1}^{(1)} g_k, \tag{11}$$

where

$$\theta_{k-1}^{(1)} = \frac{d_{k-1}^T g_k}{g_{k-1}^T g_{k-1}}.$$

Since this search direction satisfies $d_k^T g_k = \|g_k\|^2$ for all k.

In addition, they put forth the modified PR approach [Zhang, Zhou, Li, 2006b] and the modified HS approach [Zhang, Zhou, Li, 2007], which are defined as

$$d_k = -g_k + \beta^{\mathrm{PRP}} d_{k-1} - \theta_{k-1}^{(1)} y_{k-1}, \tag{12}$$

$$d_k = -g_k + \beta^{\mathrm{HS}} d_{k-1} - \theta_{k-1}^{(1)} y_{k-1}, \tag{13}$$

where

$$\theta_{k-1}^{(2)} = \frac{d_k^T g_k}{g_{k-1}^T g_{k-1}},$$

and

$$\theta_k^{(3)} = \frac{d_{k-1}^T g_k}{d_{k-1}^T y_{k-1}}.$$

The global convergence features of their line searches were demonstrated. This implies the sufficient descent requirement for $k = 1$, and we notice that these procedures always meet $d_{k-1}^T g_{k-1} = -\|g_{k-1}\|^2 < 0$ for every k.

We will mention here the contributions we made in this paper as follows:

1. To counteract the slowness of the FR and TTFR approaches, we propose using a hybrid CG parameter derived from convex combinations in the search direction of a three-term conjugate gradient.

2. The derivation of a new value for using the pure conjugate condition to speed up the new proposed method.

3. Improve the numerical results with the proposed new method by finding a new value that satisfies the pure conjugate criterion.

4. Under certain hypotheses, the new proposed approach would be convergent both locally and globally.

5. We use the new proposed method in image restoration to demonstrate its efficiency compared to other methods in the same field.

This paper follows the following structure: Section 1 includes a general introduction to the image restoration and noise removal algorithms. With a general introduction to numerical optimization and some classical conjugate gradient algorithms. In Section 2, a completely new three-term gradient conjugate algorithm is introduced. In Section 3, we study the descent property of the new algorithm. In Section 4, we study global convergence results using a standard Wolfe line search. In Section 5, we examine the numerical results and comparisons we used to solve the image restoration problems. In Section 6, the concluding section, we present the conclusions we reached in the paper.

2. New three-term conjugate gradient algorithm

In this research, we focus on developing effective algorithms for large-scale unconstrained optimization and picture restoration issues by making greater use of the information of the objective function at the present iteration. Taking inspiration from the research presented in [Jiang et al., 2022; Liu, Zhao, Wu, 2020; Narushima, Yabe, Ford, 2011], the convex combination method utilized in [Yuan, Li, Hu, 2020], and the hybrid CGM proposed in [Fletcher, 1964], we provide a novel family of hybrid TTCGMs with the following search strategy:

$$d_k = \begin{cases} -g_k, & \text{if } k = 0, \\ -g_k + (1-\lambda_k)\beta_k^{\text{HYCGM}} d_{k-1} + \eta_k \lambda_k \theta_k g_{k-1}, & \text{if } k \geqslant 1, \end{cases} \tag{14}$$

$$\beta_k^{\text{HYCGM}} = \max\left\{0, \min\left\{\beta_k^{\text{HA}}, \beta_k^{\text{FR}}\right\}\right\}, \quad \theta_k = \vartheta \frac{g_k^T g_{k-1}}{\|g_{k-1}\|^2}, \tag{15}$$

where $0 < \vartheta < 1$,

$$\beta^{\text{HA}} = \frac{\theta_k \|g_{k+1}\|^2 + (1-\theta_k)\|g_{k+1}\|_F^2}{g_k^T g_k},$$

where $\|\cdot\|_F^2$ is the Frobenius norm of an m-by-n matrix:

$$\|X\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|a_{ij}|^2} = \sqrt{\operatorname{trace}(X^T X)}.$$

Using the pure conjugacy condition [Dai, 2001; Li, Tang, Wei, 2007], we find a new value for λ_k:

$$-y_k^T g_{k+1} + (1-\lambda_k)\beta_k^{\text{HYCGM}} y_k^T d_k + \lambda_k\theta_k y_k^T g_{k+1} = 0,$$

$$\lambda_k = \frac{y_k^T g_{k+1} - \beta_k^{\text{HYCGM}} y_k^T d_k}{\theta_k y_k^T g_{k+1} - \beta_k^{\text{HYCGM}} y_k^T d_k}.$$

Also, from using the pure conjugacy condition [Dai, 2001; Li, Tang, Wei, 2007], we find a new value for η_k:

$$-y_k^T g_{k+1} + (1-\lambda_k)\beta_k^{\text{HYCGM}} y_k^T d_k + \eta_k\lambda_k\theta_k y_k^T g_{k+1} = 0,$$

$$\eta_k = \frac{y_k^T g_{k+1} - (1-\lambda_k)\beta_k^{\text{HYCGM}} y_k^T d_k}{\theta_k\lambda_k y_k^T g_{k+1}},$$

where β_k^{HYCGM} is any conjugate parameter and $0 < \vartheta < 1$.

In the upcoming analysis (see Lemma 1 and the theorem below), it is important to note that the descent property for the search direction defined in (14) and the global convergence of the proposed technique are unaffected by the selection of β_k^{HYCGM} and line searches. More theoretical and practical leeway will be possible as a result of these truths.

Using equation (15) and the weak Wolfe line search (6), and (7), we now officially present the method's steps (HYCGM).

Algorithm HYCGM

Step 0 (initialization). Given an initial point $x_1 \in R^n$, parameters $0 < \vartheta < 1$, $0 < \delta < \sigma < 1$ and $\varepsilon > 0$. Set $d_1 = -g_1$ and $k = 1$.

Step 1. If $\|g_k\| \leqslant \varepsilon$, stop.

Step 2. Find a step size α_k using the weak Wolfe line search (equations (6) and (7)).

Step 3. Generate the next iteration by $x_{k+1} = x_k + \alpha_k d_k$.

Step 4. Choose an appropriate conjugate parameter β_k for β_k^{HYCGM} in the recurrence relation (15), and then compute d_k by (14).

Step 5. Start with Step 1 and $k = k + 1$.

3. Descent property

Here, we first examine how HYCGM's search direction generates a descent property. Then, we move on to demonstrating its universal convergence.

An estimation of $(1-\lambda_k)\beta_k^{\text{HYCGM}}$ is provided, which is necessary for the convergence analysis that follows, and the following lemma demonstrates that d_{k+1} described in (14) is a descent direction.

Lemma 1. *Let* $\{d_k\}$ *be a HYCGM-generated sequence*; *then the following holds*:

$$g_k^T d_{k+1} < 0, \quad k \geqslant 1, \tag{16}$$

which means that the descending search direction is what HYCGM returns. Furthermore, for all $k \geqslant 2$, *we get*

$$0 \leqslant (1-\lambda_k)\beta_k^{\text{HYCGM}} \leqslant \frac{g_k^T d_k}{g_{k-1}^T d_{k-1}}. \tag{17}$$

Proof. This is similar to Jiang proof [Jiang et al., 2022].

4. Global convergence

The following assumptions for the objective function are necessary for analyzing the HYCGM's global convergence property.

A1. $\Lambda = \left\{x \in R^n \mid \mathcal{G}(x) \leqslant \mathcal{G}(x_1)\right\}$ is a bounded level set.

A2. Since $\mathcal{G}(x)$ is differentiable and $g(x)$ is Lipschitz continuous in some region U of, there exists a constant $L > 0$ such that

$$\|g(x) - g(y)\| \leqslant L\|x - y\|, \ \forall x, \ y \in U$$

holds true in this region. Based on the results of the weak Wolfe line search (equations (6) and (7)), we may conclude that the sequence $\{\mathcal{G}(x_k)\}$ is monotonically decreasing. When we add this to assumption A1, we find that $\{x_k\}$ is a finite sequence. This famous Zoutendijk condition is proved by the following lemma. For further information on its significance in the CGM's convergence analysis, see [Zoutendijk, 1970].

Lemma 2. *In this case, we can iterate in the manner* (2)*, where* d_{k+1} *meets the descent criterion* $g_k^T d_{k+1} < 0$ *and* k *satisfies the weak Wolfe line search* (6) *and* (7)*. Assuming* A1 *and* A2 *hold, we get*

$$\sum_{k=1}^{\infty} \frac{\left(g_k^T d_k\right)^2}{\|d_k\|^2} < +\infty.$$

Lemmas 1 and 2 allow us to provide a convergence analysis for HYCGM.

Theorem. *Let* $\{x_k\}$ *denote a sequence produced by HYCGM. Assumptions* A1 *and* A2 *imply that* $\liminf_{k\to\infty} \|g_k\| = 0$.

Proof. This is similar to Jiang's proof [Jiang et al., 2022].

5. Numerical results and compares

Here, we show how well the proposed HYCGM works in restoring the original gray level images (x) of the Camera Man, Lena, Baboon, Barbara, Brain, Man(512), Peppers (256 256), Zainb1, Zainb2, Hisham, and M&M that were distorted by salt-and-pepper impulse noise. For more on this, see [Aji et al., 2022; Hassan Ibrahim et al., 2022; Jiang et al., 2022; Khudhur, Hassan, Aji, 2024; Khudhur, Mohammed, 2024; Souli et al., 2024]. The values of $\delta = 0.01$ and $\sigma = 0.1$ were taken, respectively, based on the strategy in [Sellami, Laskri, Benzine, 2015]. We evaluate the proposed HYCGM against the FR and TTFR approaches using three metrics: the number of iterations (NOI), the amount of processing time (CPUtime), and the maximum signal-to-noise ratio (PSNR). An Intel Core i7 machine with 16 GB of RAM was used to run MATLAB and carry out all of the procedures [Jiang et al., 2022; Khudhur, Hassan, Aji, 2024]. For more information about the source code and images, see the SI file. Images are restored to 30, 50, 70, and 90 percent of their original quality depending on the amount of noise present. The number of iterations Iterations (NOI), central processing unit time (CPUtime), and peak signal-to-noise ratio (PSNR) are three metrics used in this study, and the results shown in Tables 1 show that the suggested method outperformed the other methods investigated. Figures 1–6 further demonstrate that the proposed method outperformed the alternatives in terms of removing noise from the flawed Lena, Brain, Camera man, Peppers, Zainb1, and Hisham images. The outcomes lead us to believe that the suggested HYCGM is useful and appropriate.

Table 1. Comparisons and numerical results of the algorithms (HYCGM, FR, and TTFR)

		HYCGM algorithm			FR algorithm			TTFR algorithm		
Problems	Noise Ratio	NOI	CPU time	PSNR	NOI	CPU time	PSNR	NOI	CPU time	PSNR
lena.png	30 %	15	7.73	37.04	245	74.13	34.22	89	15.8	35.28
lena.png	50 %	21	17.32	34.44	99	54.24	26.24	166	42.77	32.01
lena.png	70 %	22	23.35	31.05	5	14.45	14.19	250	81.92	29.22
lena.png	90 %	30	31.25	26.2	130	117.88	23.43	144	69.55	25.18
brain.bmp	30 %	20	2.31	29.72	75	5.43	24.43	225	7.76	29.07
brain.bmp	50 %	21	3.14	28.15	238	26.98	26.22	204	11.09	27.30
brain.bmp	70 %	23	4.49	25.93	175	25.56	23.98	269	18.76	25.30
brain.bmp	90 %	38	6.74	22.27	5	4.17	8.58	292	24.28	22.02
cameraman.png	30 %	22	2.52	29.61	148	12.08	27.91	115	4.59	29.08
cameraman.png	50 %	21	3.31	27.35	100	10.74	24.62	183	10.22	26.63
cameraman.png	70 %	23	4.84	24.76	5	3.15	13.17	207	14.87	24.22
cameraman.png	90 %	34	6.25	21.19	106	21.01	19.88	148	13.64	20.83
peppers.bmp	30 %	16	2.13	33.13	95	7.66	28.92	104	4.15	32.18
peppers.bmp	50 %	22	4.26	30.37	204	22.39	28.05	165	9.56	29.23
peppers.bmp	70 %	24	4.42	27.08	110	20.27	23.86	221	15.97	26.33
peppers.bmp	90 %	32	6.99	22.74	119	26.92	21.18	145	14.81	22.27
zainb1.jpg	30 %	17	8.51	32.27	166	63.04	28.48	171	29.28	31.69
zainb1.jpg	50 %	22	17.32	30.09	72	43.98	22.99	184	49.95	28.94
zainb1.jpg	70 %	25	22.97	27.45	161	112.93	24.66	273	92.72	26.47
zainb1.jpg	90 %	33	32.61	24.06	196	206.5	22.35	285	129.09	23.54
Hisham.jpg	30 %	24	17.01	37.8	178	68.51	30.69	186	38.88	33.70
Hisham.jpg	50 %	21	23	35.28	169	118.55	28.09	222	73.54	31.97
Hisham.jpg	70 %	26	29.45	31.96	301	267.62	28.22	223	108.36	30.22
Hisham.jpg	90 %	33	56.06	26.7	118	140.56	22.58	181	109.10	25.19

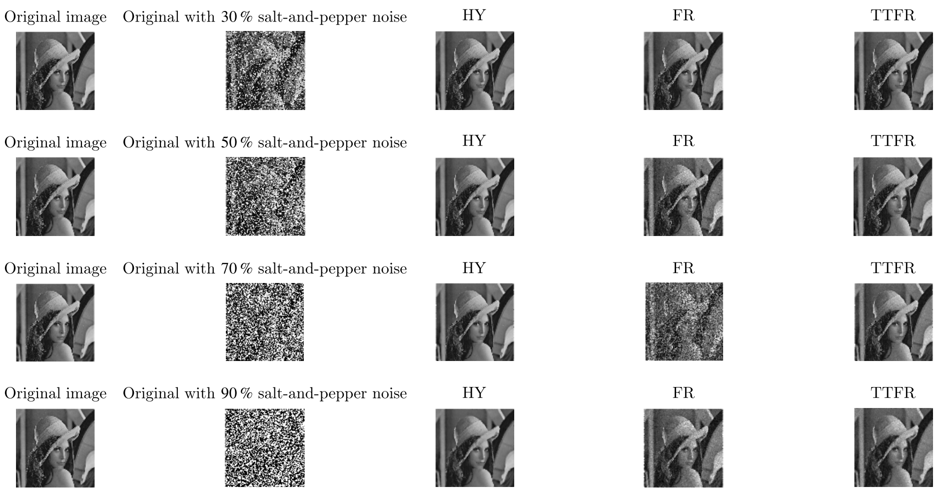

Figure 1. Comparing Lena image results of algorithms: The 1st column is the original image, the 2 column is the image after the noise, the 3, 4, and 5 columns are HYCGM, FR, and TTFR algorithms, respectively

6. Conclusions and future works

In this article, we provide a hybrid three-term conjugate gradient method (HYCGM) in which the search direction meets the descent requirement in every single instance. This is true regardless of the conjugate parameters that are selected and the lines that are searched. A minimal amount of work is required for the suggested approach to converge everywhere. We apply it to the issue of

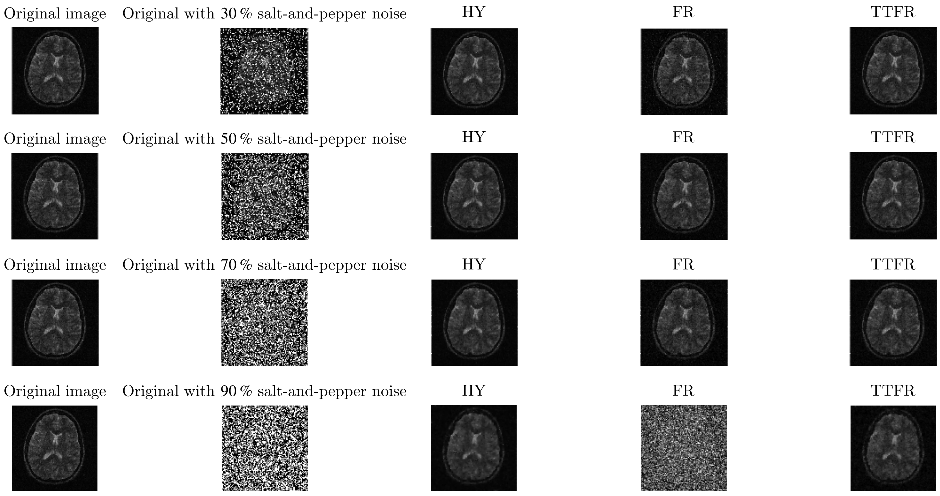

Figure 2. Comparing Brain image results of algorithms: The 1st column is the original image, the 2 column is the image after the noise, the 3, 4, and 5 columns are HYCGM, FR, and TTFR algorithms, respectively

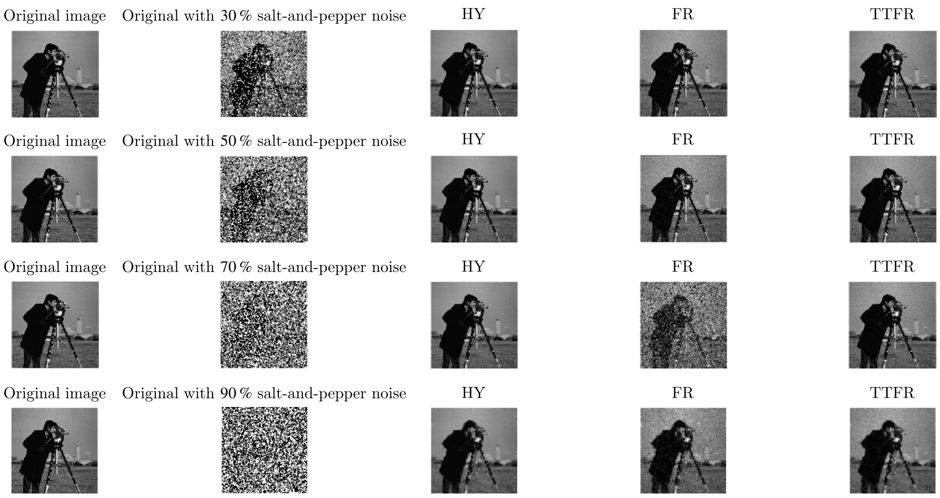

Figure 3. Comparing Cameraman image results of algorithms: The 1st column is the original image, the 2 column is the image after the noise, the 3, 4, and 5 columns are HYCGM, FR, and TTFR algorithms, respectively

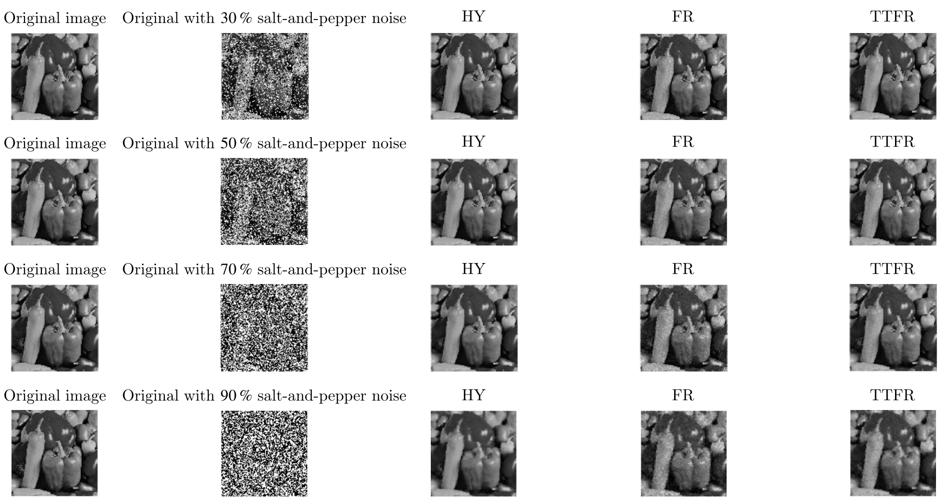

Figure 4. Comparing Peppers image results of algorithms: The 1st column is the original image, the 2 column is the image after the noise, the 3, 4, and 5 columns are HYCGM, FR, and TTFR algorithms, respectively

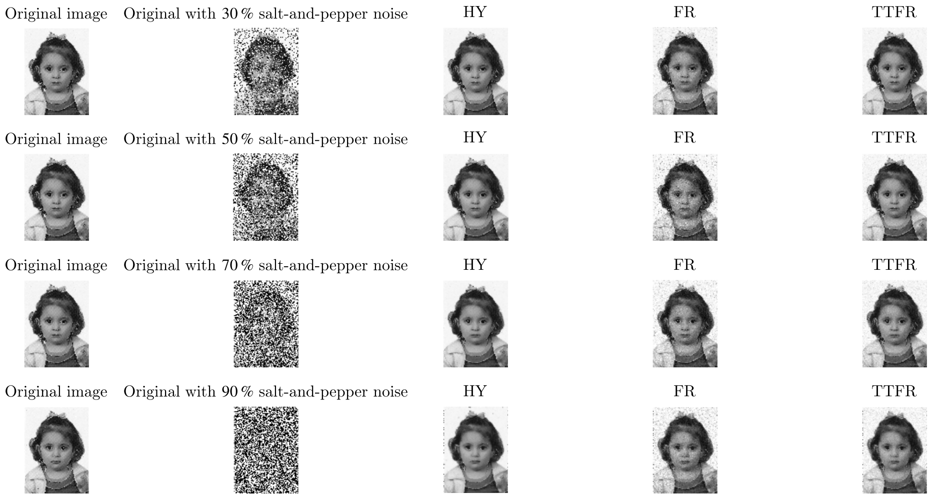

Figure 5. Comparing zainb1image results of algorithms: The 1st column is the original image, the 2 column is the image after the noise, the 3, 4, and 5 columns are HYCGM, FR, and TTFR algorithms, respectively

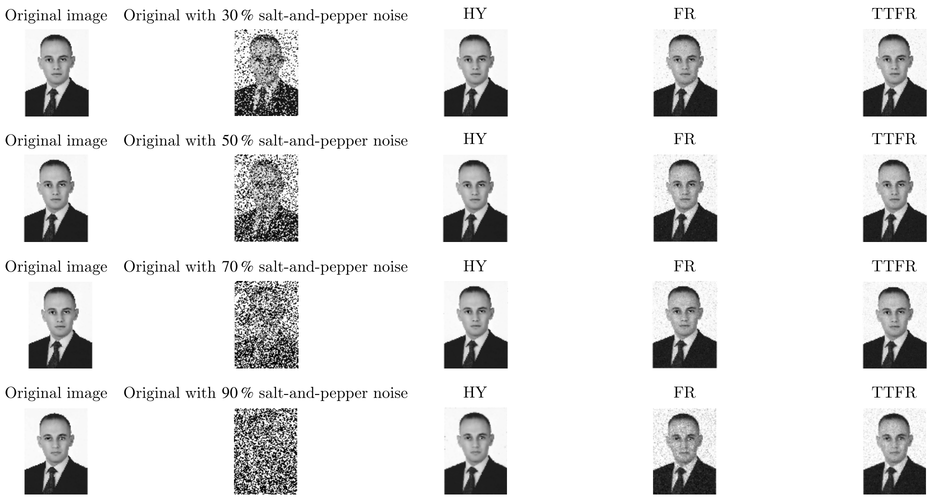

Figure 6. Comparing Hisham image results of algorithms: The 1st column is the original image, the 2 column is the image after the noise, the 3, 4, and 5 columns are HYCGM, FR, and TTFR algorithms, respectively

picture restoration, and the first numerical results show promise in terms of efficiency and applicability, even when compared to the most effective approaches that are now available. Contributions made to the study in which the slowness of the Fletcher and Revees (FR) and three-term Fletcher and Revees (TTFR) algorithms has been overcome, and the numerical results have been much improved in comparison to the Fletcher and Revees (FR) and three-term Fletcher and Revees (TTFR) algorithms, with reference to the fact that the approach satisfies the criterion of global convergence. We can also use the new algorithm in other applications (for example, artificial neural networks, fuzzy neural networks, and in the swarm intelligence algorithms, etc.) in future works.

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