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Sensitivity analysis and semi-analytical solution for analyzing the dynamics of coffee berry disease

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Coffee berry disease (CBD), resulting from the *Colletotrichum kahawae* fungal pathogen, poses a severe risk to coffee crops worldwide. Focused on coffee berries, it triggers substantial economic losses in regions relying heavily on coffee cultivation. The devastating impact extends beyond agricultural losses, affecting livelihoods and trade economies. Experimental insights into coffee berry disease provide crucial information on its pathogenesis, progression, and potential mitigation strategies for control, offering valuable knowledge to safeguard the global coffee industry. In this paper, we investigated the mathematical model of coffee berry disease, with a focus on the dynamics of the coffee plant and *Colletotrichum kahawae* pathogen populations, categorized as susceptible, exposed, infected, pathogenic, and recovered (SEIPR) individuals. To address the system of nonlinear differential equations and obtain semi-analytical solution for the coffee berry disease model, a novel analytical approach combining the Shehu transformation, Akbari – Ganji, and Pade approximation method (SAGPM) was utilized. A comparison of analytical results with numerical simulations demonstrates that the novel SAGPM is excellent efficiency and accuracy. Furthermore, the sensitivity analysis of the coffee berry disease model examines the effects of all parameters on the basic reproduction number R_0. Moreover, in order to examine the behavior of the model individuals, we varied some parameters in CBD. Through this analysis, we obtained valuable insights into the responses of the coffee berry disease model under various conditions and scenarios. This research offers valuable insights into the utilization of SAGPM and sensitivity analysis for analyzing epidemiological models, providing significant utility for researchers in the field.

Keywords: coffee berry disease (CBD), *Colletotrichum kahawae* pathogen, epidemic mathematical model, sensitivity analysis, Shehu transformation, Akbari – Ganji's method (AGM), Pade approximation method, numerical simulation

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1. Introduction

The first instance of coffee berry disease was reported in Kenya in 1926, and it has since appeared in almost every Arabica coffee producing country in the continent. *Colletotrichum kahawae* is a significant fungal disease known as coffee berry disease (CBD), which affects coffee plants. It mostly affects coffee berries and can lead to significant drops in coffee production, which will have an effect on the quantity and quality of coffee beans. The symptoms of coffee berry disease include the emergence of tiny black lesions on coffee berries. These lesions produce spores that can spread the disease to other berries, causing rotting and reducing the quality and production of coffee beans [Kebati et al., 2016; Mouen Bedimo et al., 2007; Mohammed, 2015]. Temperature and rainfall play crucial roles in facilitating the occurrence of coffee berry disease (CBD) epidemics. The quantity of raining days and maximum temperatures are highlighted as vital climatic elements influencing the CBD epidemic among Arabica coffee trees. According to [Mouen Bedimo et al., 2010], these climate factors could potentially contribute to the development of predictive models, aiding in the efficient management of CBD in regions experiencing high disease prevalence. CBD's effects were significantly evident in Kenya during the 1962/1963 and 1967/1968 crop years, resulting in an 80 % rise in coffee production losses. A study was conducted to assess the performance of five advanced breeder's lines across diverse environments and spacing, emphasizing the enhancement of coffee production in Kenya, and it confirms that the new lines exhibited superior or comparable traits to existing varieties across all measured parameters, signifying the successful achievement of breeding objectives. Furthermore, it was observed that closer spacing facilitated a more favorable expression of most parameters compared to wider spacing [Gichimu, Omondi, 2010].

Statistical modeling is an important epidemiological framework for demonstrating the spread of diseases. This method uses mathematical approaches to analyze and predict the distribution and transmission of diseases within a population. Various mathematical models have been developed to investigate the effects of protective and control measures on plant disease spread dynamics [Melese, Makinde, Obsu, 2023; Cerda et al., 2017; Melese, 2024]. Several mathematical models for coffee berry disease dynamics have been developed, which also include calculations of disease-free equilibrium points, derivation of the basic reproduction number, analysis of local and global stability of the equilibria, bifurcation analysis, and sensitivity indices [Melese, Makinde, Obsu, 2022; Muhumuza, 2018]. A recent study [Nyaberi et al., 2023] investigated the impact of coffee berry disease on coffee results and used a mathematical model to analyze disease dynamics.

Analytical techniques are important in tackling complex mathematical problems because they provide a combination of numerical analysis and analytical accuracy. These methods are particularly useful when dealing with nonlinearity or system complexity. The combination of analytical expressions and numerical computations enables an accurate and effective solution to real-world challenges across several domains. A variety of semi-analytical methods, including the differential transform method (DTM), Adomian decomposition method (ADM), Homotopy perturbation method (HPM), Laplace Adomian decomposition method (LADM), Taylor series method (TSM), and variation iteration method (VIM), have been used to address complex mathematical problems [Ebiwareme, Kormane, Odok, 2022; Xu et al., 2021; Thongmoon, Pusjuso, 2010; Harir et al., 2020; Adeniji et al., 2023; Suganya, Senthamarai, 2022; Ali et al., 2021]. These analytical approaches have undergone numerous modifications over time to improve their efficacy in solving various problems. One such development is the fusion approach, which includes the Shehu transformation, Akbari – Ganji, and Pade approximation method (SAGPM) [Al-Sadi, Al-Saif, 2023a; Al-Sadi, Al-Saif, 2023b].

The basic reproduction number R_0 is the average number of secondary infections caused by a single infected individual in a community that is completely susceptible. This number provides an indicator of a disease's potential spread and is widely used for evaluating the initial scale of

an epidemic. R_0 is often calculated using mathematical models and considers characteristics such as transmission rate, infectious duration, and contact rate [Delamater et al., 2019; Alimohamadi, Taghdir, Sepandi, 2020; Rodrigues et al., 2012]. Sensitivity analysis of R_0 examines how changes in model parameters affect the resultant basic reproduction number. This investigation helps to determine which parameters have the most effect on disease propagation. By systematically adjusting input values, researchers may study the durability of R_0 and gain insights into important aspects affecting infectious disease dynamics [Rodrigues, Monteiro, Torres, 2013; Mazhar et al., 2023; Ndaïrou et al., 2020].

The primary goal of this paper was to solve the system of nonlinear differential equations in a coffee berry disease (CBD) model using the novel fusion of Shehu transformation, Akbari – Ganji, and Pade approximation method (SAGPM). Additionally, a sensitivity analysis of CBD will be performed to determine the importance of model parameters in influencing disease transmission. The paper is organized as follows: Section 2 discusses the preliminary aspects of the novel SAGPM, and Section 3 describes the methodology. Section 4 presents a mathematical model of coffee berry disease (CBD) through an in-depth description of governing equations. Section 5 describes the analytical solution of CBD model using the SAGPM. In Section 6, sensitivity analysis of CBD is discussed. Section 7 summarizes the discussion of the findings. Section 8 outlines the conclusions of the paper.

2. Preliminaries

2.1. Shehu transformation method

The Shehu transformation serves as a generalization of both the Laplace and the Sumudu integral transforms. It was proposed by Maitama and Zhao in 2019 [Maitama, Zhoa, 2019]. Several properties of the Shehu transformation are given and its applications are directly employed for the solution of ordinary and partial differential equations.

The Shehu transformation is derived over the set G:

$$G = \left\{w(t)\colon \exists N,\, n_1,\, n_2 > 0,\ |w(t)| < Ne^{|t|/n_j},\ t \in (-1)^j \times [0,\, \infty)\right\}$$

using the subsequent integral:

$$\mathcal{S}[w(t)] = w^*(v,\, s) = \lim_{a\to\infty} \int_0^a e^{-vt/s} w(t)\, dt; \quad v,\, s > 0. \tag{1}$$

The inverse Shehu transformation is expressed as:

$$w(t) = \mathcal{S}^{-1}\left[w^*(v,\, s)\right] = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} \frac{1}{s} e^{vt/s} w^*(v,\, s)\, ds. \tag{2}$$

Here, v and s represent the Shehu transform variables, while a is a real constant. The integral in Equation (2) is computed along $v = a$ in the complex plane, where $v = x + iy$.

The fundamental properties and derivatives of the Shehu transformation, considered at $\mathcal{S}\{w(t)\} = w^*(v,\, s)$ are shown in Table 1.

2.2. Akbari – Ganji's method (AGM)

Differential equation problems can be successfully solved using Akbari – Ganji's method (AGM) [Attar et al., 2022; Jalili et al., 2022]. The solution is reached by a sequence of algebraic problems by use of a finite series.

Table 1. Properties of the Shehu transformation method

Original functions	Transformed functions
$\mathcal{S}\left\{\frac{dw}{dt}\right\}$	$\frac{v}{s}w^*(v,\ s) - w(0)$
$\mathcal{S}\left\{w^{(n)}(t)\right\}$	$\frac{v^n}{s^n}w^*(v,\ s) - \sum\limits_{k=0}^{n-1}\left(\frac{v}{s}\right)^{v-(k+1)} w^{(k)}$
$\mathcal{S}\{t\}$	$\frac{v^2}{s^2}$
$\mathcal{S}\{e^{pt}\}$	$\frac{v}{s-pv}$
$\mathcal{S}\left\{\frac{t^n}{n!}\right\}$	$\left(\frac{v}{n}\right)^{n+1}$, $n = 0,\ 1,\ 2,\ \ldots$

In applying AGM, the differential equation and its mth-order derivatives for a function $y(t)$ can be represented as:

$$h = g\left(y,\ y',\ \ldots,\ y^{(m)}\right) = 0, \quad y = y(t) \tag{3}$$

with boundary conditions:

$$\begin{aligned} &y(t) = y_0, \quad y'(t) = y_1, \quad \ldots, \quad y^{(m-1)}(t) = y_{m-1}; \quad t = 0, \\ &y(t) = y_{L_0}, \quad y'(t) = y_{L_1}, \quad \ldots, \quad y^{(m-1)}(t) = y_{L_{m-1}}; \quad t = L. \end{aligned} \tag{4}$$

Let's consider the solutions to Equation (3) as follows:

$$y(t) = \sum_{i=0}^{n} b_i t^i. \tag{5}$$

By increasing the number of terms in series (5), it can be more accurate. With (n) degrees in the series, there are ($n+1$) unknown coefficients that need to be determined to obtain the solution, and by applying the boundary conditions (4) in (5), we have the following:
at $t = 0$,

$$\begin{aligned} y(0) &= b_0 = y_0, \\ y'(0) &= b_1 = y_1 \\ &\vdots \end{aligned} \tag{6}$$

at $t = L$,

$$\begin{aligned} y(L) &= b_0 + b_1 L + \cdots + b_n L^n = y_{L_0}, \\ y'(L) &= b_1 + 2b_2 L + \cdots + nb_n L^{n-1} = y_{L_1}, \\ &\vdots \end{aligned} \tag{7}$$

Substituting (6) and (7) with boundary conditions into (3), we obtain:

$$h = \begin{cases} g\left(y(0),\ y'(0),\ \ldots,\ y^{(m)}(0)\right), \\ g\left(y(L),\ y'(L),\ \ldots,\ y^{(m)}(L)\right), \\ \quad \vdots \\ g\left(y^n(0),\ y^{n+1}(0),\ \ldots,\ y^{(m+n)}(0)\right), \\ g\left(y^n(L),\ y^{n+1}(L),\ \ldots,\ y^{(m+n)}(L)\right). \end{cases} \tag{8}$$

After solving the system of equations, the system produces (n + 1) equations that make it easier to calculate the unknown coefficients $b_0, b_1, b_2, \ldots, b_n$.

2.3. Pade approximation method

Pade approximation relies on the characteristics of rational power approximation and the use of a quotient of two polynomials with varying degrees. This method provides a more effective approximation of the function compared to the Taylor series method [Wu, Qian, 2016; Wazwaz, 1999].

Pade approximation involves the utilization of a ratio of two polynomials derived from the Taylor series expansion of a function $y(t)$ and is defined by

$$P_m^l = \frac{\sum\limits_{n=0}^{l} a_n t^n}{\sum\limits_{n=0}^{m} b_n t^n} \tag{9}$$

where $b_0 = 1$.

The function $y(t)$ can be written as $y(t) = \sum\limits_{n=0}^{\infty} e_n t^n$.

Also, we have that $y(t) - P_m^l = o\left(t^{l+m+1}\right)$.

Consequently, we conclude

$$\sum_{n=0}^{\infty} e_n t^n = \frac{\sum\limits_{n=0}^{l} a_n t^n}{\sum\limits_{n=0}^{m} b_n t^n}. \tag{10}$$

From (10), we obtain:

$$\begin{gathered} a_0 = e_0, \\ a_1 = e_1 + e_0 b_1, \\ a_2 = e_2 + e_1 b_1 + e_0 b_2 \\ \vdots \end{gathered} \tag{11}$$

In order to derive the values for a_n and b_n in the given system, we set the degree of the numerator to l and the degree of the denominator to m. Furthermore, we consider the Taylor series expansion of $y(t)$ up to t^{l+m}.

The basic idea of the fusion of Shehu transformation – Akbari – Ganji – Pade approximation method (SAGPM) is explained in the following steps.

3. SAGPM methodology

In this section, we discuss the novel SAGPM approach for the system of differential equations problems [Al-Sadi, Al-Saif, 2023a; Al-Sadi, Al-Saif, 2023b].

We consider the system of nonlinear differential equations:

$$\begin{aligned} \frac{dw_1}{dt} &= f(w_1(t),\, w_2(t),\, w_3(t),\, p_1(t),\, \ldots,\, p_n(t)), \\ \frac{dw_2}{dt} &= f(w_1(t),\, w_2(t),\, w_3(t),\, r_1(t),\, \ldots,\, r_n(t)), \\ \frac{dw_3}{dt} &= f(w_1(t),\, w_2(t),\, w_3(t),\, u_1(t),\, \ldots,\, u_n(t)). \end{aligned} \tag{12}$$

The initial conditions are

$$w_1(0) = \gamma_1, \quad w_2(0) = \gamma_2, \quad w_3(0) = \gamma_3. \tag{13}$$

Step 1: Applying the Shehu transformation to both sides of (12), we get:

$$\begin{aligned}
\mathcal{S}\left(\frac{dw_1}{dt}\right) &= \mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, p_1(t),\, \ldots,\, p_n(t))),\\
\mathcal{S}\left(\frac{dw_2}{dt}\right) &= \mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, r_1(t),\, \ldots,\, r_n(t))),\\
\mathcal{S}\left(\frac{dw_3}{dt}\right) &= \mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, u_1(t),\, \ldots,\, u_n(t))).
\end{aligned} \tag{14}$$

With the application of the Shehu transformation properties and the given initial conditions (13), the following is obtained:

$$\begin{aligned}
w_1^*(u,\, s) &= \frac{\gamma_1 s}{u} + \frac{s}{u}\mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, p_1(t),\, \ldots,\, p_n(t))),\\
w_2^*(u,\, s) &= \frac{\gamma_2 s}{u} + \frac{s}{u}\mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, r_1(t),\, \ldots,\, r_n(t))),\\
w_3^*(u,\, s) &= \frac{\gamma_3 s}{u} + \frac{s}{u}\mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, u_1(t),\, \ldots,\, u_n(t))).
\end{aligned} \tag{15}$$

Step 2: After performing the inverse Shehu transformation to both sides of (15), we get:

$$\begin{aligned}
w_1(t) &= \gamma_1 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, p_1(t),\, \ldots,\, p_n(t)))\right),\\
w_2(t) &= \gamma_2 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, r_1(t),\, \ldots,\, r_n(t)))\right),\\
w_3(t) &= \gamma_3 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}(f(w_1(t),\, w_2(t),\, w_3(t),\, u_1(t),\, \ldots,\, u_n(t)))\right).
\end{aligned} \tag{16}$$

Step 3: Consider the Akbari – Ganji's polynomial series with constant coefficient:

$$w_1(t) = \sum_{i=0}^{n} a_i t^i, \quad w_2(t) = \sum_{i=0}^{n} b_i t^i, \quad w_3(t) = \sum_{i=0}^{n} c_i t^i. \tag{17}$$

After substituting (17) into (16), we obtain:

$$\begin{aligned}
\sum_{i=0}^{n} a_i t^i &= \gamma_1 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}\left(f\left(\sum_{i=0}^{n} a_i t^i,\, \sum_{i=0}^{n} b_i,\, \sum_{i=0}^{n} c_i t^i,\, p_1(t),\, \ldots,\, p_n(t)\right)\right)\right),\\
\sum_{i=0}^{n} b_i &= \gamma_2 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}\left(f\left(\sum_{i=0}^{n} a_i t^i,\, \sum_{i=0}^{n} b_i,\, \sum_{i=0}^{n} c_i t^i,\, r_1(t),\, \ldots,\, r_n(t)\right)\right)\right),\\
\sum_{i=0}^{n} c_i t^i &= \gamma_3 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}\left(f\left(\sum_{i=0}^{n} a_i t^i,\, \sum_{i=0}^{n} b_i,\, \sum_{i=0}^{n} c_i t^i,\, u_1(t),\, \ldots,\, u_n(t)\right)\right)\right).
\end{aligned} \tag{18}$$

By applying the initial conditions, we can attain initial values of the coefficients. Further, through the derivation of (18) and substitution of the initial conditions, we can derive the remaining values of a_i, b_i, and c_i.

Step 4: The Pade approximation of an order $\left[\frac{l}{m}\right]$ is applied to a power series solution derived using Shehu transformation and Akbari – Ganji's method, with arbitrary selections for the values of l and m. This process results in the final solutions of Shehu transformation – Akbari – Ganji – Pade approximation method (SAGPM).

4. Governing system of equations

In this section, we consider the coffee berry disease (CBD) model developed by H. O. Nyaberi [Nyaberi et al., 2023]. The coffee plants within the plantation are categorized into five groups: susceptible $S(t)$, exposed to *Colletotrichum kahawae* but not exhibiting symptoms $E(t)$, infected with CBD $I(t)$, and recovered coffee plants $R(t)$. Additionally, the number of *Colletotrichum kahawae* pathogens in the plantation is denoted as $P(t)$:

$$\begin{aligned}
\frac{dS}{dt} &= \Upsilon - (\omega P + \xi)S, \\
\frac{dE}{dt} &= \omega PS - (\alpha + \xi + \eta)E, \\
\frac{dI}{dt} &= \eta E - (\tau + \xi + \delta)I, \\
\frac{dP}{dt} &= \theta_1 E + \theta_2 I - \phi P, \\
\frac{dR}{dt} &= \alpha E + \tau I - \xi R
\end{aligned} \tag{19}$$

with the initial conditions:

$$S(0) = m_1, \quad E(0) = m_2, \quad I(0) = m_3, \quad P(0) = m_4, \quad R(0) = m_5. \tag{20}$$

Susceptible coffee trees $S(t)$ enter the system at a rate of Υ. Some trees leave all classes due to natural mortality at a constant rate ξ. Susceptible trees encounter the coffee berry disease through contact with *Colletotrichum kahawae* at a rate of ω, causing them to move to the $E(t)$. Some trees in the $E(t)$ progress to the $I(t)$ at the rate of η, while others progress to the $R(t)$ at the rate of α. Additionally, some trees in the $I(t)$ recover and proceed to the $R(t)$ at the rate of τ. A portion of trees in the $I(t)$ suffer CBD-induced mortality at a rate of δ. Moreover, the trees in the $E(t)$ and $I(t)$ classes contribute to the increase of the P pathogen in the environment at rates of θ_1 and θ_2, respectively. Finally, the pathogens in the $P(t)$ decay at a rate of ϕ.

5. Semianalytical expression of CBD model using SAGPM

In this section, we apply the novel technique of SAGPM for coffee berry disease (CBD). By taking the Shehu transformation on Equation (19) with Initial condition (20), we obtain:

$$\begin{aligned}
S^*(u, s) &= \frac{m_1 s}{u} + \frac{s}{u}\mathcal{S}(\Upsilon - (\omega P + \xi)S), \\
E^*(u, s) &= \frac{m_2 s}{u} + \frac{s}{u}\mathcal{S}(\omega PS - (\alpha + \xi + \eta)E), \\
I^*(u, s) &= \frac{m_3 s}{u} + \frac{s}{u}\mathcal{S}(\eta E - (\tau + \xi + \delta)I), \\
P^*(u, s) &= \frac{m_4 s}{u} + \frac{s}{u}\mathcal{S}(\theta_1 E + \theta_2 I - \phi P), \\
R^*(u, s) &= \frac{m_5 s}{u} + \frac{s}{u}\mathcal{S}(\alpha E + \tau I - \xi R),
\end{aligned} \tag{21}$$

where $S^*(u, s) = \mathcal{S}\{S(t)\}$, $E^*(u, s) = \mathcal{S}\{E(t)\}$, $I^*(u, s) = \mathcal{S}\{I(t)\}$, $P^*(u, s) = \mathcal{S}\{P(t)\}$, $R^*(u, s) = \mathcal{S}\{R(t)\}$.

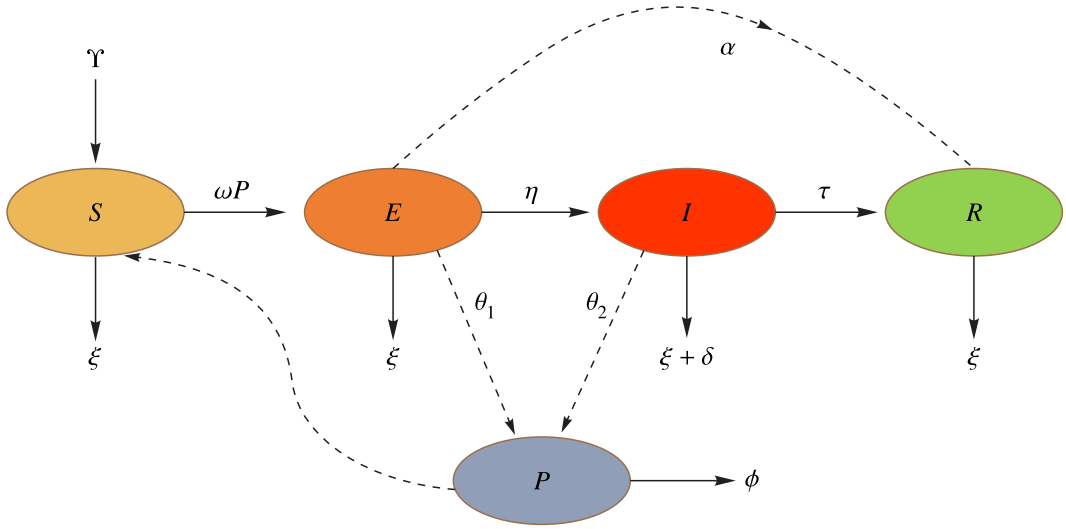

Figure 1. Flow chart of the dynamics of the coffee berry disease model. Susceptible coffee plants $S(t)$ enter the system at a rate of Υ. Natural mortality affects all classes at a constant rate ξ. Trees encountering *Colletotrichum kahawae* transition to exposed coffee plants $E(t)$ at ω. Trees in exposed coffee plants $E(t)$ progress to infected coffee plants $I(t)$ at η and to recovered coffee plants $R(t)$ at α, respectively. Trees in infected coffee plants $I(t)$ recover to recovered coffee plants $R(t)$ at τ, and some portion of infected coffee plants $I(t)$ suffer CBD-induced mortality at δ. Exposed coffee plants $E(t)$ and infected coffee plants $I(t)$ contribute to pathogen P at θ_1 and θ_2 rates, respectively, while *Colletotrichum kahawae* $P(t)$ decays at ϕ

Table 2. Parameters and their numerical values

Parameters	Values (unit: per day)	Source
Υ	0.00133	[Muhumuza, 2018]
ω	0.0007954551	[Nyaberi et al., 2023]
ξ	0.00056	[Muhumuza, 2018]
α	0.001	[Nyaberi et al., 2023]
η	0.01	[Nyaberi et al., 2023]
τ	0.005	[Nyaberi et al., 2023]
δ	0.0001	[Nyaberi et al., 2023]
θ_1	0.0587364	[Nyaberi et al., 2023]
θ_2	0.0487364	[Nyaberi et al., 2023]
ϕ	0.00900982	[Nyaberi et al., 2023]

Taking the inverse Shehu transformation in Equation (21), we get

$$\begin{aligned}
S(t) &= m_1 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}(\Upsilon - (\omega P + \xi)S)\right),\\
E(t) &= m_2 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}(\omega PS - (\alpha + \xi + \eta)E)\right),\\
I(t) &= m_3 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}(\eta E - (\tau + \xi + \delta)I)\right),\\
P(t) &= m_4 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}(\theta_1 E + \theta_2 I - \phi P)\right),\\
R(t) &= m_5 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}(\alpha E + \tau I - \xi R)\right).
\end{aligned} \tag{22}$$

By applying Akbari – Ganji's method, we substitute the polynomials (17) into (22) to obtain (23):

$$\begin{aligned}
\sum_{i=0}^{n} a_i t^i &= m_1 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}\left(\Upsilon - \left(\omega \sum_{i=0}^{n} d_i t^i + \xi\right)\sum_{i=0}^{n} a_i t^i\right)\right),\\
\sum_{i=0}^{n} b_i t^i &= m_2 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}\left(\omega \sum_{i=0}^{n} d_i t^i \sum_{i=0}^{n} a_i t^i - (\alpha + \xi + \eta)\sum_{i=0}^{n} b_i t^i\right)\right),\\
\sum_{i=0}^{n} c_i t^i &= m_3 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}\left(\eta \sum_{i=0}^{n} b_i t^i - (\tau + \xi + \delta)\sum_{i=0}^{n} c_i t^i\right)\right),\\
\sum_{i=0}^{n} d_i t^i &= m_4 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}\left(\theta_1 \sum_{i=0}^{n} b_i t^i + \theta_2 \sum_{i=0}^{n} c_i t^i - \phi \sum_{i=0}^{n} d_i t^i\right)\right),\\
\sum_{i=0}^{n} e_i t^i &= m_5 + \mathcal{S}^{-1}\left(\frac{s}{u}\mathcal{S}\left(\alpha \sum_{i=0}^{n} b_i t^i + \tau \sum_{i=0}^{n} c_i t^i - \xi \sum_{i=0}^{n} e_i t^i\right)\right),
\end{aligned} \tag{23}$$

when $n = 6$, by further simplification of Equation (23), we get:

$$\begin{aligned}
f(S(t)):\ & 6a_6t^5 + 5a_5t^4 + 4a_4t^3 + 3a_3t^2 + 2a_2t + a_1 - \Upsilon + \omega q_1 q_4 + \xi q_1 = 0,\\
g(E(t)):\ & 6b_6t^5 + 5b_5t^4 + 4b_4t^3 + 3b_3t^2 + 2b_2t + b_1 - \omega q_1 q_4 + (\alpha + \xi + \eta) q_2 = 0,\\
h(I(t)):\ & 6c_6t^5 + 5c_5t^4 + 4c_4t^3 + 3c_3t^2 + 2c_2t + c_1 - \eta q_2 + (\tau + \xi + \delta) q_3 = 0,\\
j(P(t)):\ & 6d_6t^5 + 5d_5t^4 + 4d_4t^3 + 3d_3t^2 + 2d_2t + d_1 - \theta_1 q_2 - \theta_2 q_3 + \phi q_4 = 0,\\
k(R(t)):\ & 6e_6t^5 + 5e_5t^4 + 4e_4t^3 + 3e_3t^2 + 2e_2t + e_1 - \alpha q_2 - \tau q_3 + \xi q_5 = 0,
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
q_1 &= a_0 + a_1 t + 2a_2t^2 + 3a_3t^3 + 4a_4t^4 + 5a_5t^5 + 6a_6t^6,\\
q_2 &= b_0 + b_1 t + 2b_2t^2 + 3b_3t^3 + 4b_4t^4 + 5b_5t^5 + 6b_6t^6,\\
q_3 &= c_0 + c_1 t + 2c_2t^2 + 3c_3t^3 + 4c_4t^4 + 5c_5t^5 + 6c_6t^6,\\
q_4 &= d_0 + d_1 t + 2d_2t^2 + 3d_3t^3 + 4d_4t^4 + 5d_5t^5 + 6d_6t^6,\\
q_5 &= e_0 + e_1 t + 2e_2t^2 + 3e_3t^3 + 4e_4t^4 + 5e_5t^5 + 6e_6t^6.
\end{aligned}$$

Table 3. Initial conditions of the coffee berry disease model at $t = 0$

Initial values	Values
$S(0)$	10000
$E(0)$	2000
$I(0)$	100
$P(0)$	1600
$R(0)$	1000

The unknown constant coefficients a_0, b_0, c_0, d_0 and e_0 can be found by solving the above equations with initial condition values in Table 3:

$$\begin{aligned}
S(t=0):\ & m_1 = 10000 \Rightarrow a_0 = 10000,\\
E(t=0):\ & m_2 = 2000 \Rightarrow b_0 = 2000,\\
I(t=0):\ & m_3 = 100 \Rightarrow c_0 = 100,\\
P(t=0):\ & m_4 = 1600 \Rightarrow d_0 = 1600,\\
R(t=0):\ & m_5 = 1000 \Rightarrow e_0 = 1000.
\end{aligned} \tag{25}$$

In order to find more iterations of the proposed method, the derivatives of Equation (24) are given below:

$$\begin{aligned}
f(S(t=0)):\ & a_1 + a_0 d_0 \omega + a_0 \xi - \Upsilon = 0,\\
g(E(t=0)):\ & b_1 - \omega d_0 a_0 + (\alpha + \xi + \eta) b_0 = 0,\\
h(I(t=0)):\ & c_1 - \eta b_0 + (\tau + \xi + \delta) c_0 = 0,\\
j(P(t=0)):\ & d_1 - b_0 \theta_1 - c_0 \theta_2 + d_0 \phi = 0,\\
k(R(t=0)):\ & e_1 - \alpha b_0 - c_0 \tau + e_0 \xi = 0,
\end{aligned} \tag{26}$$

$$\begin{aligned}
f'(S(t=0)):\ & 2a_2 + a_0 d_1 \omega + a_1 d_0 \omega + a_1 \xi = 0,\\
g'(E(t=0)):\ & 2b_2 - \omega d_1 a_0 - \omega d_0 a_1 + (\alpha + \xi + \eta) b_1 = 0,\\
h'(I(t=0)):\ & 2c_2 - \eta b_1 + (\tau + \xi + \delta) c_1 = 0,\\
j'(P(t=0)):\ & 2d_2 - b_1 \theta_1 - c_1 \theta_2 + d_1 \phi = 0,\\
k'(R(t=0)):\ & 2e_2 - \alpha b_1 - c_1 \tau + e_1 \xi = 0,
\end{aligned} \tag{27}$$

$$\begin{aligned}
f''(S(t=0)):\ & 6a_3 + 2a_0 d_2 \omega + 2a_1 d_1 \omega + 2a_2 d_0 \omega + 2a_2 \xi = 0,\\
g''(E(t=0)):\ & 6b_3 - 2\omega d_2 a_0 - 2\omega d_1 a_1 - 2\omega d_0 a_2 + 2(\alpha + \xi + \eta) b_2 = 0,\\
h''(I(t=0)):\ & 6c_3 - 2\eta b_2 + 2(\tau + \xi + \delta) c_2 = 0,\\
j''(P(t=0)):\ & 6d_3 - 2b_2 \theta_1 - 2c_2 \theta_2 + 2d_2 \phi = 0,\\
k''(R(t=0)):\ & 6e_3 - 2\alpha b_2 - 2c_2 \tau + 2e_2 \xi = 0,
\end{aligned} \tag{28}$$

$$\begin{aligned}
f'''(S(t=0)):\ & 24a_4 + 6a_0 d_3 \omega + 6a_1 d_2 \omega + 6a_2 d_1 \omega + 6a_3 d_0 \omega + 6a_3 \xi = 0,\\
g'''(E(t=0)):\ & 24b_4 - 6\omega d_3 a_0 - 6\omega d_2 a_1 - 6\omega d_1 a_2 - 6\omega d_0 a_3 + 6(\alpha + \xi + \eta) b_3 = 0,\\
h'''(I(t=0)):\ & 24c_4 - 6\eta b_3 + 6(\tau + \xi + \delta) c_3 = 0,\\
j'''(P(t=0)):\ & 24d_4 - 6b_3 \theta_1 - 6c_3 \theta_2 + 6d_3 \phi = 0,\\
k'''(R(t=0)):\ & 24e_4 - 6\alpha b_3 - 6c_3 \tau + 6e_3 \xi = 0,
\end{aligned} \tag{29}$$

$$\begin{aligned}
f^{iv}(S(t=0)):\ & 120a_5 + 24a_0 d_4 \omega + 24a_1 d_3 \omega + 24a_2 d_2 \omega + 24a_3 d_1 \omega + 24a_4 d_0 \omega +\\
& + 24a_4 \xi = 0,\\
g^{iv}(E(t=0)):\ & 120b_5 - 24\omega d_4 a_0 - 24\omega d_3 a_1 - 24\omega d_2 a_2 - 24\omega d_1 a_3 -\\
& - 24\omega d_0 a_4 + 24(\alpha + \xi + \eta) b_4 = 0,\\
h^{iv}(I(t=0)):\ & 120c_5 - 24\eta b_4 + 24(\tau + \xi + \delta) c_4 = 0,\\
j^{iv}(P(t=0)):\ & 120d_5 - 24b_4 \theta_1 - 24c_4 \theta_2 + 24d_4 \phi = 0,\\
k^{iv}(R(t=0)):\ & 120e_5 - 24\alpha b_4 - 24c_4 \tau + 24e_4 \xi = 0,
\end{aligned} \tag{30}$$

$$\begin{aligned}
f^{v}(S(t=0)):\ & 720a_6 + 120a_0 d_5 \omega + 120a_1 d_4 \omega + 120a_2 d_3 \omega + 120a_3 d_2 \omega + 120a_4 d_1 \omega +\\
& + 120a_5 d_0 \omega + 120a_5 \xi = 0,\\
g^{v}(E(t=0)):\ & 720b_6 - 120\omega d_5 a_0 - 120\omega d_4 a_1 - 120\omega d_3 a_2 - 120\omega d_2 a_3 - 120\omega d_1 a_4 -\\
& - 120\omega d_0 a_5 + 120(\alpha + \xi + \eta) b_5 = 0,\\
h^{v}(I(t=0)):\ & 720c_6 - 120\eta b_5 + 120(\tau + \xi + \delta) c_5 = 0,\\
j^{v}(P(t=0)):\ & 720d_6 - 120b_5 \theta_1 - 120c_5 \theta_2 + 120d_5 \phi = 0,\\
k^{v}(R(t=0)):\ & 720e_6 - 120\alpha b_5 - 120c_5 \tau + 120e_5 \xi = 0.
\end{aligned} \tag{31}$$

The unknown constant coefficients a_i, b_i, c_i, d_i and e_i (i = 1, 2, 3, 4, 5, 6) can be found by solving the above equations with substitution of the parameter values provided in Table 2, and Equation (25).

The solutions are derived as:

$$\begin{aligned}
S(t) &= 10000 - 12732.88027t + 7677.042605t^2 - 3883.215822t^3 + 2317.835250t^4 - \\
&\quad - 1378.092714t^5 + 739.1028826t^6, \\
E(t) &= 2000 + 12704.16160t - 7746.907455t^2 + 3911.634190t^3 - 2328.596222t^4 + \\
&\quad + 1383.216831t^5 - 741.6392585t^6, \\
I(t) &= 100 + 19.43400000t + 63.46580980t^2 - 25.94276368t^3 + 9.815794488t^4 - \\
&\quad - 4.668303923t^5 + 2.309765151t^6, \\
P(t) &= 1600 + 107.9307280t + 373.0857121t^2 - 151.7645984t^3 + 57.46458133t^4 - \\
&\quad - 27.36254363t^5 + 13.54403206t^6, \\
R(t) &= 1000 + 1.940000000t + 6.400122600t^2 - 2.477720825t^3 + 0.9458269738t^4 - \\
&\quad - 0.4560093825t^5 + 0.2266884460t^6.
\end{aligned} \tag{32}$$

By using the Pade approximation, we arbitrarily set the degree of the numerator $l = 3$ and the degree of the denominator $m = 3$ for the concentrations $S(t)$, $E(t)$, $I(t)$, $P(t)$, and $R(t)$.

The semianalytical solutions for the system (19) using SAGPM are obtained as follows:

$$S_{[3,3]}(t) = \frac{10000 - 11045.31142t + 5321.182766t^2 - 1143.144683t^3}{1 + 0.1687568845t - 0.02070986341t^2 + 0.1180821105t^3}, \tag{33}$$

$$E_{[3,3]}(t) = \frac{2000 + 13002.06516t - 5924.273296t^2 + 2547.524265t^3}{1 + 0.1489517822t - 0.03483667669t^2 + 0.1161882903t^3}, \tag{34}$$

$$I_{[3,3]}(t) = \frac{100 + 73.31697224t + 80.03324992t^2 + 10.95287149t^3}{1 + 0.5388297224t + 0.06095823305t^2 + 0.01513708186t^3}, \tag{35}$$

$$P_{[3,3]}(t) = \frac{1600 + 971.5967670t + 529.2450014t^2 + 80.37101546t^3}{1 + 0.5397912744t + 0.06118701506t^2 + 0.01508952675t^3}, \tag{36}$$

$$R_{[3,3]}(t) = \frac{1000 + 551.0839440t + 68.15651215t^2 + 14.74648973t^3}{1 + 0.5491439440t + 0.06069105030t^2 + 0.01359188135t^3}. \tag{37}$$

6. Sensitivity analysis

In epidemiology, the average number of secondary infections in a population that is completely susceptible to infection due to a single infected individual is known as the basic reproduction number R_0. The basic reproduction number R_0 for the coffee berry disease model is provided in [Nyaberi et al., 2023].

$$R_0 = \frac{w\Upsilon((\tau + \xi + \delta)\theta_1 + \eta\theta_2)}{\phi\xi(\alpha + \xi + \eta)(\tau + \xi + \delta)}. \tag{38}$$

Implementing the basic reproduction number R_0, Equation (38) for sensitivity analysis provides significant insight into the dynamics and control of infectious illnesses, which enables the development of effective ways to reduce their disease impact in the coffee berry model. This method aids in analyzing the effects of changing parameter values such as Υ, ω, ξ, α, η, τ, δ, θ_1, θ_2, and ϕ. Continuously modifying these parameters within a suitable range and evaluating the resulting adjustments in R_0 yields significant insights into the components of disease dynamics that are most sensitive to parameter changes. More specifically, when a parameter is changed, sensitivity indices allow one to quantify the proportionate change in a variable. In order to do this, we use the normalized forward sensitivity index of a variable with respect to a certain parameter. This index can be defined as the ratio of the relative change in the parameter to the relative change in each variable. The normalized forward sensitivity

index of the reproduction number R_0 with respect to the parameter ω can be described in the following manner:

$$\Psi_{\omega}^{R_0} = \frac{\partial R_0}{\partial \omega}\frac{\omega}{R_0}. \tag{39}$$

Likewise, we can apply this normalized forward sensitivity index of the reproduction number R_0 to all the parameters within the coffee berry disease model. This process is employed to assess the correlation between parameters and reproduction numbers, as detailed in Table 4 and depicted in Figure 2. Normalized forward sensitivity indices indicate that when parameter values are below zero, like $\Psi_{\tau}^{R_0}$, $\Psi_{\xi}^{R_0}$, $\Psi_{\delta}^{R_0}$, $\Psi_{\eta}^{R_0}$, $\Psi_{\alpha}^{R_0}$, and $\Psi_{\phi}^{R_0}$, there exists an inverse relationship with reproduction numbers, leading to a decrease in infections as they increase. Conversely, values greater than zero, such as $\Psi_{\omega}^{R_0}$, $\Psi_{\Upsilon}^{R_0}$, $\Psi_{\theta_1}^{R_0}$, and $\Psi_{\theta_2}^{R_0}$, demonstrate a direct correlation, resulting in an increase in infections as the parameters increase. While the normalized forward sensitivity index provides fundamental insights into factors influencing infection control, it does not yield precise parameter sensitivity values.

Table 4. The sensitivity index of the basic reproduction number R_0 was determined using the parameter values provided in Table 2

Sensitivity index of R_0	Values
$\Psi_{\omega}^{R_0}$	1
$\Psi_{\Upsilon}^{R_0}$	1
$\Psi_{\tau}^{R_0}$	−0.5251613253
$\Psi_{\xi}^{R_0}$	−1.107260975
$\Psi_{\delta}^{R_0}$	−0.01050322650
$\Psi_{\theta_1}^{R_0}$	0.4055173799
$\Psi_{\theta_2}^{R_0}$	0.5944826201
$\Psi_{\eta}^{R_0}$	−0.2705692834
$\Psi_{\alpha}^{R_0}$	−0.08650519031
$\Psi_{\phi}^{R_0}$	−1

7. Discussion of results

A novel analytical approach of Shehu transformation – Akbari – Ganji – Pade approximation method (SAGPM) was used to solve the system of nonlinear differential equations in the coffee berry disease (CBD) model. Semianalytical solutions for the concentrations of S, E, I, P, and R are represented in Equations (33), (34), (35), (36), and (37), respectively. The *ode*45 solver was used to numerically solve the system of equations (19), with MATLAB program used for the numerical simulations. The analysis considered the initial values and parameters outlined in Tables 2 and 3, respectively. Figure 3 compares the SAGPM solution with the numerical simulation for susceptible coffee plants S over time. The susceptible coffee plants that are in contact with *Colletotrichum kahawae* pathogens are exposed to the disease. We noticed a decline in the number of susceptible coffee plants as they moved to exposed coffee plants. Figure 4 compares the SAGPM solution with the numerical simulation for the exposed coffee plants E over time. We notice an increase in the number of exposed coffee plants. This is because coffee plants may lose their immunity to *Colletotrichum kahawae*

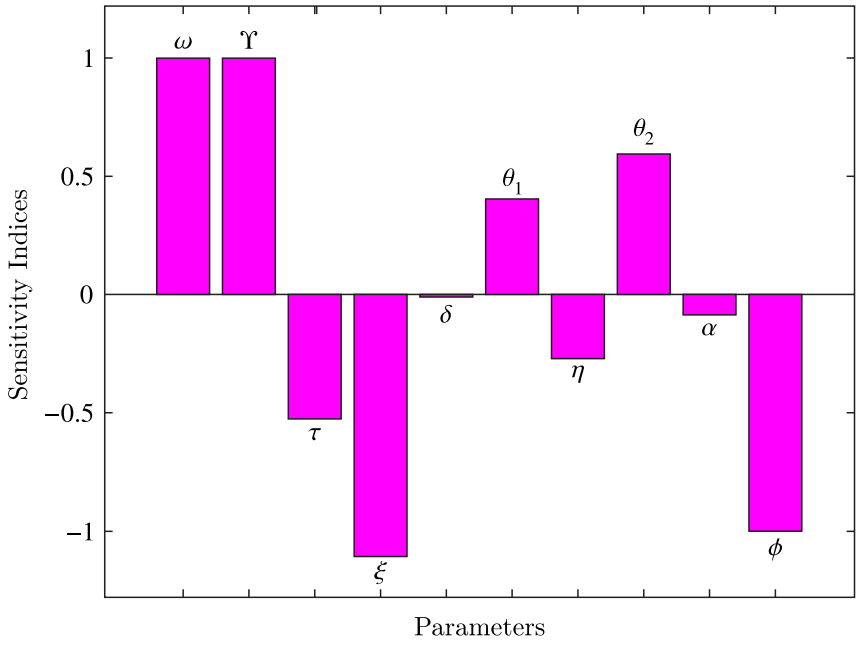

Figure 2. Normalized forward sensitivity indices of the basic reproduction number R_0 concerning various coffee berry disease model parameters: Υ (rate of entry of susceptible trees), ξ (natural mortality rate), ω (encounter rate with *Colletotrichum kahawae*), η (transition rate from exposed coffee plants $E(t)$ to infected coffee plants $I(t)$), α (transition rate from exposed coffee plants $E(t)$ to recovered coffee plants $R(t)$), τ (recovery rate from infected coffee plants $I(t)$ to recovered coffee plants $R(t)$), δ (CBD-induced mortality rate in infected coffee plants $I(t)$), θ_1 (contribution rate of exposed coffee plants $E(t)$ to P pathogen), θ_2 (contribution rate of infected coffee plants $I(t)$ to P pathogen), and ϕ (decay rate of pathogens $P(t)$)

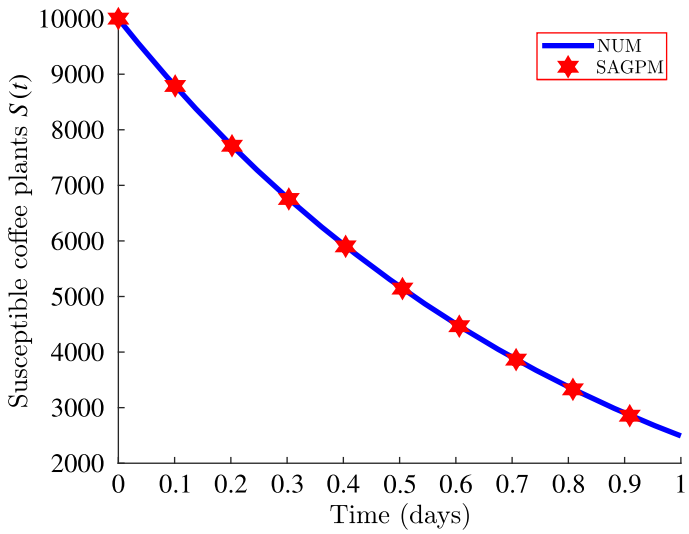

Figure 3. Graphical representation of the susceptible coffee plants $S(t)$ by using the SAGPM (Eq. (33)) and numerical simulation for $\Upsilon = 0.00133$, $\omega = 0.0007954551$, $\xi = 0.00056$, $\alpha = 0.001$, $\eta = 0.01$, $\tau = 0.005$, $\delta = 0.0001$, $\theta_1 = 0.0587364$, $\theta_2 = 0.0487364$, $\phi = 0.00900982$

pathogens and may come into contact with infected coffee plants. Figure 5 compares the SAGPM solution with the numerical simulation for the infected coffee plants I over time. We noticed an increase in the number of infected coffee plants, as the number of exposed coffee plants is also on the rise.

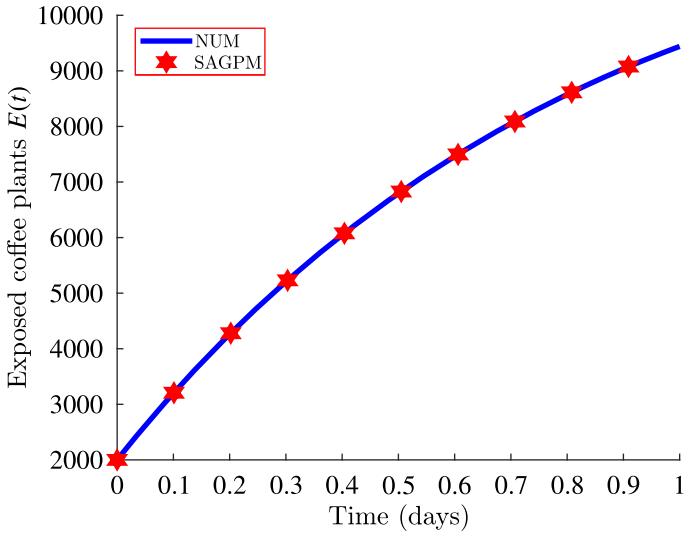

Figure 4. Graphical representation of the exposed coffee plants $E(t)$ by using the SAGPM (Eq. (34)) and numerical simulation for $\Upsilon = 0.00133$, $\omega = 0.0007954551$, $\xi = 0.00056$, $\alpha = 0.001$, $\eta = 0.01$, $\tau = 0.005$, $\delta = 0.0001$, $\theta_1 = 0.0587364$, $\theta_2 = 0.0487364$, $\phi = 0.00900982$

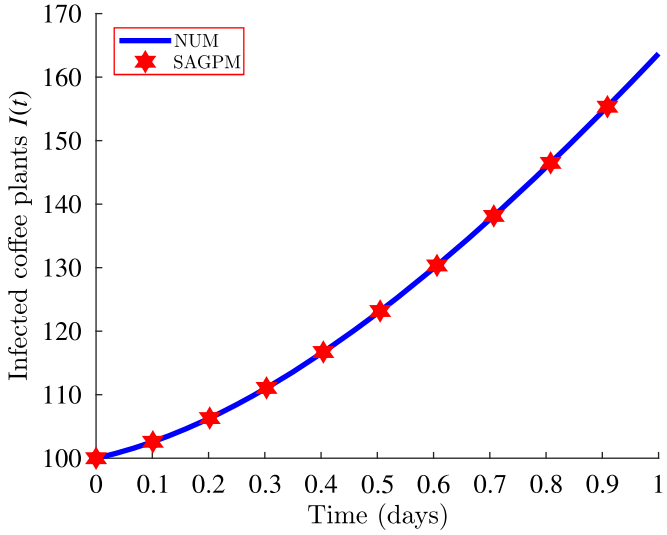

Figure 5. Graphical representation of the infected coffee plants $I(t)$ by using the SAGPM (Eq. (35)) and numerical simulation for $\Upsilon = 0.00133$, $\omega = 0.0007954551$, $\xi = 0.00056$, $\alpha = 0.001$, $\eta = 0.01$, $\tau = 0.005$, $\delta = 0.0001$, $\theta_1 = 0.0587364$, $\theta_2 = 0.0487364$, $\phi = 0.00900982$

Figure 6 compares the SAGPM solution with the numerical simulation for the *Colletotrichum kahawae* pathogens P over time. We noticed an increase in the number of *Colletotrichum kahawae* pathogens, as the coffee plants were affected as expected. Figure 7 compares the SAGPM solution with the numerical simulation for the recovered coffee plants R over time. We observed a slow increase in the number of recovered coffee plants. This is because infected and exposed coffee plants may have better progression rates as a result of reducing the contact rate of the disease. The accuracy of SAGPM solutions for S, E, I, P, and R concentrations is shown in Tables 5, 6, 7, 8, and 9, respectively. The overall error between the numerical simulation and the novel SAGPM was consistently less than 0.15 %.

Figure 2 and Table 4 indicate that the parameters ω, Υ, θ_1, and θ_2 have positive sensitivity values. ω and Υ have a direct impact on R_0 with a sensitivity value of +1. Figures 11, *a* and 11, *b* show

Table 5. Comparative analysis of SAGPM (Eq. (33)) and numerical simulations for the susceptible coffee plants $S(t)$ with parameter values from Figure 3

Susceptible coffee plants $S(t)$			
t	Num.	SAGPM	Error %
0	10000.0000	10000.0000	0.0000
0.1	8800.0258	8799.8179	0.0024
0.2	7733.1433	7732.7499	0.0051
0.3	6782.7064	6782.1081	0.0088
0.4	5935.1551	5934.3360	0.0138
0.5	5179.3156	5178.1971	0.0216
0.6	4505.8192	4504.1546	0.0369
0.7	3906.6399	3903.8962	0.0702
0.8	3374.7510	3369.9769	0.1415
0.9	2903.8995	2895.5616	0.2871
1	2488.5000	2474.2553	0.5724
Mean			0.1054

Table 6. Comparative analysis of SAGPM (Eq. (34)) and numerical simulations for the exposed coffee plants $E(t)$ with parameter values from Figure 4

Exposed coffee plants $E(t)$			
t	Num.	SAGPM	Error %
0	2000.0000	2000.0000	0.0000
0.1	3196.3014	3196.6390	0.0106
0.2	4258.2673	4258.9234	0.0154
0.3	5202.6840	5203.6796	0.0191
0.4	6043.2364	6044.5870	0.0223
0.5	6791.2109	6792.9936	0.0263
0.6	7456.0786	7458.5425	0.0330
0.7	8045.9586	8049.6533	0.0459
0.8	8567.9636	8573.8853	0.0691
0.9	9028.4251	9038.2020	0.1083
1	9433.0000	9449.1458	0.1712
Mean			0.0473

Table 7. Comparative analysis of SAGPM (Eq. (35)) and numerical simulations for the infected coffee plants $I(t)$ with parameter values from Figure 5

Infected coffee plants $I(t)$			
t	Num.	SAGPM	Error %
0	100.0000	102.5531	0.0000
0.1	102.5533	102.5531	0.0002
0.2	106.2326	106.2322	0.0004
0.3	110.9117	110.9113	0.0004
0.4	116.4795	116.4792	0.0003
0.5	122.8373	122.8373	0.0000
0.6	129.8972	129.8972	0.0000
0.7	137.5796	137.5791	0.0004
0.8	145.8129	145.8105	0.0016
0.9	154.5320	154.5253	0.0043
1	163.6781	163.6628	0.0093
Mean			0.0015

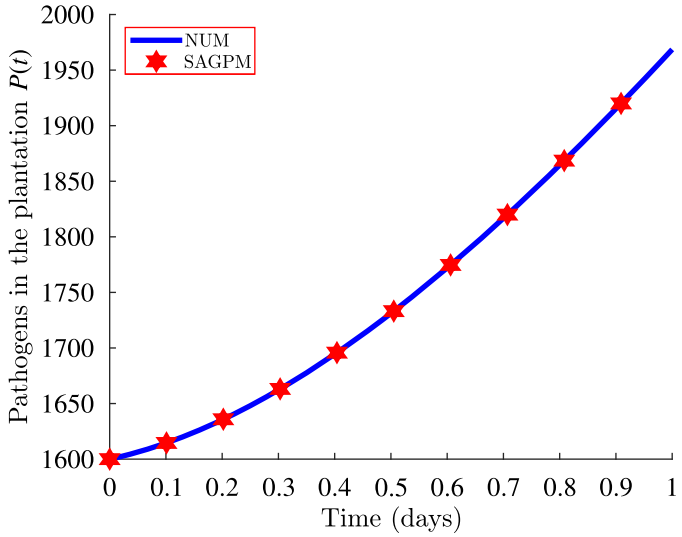

Figure 6. Graphical representation of the *Colletotrichum kahawae* pathogens $P(t)$ by using the SAGPM (Eq. (36)) and numerical simulation for $\Upsilon = 0.00133$, $\omega = 0.0007954551$, $\xi = 0.00056$, $\alpha = 0.001$, $\eta = 0.01$, $\tau = 0.005$, $\delta = 0.0001$, $\theta_1 = 0.0587364$, $\theta_2 = 0.0487364$, $\phi = 0.00900982$

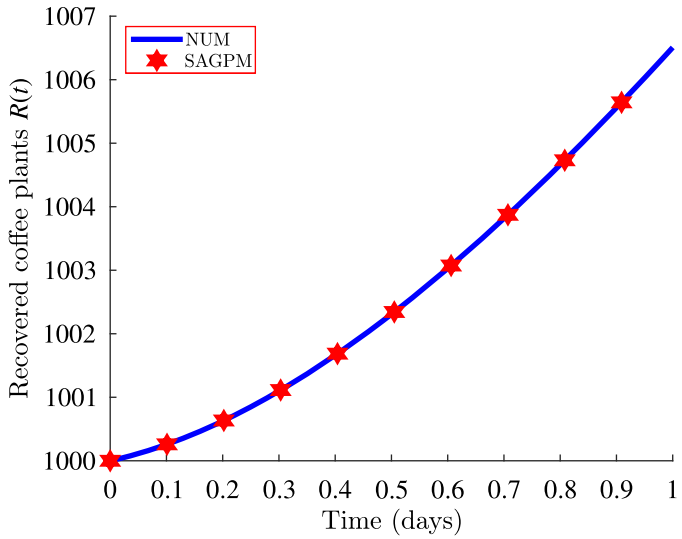

Figure 7. Graphical representation of the recovered coffee plants $R(t)$ by using the SAGPM (Eq. (37)) and numerical simulation for $\Upsilon = 0.00133$, $\omega = 0.0007954551$, $\xi = 0.00056$, $\alpha = 0.001$, $\eta = 0.01$, $\tau = 0.005$, $\delta = 0.0001$, $\theta_1 = 0.0587364$, $\theta_2 = 0.0487364$, $\phi = 0.00900982$

that raising ω and Υ by 10 % increases the basic reproduction number R_0 by 10 %, indicating linearity. The parameters θ_1 and θ_2 also seem to have a larger impact on R_0. Figures 11, *c* and 11, *d* indicate that increased sensitivity implies that small variations have a greater influence on R_0, and by raising θ_1 and θ_2 by 10 %, R_0 rises by 5.94 % and 4.05 %, respectively. The remaining parameters τ, ξ, η, δ, α, and ϕ all have negative sensitivity values, implying that these parameters tend to decrease R_0, as seen in Figures 12, *a–f*, and increasing τ, ξ, η, δ, α, ϕ by 10 % decreases the basic reproduction number R_0 by 5.25 %, 11.07 %, 2.7 %, 0.105 %, 0.86 %, and 10 %, respectively.

Recall that the progression rate from $E(t)$ to $R(t)$ is represented by the parameter α, the progression rate from $I(t)$ to $R(t)$ by the parameter τ, and the transmission rate from $E(t)$ to $I(t)$ by the parameter η. These parameters play a crucial role in combatting the coffee berry disease (CBD). As

Table 8. Comparative analysis of SAGPM (Eq. (36)) and numerical simulations for the *Colletotrichum kahawae* pathogens $P(t)$ with parameter values from Figure 6

Colletotrichum kahawae pathogens $P(t)$			
t	Num.	SAGPM	Error %
0	1600.0000	1600.0000	0.0000
0.1	1614.3732	1614.3777	0.0003
0.2	1635.3707	1635.3794	0.0005
0.3	1662.2551	1662.2669	0.0007
0.4	1694.3770	1694.3905	0.0008
0.5	1731.1599	1731.1732	0.0008
0.6	1772.0889	1772.0981	0.0005
0.7	1816.7002	1816.6983	0.0001
0.8	1864.5744	1864.5497	0.0013
0.9	1915.3310	1915.2651	0.0034
1	1968.6260	1968.4897	0.0069
Mean			0.0014

Table 9. Comparative analysis of SAGPM (Eq. (37)) and numerical simulations for the recovered coffee plants $R(t)$ with parameter values from Figure 7

Recovered coffee plants $R(t)$			
t	Num.	SAGPM	Error %
0	1000.0000	1000.0000	0.0000
0.1	1000.2556	1000.2556	0.0000
0.2	1000.6255	1000.6256	$9.9937 \cdot 10^{-6}$
0.3	1001.0977	1001.0978	$9.9890 \cdot 10^{-6}$
0.4	1001.6616	1001.6618	$1.9966 \cdot 10^{-5}$
0.5	1002.3078	1002.3080	$1.9953 \cdot 10^{-5}$
0.6	1003.0280	1003.0282	$1.9939 \cdot 10^{-5}$
0.7	1003.8144	1003.8146	$1.9924 \cdot 10^{-5}$
0.8	1004.6603	1004.6603	0.0000
0.9	1005.5595	1005.5590	$4.9723 \cdot 10^{-5}$
1	1006.5063	1006.5048	$1.4903 \cdot 10^{-4}$
Mean			$2.2652 \cdot 10^{-5}$

the rate of parameter η increases, the number of infected coffee plants I rapidly increases, as shown in Figure 8. So, it is important to control the transmission rate of η, and this may be done by significantly reducing the contact rate of coffee plants with the *Colletotrichum kahawae* pathogens. As the rates of τ and α increase, the number of recovered coffee plants population R steadily increases over time, as shown in Figures 9 and 10.

8. Conclusion

In this paper, we derived an semi-analytical expression for a system of nonlinear differential equations in the coffee berry disease (CBD) model. The proposed model is evaluated using a novel analytical method, which is a fusion of the Shehu transformation – Akbari – Ganji – Pade approximation method (SAGPM). The analytical solutions were compared with numerical results and demonstrated good accuracy. The findings from the Tables and Figures exhibit that the SAGPM technique is capable of producing computations that are highly accurate, even within a limited time interval. SAGPM takes on more manageable forms without requiring decomposition or perturbation applications and it has

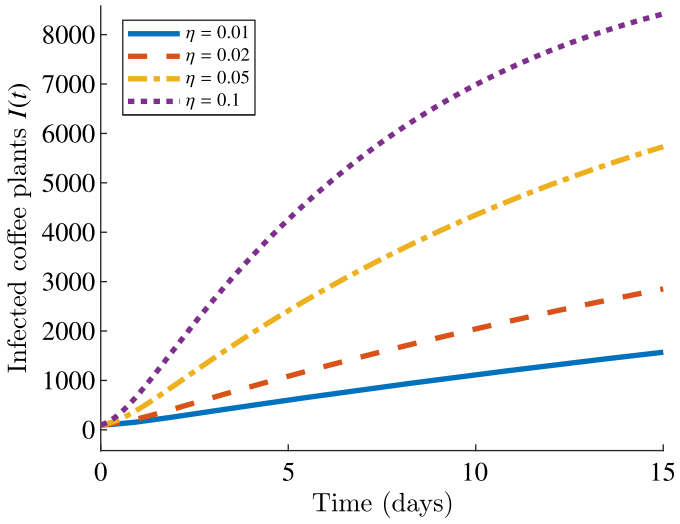

Figure 8. Impact of the rate of η on the progression from exposed coffee plants $E(t)$ to infected coffee plants $I(t)$ in infected coffee plants

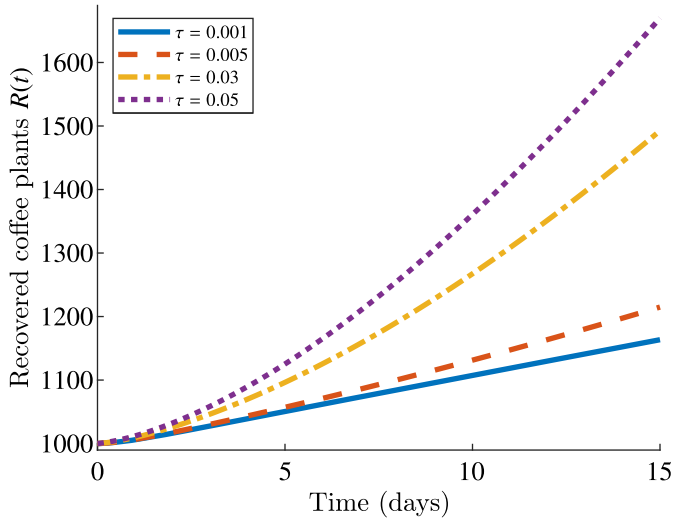

Figure 9. Impact of the recovery rate of τ on the progression from infected coffee plants $I(t)$ to recovered coffee plants $R(t)$ in recovered coffee plants

been analyzed that the proposed techniques could be applied to address a range of real-life problems for predicting disease causes and implementing preventive measures. Based on sensitivity analysis, the basic reproduction number R_0 is more significantly increased by parameters with higher positive sensitivity values, such as ω, Υ, θ_1, and θ_2. In contrast, parameters with negative sensitivity values, such as τ, ξ, η, δ, α, and ϕ, tend to decrease R_0 as they increase. Consistently maintaining favorable progression rates will lead to a reduction in the number of infected coffee plants by *Colletotrichum kahawae* and yet the recovered plants experience growth in CBD over the period of time. Therefore, it concludes that controlling the rate of contact and increasing the progression rates will help to reduce CBD.

Conflict of interest

The authors declare that there is no conflict of interest.

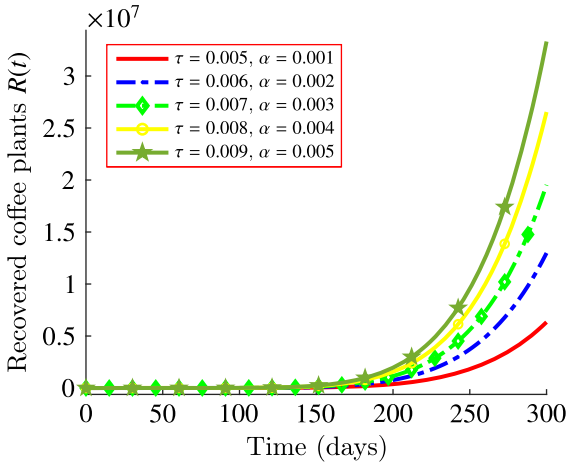

Figure 10. Impact of the recovery rate of τ on the progression from infected coffee plants $I(t)$ to recovered coffee plants $R(t)$ and rate of α on the progression from exposed coffee plants $E(t)$ to recovered coffee plants $R(t)$ in recovered coffee plants

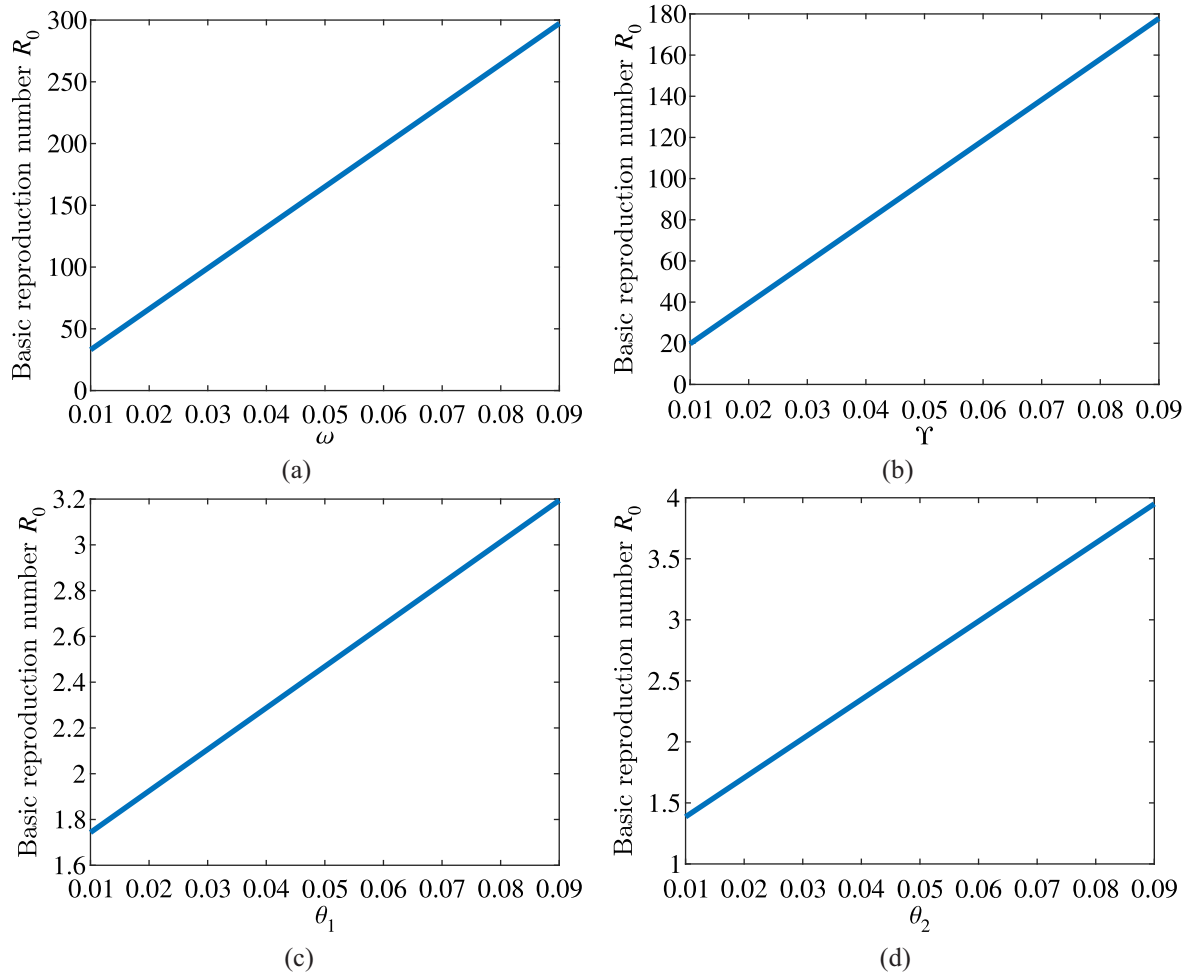

Figure 11. Sensitivity index plot of basic reproduction number R_0 and (a) impact of the rate ω on the transition of susceptible trees to the exposed coffee plants $E(t)$ stage due to contact with *Colletotrichum kahawae*, (b) impact of the rate Υ on the entry of susceptible coffee trees $S(t)$ into the system, (c) θ_1, rate of increase of the P pathogen in the environment due to trees in exposed coffee plants $E(t)$, (d) θ_2, rate of increase of the P pathogen in the environment due to trees in infected coffee plants $I(t)$

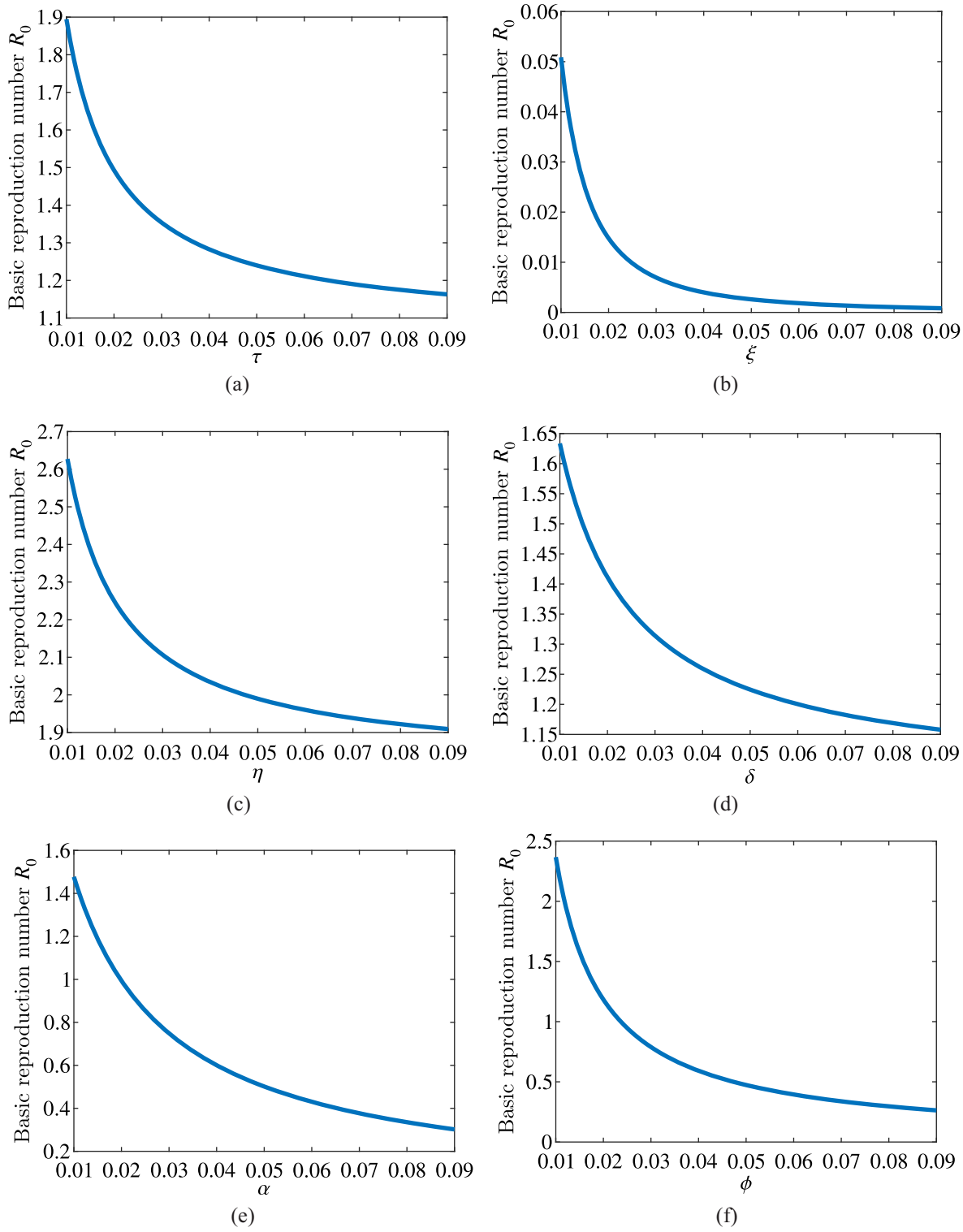

Figure 12. Sensitivity index plot of basic reproduction number R_0 and (a) impact of the recovery rate of τ on the progression from infected coffee plants $I(t)$ to recovered coffee plants $R(t)$, (b) impact of natural mortality rate ξ on trees leaving all classes, (c) impact of the rate of η on the progression from exposed coffee plants $E(t)$ to infected coffee plants $I(t)$, (d) impact of CBD-induced mortality rate δ on trees in the infected coffee plants $I(t)$ class, (e) rate of α on the progression from exposed coffee plants $E(t)$ to recovered coffee plants $R(t)$, (f) decay rate ϕ of pathogens in the *Colletotrichum kahawae* $P(t)$ class

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