

UDC: 004.94/519.21

Dynamical trap model for stimulus – response dynamics of human control

I. A. Lubashevsky^{1,2,a}, V. I. Lubashevskiy^{3,b}

¹HSE Tikhonov Moscow Institute of Electronics and Mathematics,
34 Tallinskaya st., Moscow, 123458, Russia

²Moscow Automobile and Road Construction State Technical University,
64 Leningradsky ave., Moscow, 125319, Russia

³Tokyo International University,
1-13-1 Matoba-Kita, Kawagoe, Saitama, 350-1197, Japan

E-mail: ^a ilubashevskii@hse.ru, ^b vlubashe@tiu.ac.jp

Received 04.11.2023, after completion — 18.11.2023.

Accepted for publication 21.11.2023.

We present a novel model for the dynamical trap of the stimulus–response type that mimics human control over dynamic systems when the bounded capacity of human cognition is a crucial factor. Our focus lies on scenarios where the subject modulates a control variable in response to a certain stimulus. In this context, the bounded capacity of human cognition manifests in the uncertainty of stimulus perception and the subsequent actions of the subject. The model suggests that when the stimulus intensity falls below the (blurred) threshold of stimulus perception, the subject suspends the control and maintains the control variable near zero with accuracy determined by the control uncertainty. As the stimulus intensity grows above the perception uncertainty and becomes accessible to human cognition, the subject activates control. Consequently, the system dynamics can be conceptualized as an alternating sequence of passive and active modes of control with probabilistic transitions between them. Moreover, these transitions are expected to display hysteresis due to decision-making inertia.

Generally, the passive and active modes of human control are governed by different mechanisms, posing challenges in developing efficient algorithms for their description and numerical simulation. The proposed model overcomes this problem by introducing the dynamical trap of the stimulus-response type, which has a complex structure. The dynamical trap region includes two subregions: the stagnation region and the hysteresis region. The model is based on the formalism of stochastic differential equations, capturing both probabilistic transitions between control suspension and activation as well as the internal dynamics of these modes within a unified framework. It reproduces the expected properties in control suspension and activation, probabilistic transitions between them, and hysteresis near the perception threshold. Additionally, in a limiting case, the model demonstrates the capability of mimicking a similar subject's behavior when (1) the active mode represents an open-loop implementation of locally planned actions and (2) the control activation occurs only when the stimulus intensity grows substantially and the risk of the subject losing the control over the system dynamics becomes essential.

Keywords: human control, intermittency, uncertainty, hysteresis, stochastic process, stochastic differential equations

Citation: *Computer Research and Modeling*, 2024, vol. 16, no. 1, pp. 79–87.

We appreciate the Sirius Mathematical Center for supporting Conference 028w: Transport Traffic on Networks, Sochi, April, 24–28, 2023, where this work was partly presented.

УДК: 004.94/519.21

Модель динамической ловушки для описания человеческого контроля в рамках «стимул – реакция»

И. А. Лубашевский^{1,2,a}, В. И. Лубашевский^{3,b}

¹Московский институт электроники и математики им. А. Н. Тихонова, Национальный исследовательский университет «Высшая школа экономики» (МИЭМ НИУ ВШЭ),

Россия, 123458, г. Москва, ул. Таллинская, д. 34

²Московский автомобильно-дорожный государственный технический университет (МАДИ),
Россия, 125319, Москва, Ленинградский проспект, д. 64

³Токийский международный университет,
Япония, Сайтама 350-1197, Кавагоэ-си, Матоба-кита, 1-13-1

E-mail: ^a ilubashevskii@hse.ru, ^b vlubashe@tiu.ac.jp

Получено 04.11.2023, после доработки — 18.11.2023.

Принято к публикации 21.11.2023.

В статье предлагается новая модель динамической ловушки типа «стимул – реакция», которая имитирует человеческий контроль динамических систем, где ограниченная рациональность человеческого сознания играет существенную роль. Детально рассматривается сценарий, в котором субъект модулирует контролируемую переменную в ответ на определенный стимул. В этом контексте ограниченная рациональность человеческого сознания проявляется в неопределенности восприятия стимула и последующих действий субъекта. Модель предполагает, что когда интенсивность стимула падает ниже (размытого) порога восприятия стимула, субъект приостанавливает управление и поддерживает контролируемую переменную вблизи нуля с точностью, определяемую неопределенностью ее управления. Когда интенсивность стимула превышает неопределенность восприятия и становится доступной человеческому сознанию, испытуемый активирует контроль. Тем самым, динамику системы можно представить как чередующуюся последовательность пассивного и активного режимов управления с вероятностными переходами между ними. Более того, ожидается, что эти переходы проявляют гистерезис из-за инерции принятия решений.

В общем случае пассивный и активный режимы базируются на различных механизмах, что является проблемой для создания эффективных алгоритмов их численного моделирования. Предлагаемая модель преодолевает эту проблему за счет введения динамической ловушки типа «стимул – реакция», имеющей сложную структуру. Область динамической ловушки включает две подобласти: область стагнации динамики системы и область гистерезиса. Модель основывается на формализме стохастических дифференциальных уравнений и описывает как вероятностные переходы между пассивным и активным режимами управления, так и внутреннюю динамику этих режимов в рамках единого представления. Предложенная модель воспроизводит ожидаемые свойства этих режимов управления, вероятностные переходы между ними и гистерезис вблизи порога восприятия. Кроме того, в предельном случае модель оказывается способной имитировать человеческий контроль, когда (1) активный режим представляет собой реализацию «разомкнутого» типа для локально запланированных действий и (2) активация контроля возникает только тогда, когда интенсивность стимула существенно возрастает и риск потери контроля системы становится существенным.

Ключевые слова: человеческий контроль, прерывистость, неопределенность, гистерезис, случайные процессы, стохастические дифференциальные уравнения

Мы благодарим Математический центр Сириуса за поддержку конференции 028w: Транспортный трафик на сетях, Сочи, 24–28 апреля 2023 г., где работа была частично представлена.

Introduction: concept of dynamical trap

The notion of a “dynamical trap” related to the mathematical description of human actions was initially introduced in [Lubashevsky, Gafiychuk, Demchuk, 1998]¹. It generalizes the stationary point — one of the key notions in the theory of dynamical systems — to a certain volumetric region with blurred boundaries, where each of its internal points can be treated as a stationary point of a given dynamical system. In other words, the dynamical trap region represents a multitude of neutral equilibrium states. In describing human perception from the first-person perspective, the dynamical trap becomes a basic element of its formalism due to the bounded capacity of human cognition [Lubashevsky, 2017]. The latter, in particular, manifests in the inability of humans to distinguish unambiguously between similar states of an observed object, which are characterized by different values of a certain quantitative parameter in reality.

The formalism of dynamical traps has demonstrated its efficiency in describing human intermittent control observed during experiments involving the balance of virtual pendulums [Zgonnikov et al., 2014] and driving virtual cars within the car-following scenario [Lubashevsky, Morimura, 2019]. Additionally, dynamical traps can give rise to a new type of nonequilibrium phase transitions, where the emergence of new phases does not stem from the appearance of new stationary points of the corresponding governing equations [Lubashevsky, 2016, for a review].

The purpose of this paper is to introduce and explore a special kind of dynamical trap that describes the subject’s response to some stimulus S through varying a certain system parameter η to be referred to as the “control variable”. In general, the concept of dynamical traps implies that when the stimulus intensity S becomes comparable to or falls below a certain value S_c quantifying the perception uncertainty, the subject probabilistically suspends control over the variable η . This probability increases as the ratio $\frac{S}{S_c}$ decreases [Lubashevsky, 2012]. As a result, the system dynamics can be conceived of as a sequence of two distinct alternating modes — active and passive — with probabilistic transitions between them. In this scenario, the mathematical description of the system dynamics governed by the subject’s behavior near perception threshold becomes “heterogeneous”, hindering the development of efficient numerical algorithms for its simulation. In the present paper we propose a novel model that can be categorized as the dynamical trap of stimulus – response type. This model, on the one hand, allows for the probabilistic transitions between the active and passive modes, on the other hand, it remains “homogeneous” in mathematical structure.

Dynamical trap of stimulus – response type

The following premises characterizing the subject’s perception and behavior underlie our constructions.

- The subject is able to change the control parameter η arbitrarily, including formal step-wise variations. The expected continuous dynamics of η reflects the subject’s perception of the stimulus S and their intentions in selecting action strategies.
- The perception of the stimulus S as well as the control over the variable η are characterized by uncertainty, which is quantified by the values S_c and η_c , respectively. The relationship between the uncertainties S_c and η_c is not arbitrary but stems from skill acquisition and reflects some optimal balance of attention allocation between perceiving the stimulus S and controlling the variable η .

¹ Another type of dynamical traps arises in the theory of Hamiltonian systems with complex dynamics [Zaslavsky, 1995; Zaslavsky, 2002]. In this context, a dynamical trap represents a region in the corresponding phase space with an unusually long residence time. However, such a trap fundamentally differs from the analyzed one in nature; its emergence is due to a delicate balance between several nonlinear properties of a Hamiltonian system. The dynamical trap we address in the present paper pertains to the general features of human perception.

- When recognizing the stimulus S becomes difficult, i.e., when $S \in \mathbb{Q}_S = \{S : S \lesssim S_c\}$, the subject may suspend the control over the variable η and keep the control variable η near the value $\eta = 0$. In other words, the value $\eta = 0$ is associated with the passive mode of system dynamics — the no-control phase of the subject's actions. In this case, possible time variations of the variable $\eta \in \mathbb{Q}_\eta = \{\eta : |\eta| \lesssim \eta_c\}$ are caused by uncontrollable random factors only.
- When the stimulus S is clearly recognized, namely, for its intensity $S \gg S_c$, the subject's response gives rise to the dependence of the control variable on the stimulus intensity that is approximated by the proportionality $\eta \propto S$. Additionally, the subject's control is assumed to be effective. Therefore, at the boundary of stimulus perception, i.e., for $S \sim S_c$, the corresponding value of the control variable $\eta_* = \eta|_{S \sim S_c}$ is expected to meet the inequality $\eta_* \gg \eta_c$.
- Such features of human behavior as delay in human reaction, decision-making inertia caused by uncertainty [Sautua, 2017], and the well-known hysteresis of judgments on sensory stimulus intensity [Stevens, 1957] suggest that the control variable η should exhibit different dependencies $\eta_\uparrow(S)$ and $\eta_\downarrow(S)$ on the stimulus intensity S when it increases or decreases gradually.

Summarizing the aforementioned items, Figure 1 illustrates the gist of the proposed dynamical trap model and exhibits the expected dependence of the control variable η on the stimulus intensity S .

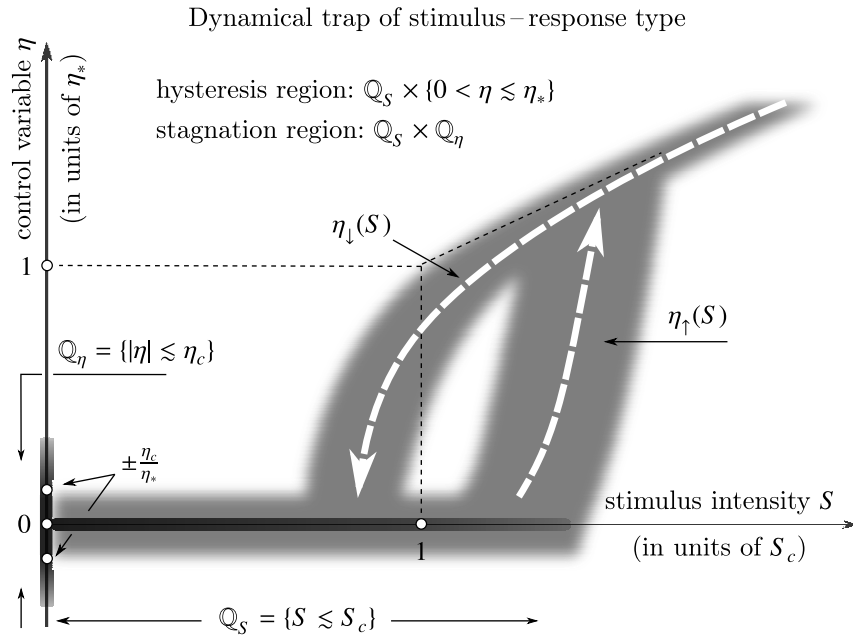


Figure 1. The proposed model for the dynamical trap of the stimulus-response type schematically shown on the $S\eta$ -plane. The dynamical trap region with blurred boundaries is the Cartesian product of its two constituent components, the region \mathbb{Q}_S and the region \mathbb{Q}_η , representing the uncertainties S_c and η_c in the subject's perception of the stimulus intensity S and the control over the variable η . The shaded region illustrates how the control variable η varies with the stimulus intensity S gradually decreasing, $\eta_\downarrow(S)$, and increasing, $\eta_\uparrow(S)$. The symbol η_* denotes the mean value $\eta|_{S \sim S_c}$ of the control variable, where the dynamical trap effect becomes pronounced in the dependence $\eta_\downarrow(S)$

As is clear from Fig. 1, the dynamical trap of the stimulus-response type is characterized by a complex structure. By definition, the dynamical trap region is the domain on the $S\eta$ -plane, where uncertainty in the subject's perception and control affects the system dynamics substantially. Its complexity manifests in that there can be singled out two subregions with distinct properties from this

dynamical trap region. One of them is the stagnation region, $\mathbb{Q}_S \times \mathbb{Q}_\eta$, where the subject active behavior is completely suspended. The other is the hysteresis region $\mathbb{Q}_S \times \{0 < \eta \lesssim \eta_*\}$, where uncertainty in the stimulus perception becomes substantial and the subject's response gives rise to the hysteresis in the dependence of the control variable η on the stimulus intensity S . This hysteresis enables us to differentiate between the suspension of the subject's control over the variable η and the initiation of control, treating them as distinct phenomena.

Model and results of numerical simulation

The proposed model for the analyzed type of dynamical trap may be treated as a certain generalization of the models employed in studying nonequilibrium phase transition in chains of oscillators with dynamical traps [Lubashevsky, 2016] as well as in simulating the balancing of overdamped pendulums [Zgonnikov et al., 2014] and the car driving within the car-following scenario [Lubashevsky, Morimura, 2019].

The gist of the proposed model is the following stochastic differential equation governing the subject's response to the stimulus intensity S :

$$d\eta = \left[\Omega(\eta, S) \left(\frac{\eta_*}{S_c} \right) S - \eta \right] \frac{dt}{\tau} + \frac{1}{\sqrt{\tau}} \eta_c dW, \quad (1a)$$

where the time scale τ quantifies the subject's reaction delay, dW is the infinitesimal increment of Wiener process meeting the condition $\langle dW^2 \rangle = dt$, the dynamical trap function $\Omega(\eta, S)$ is specified as

$$\Omega(\eta, S) = \frac{\vartheta^2}{1 + \vartheta^2} \quad \text{for} \quad \vartheta = g \frac{\eta}{\eta_*} \left(\frac{S}{S_c} \right)^\beta \quad (1b)$$

and the constants $g \sim 1$ and $\beta > 0$ are its parameters.

To justify that model (1) does describe the expected subject's response to the stimulus S represented in Fig. 1, Eq. (1a) has been solved numerically using the order 1.0 strong stochastic Runge – Kutta algorithm SRI2 elaborated in [Rößler, 2010] and implemented in the Python library “sdeint” 0.3.0. In the numerical simulation, smooth variations in the stimulus intensity were modeled as

$$S(t) = 4S_c \left[\cos \left(\frac{t}{T} \right) \right]^2 \quad \text{for} \quad T = 100\tau, \quad (2)$$

the integration time step dt , the parameter g , and the ratio $\frac{\eta_c}{\eta_*}$ were set equal to $dt = 0.001\tau$, $g = 2$, and $\frac{\eta_c}{\eta_*} = 0.1$. The total integration time was $10^4\tau$.

The results obtained are shown in Fig. 2 for several values of the exponent $\beta = 0, 0.5, 1, 2$. As can be seen, for $\beta = 1, 2$ the $\eta(S)$ -dependence is similar to the expected one (Fig. 1) and admits the interpretation suggesting the subject either to suspend or activate the control over the variable η depending on decrease or increase in stimulus intensity within a certain neighborhood of the \mathbb{Q}_S -boundary. As it should be, the control suspension and activation form the hysteresis. By comparing these results with those for $\beta = 0, 0.5$, we can draw a conclusion that the localization of the control suspension and activation near the \mathbb{Q}_S -boundary is due to the essential dependence (Eq. 1b) of the dynamical trap function $\Omega(\eta, S)$ on the stimulus intensity S . Indeed, when the deviation of the stimulus intensity S from the critical value S_c becomes remarkable, $|S - S_c| \lesssim S_c$, the induced decrease or increase in the cumulative argument ϑ of the trap function $\Omega(\vartheta)$ enhances either the system's capture by the dynamical trap or the system's escape from it, respectively.

For $\beta = 0$ and 0.5 , especially for $\beta = 0$, a distinct interpretation is necessary for the found $\eta(S)$ -dependence because the region of the η -control activation essentially exceeds in size the

region \mathbb{Q}_S which represents uncertainty in the subject's perception of the stimulus S . Broadly speaking, the findings for $\beta = 0$ suggest that the subject suspends the η -control only when the variable η approaches the region \mathbb{Q}_η due to $S \rightarrow 0$. This is demonstrated by a weakly pronounced kink in the dependence $\eta_\downarrow(S)$; for $\beta = 2$ this kink is much more pronounced. The given behavior of the $\eta_\downarrow(S)$ -dependence meets the concept of the active phase as the implementation of open-loop control when locally planned actions come close to their completion and the desired state $S = 0$ is actually achieved. In turn, the η -control is activated only when the stimulus intensity S becomes comparable with a value $S_l \gg S_c$, which indicates that the risk of the subject losing the control over the system dynamics is essential. A more detailed discussion of this issue can be found in [Zgonnikov et al., 2014; Lubashevskiy, Lubashevsky, 2023].

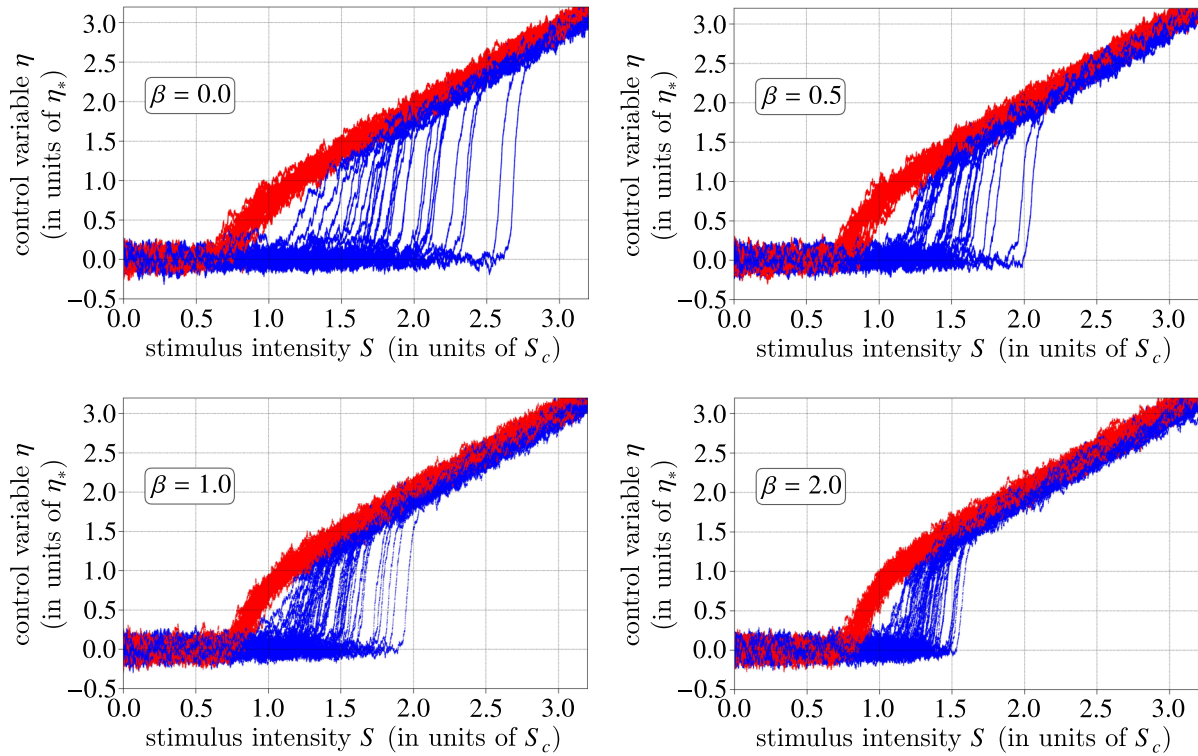


Figure 2. The η -dynamics governed by the proposed model for the dynamical trap of the stimulus–response type which has been obtained by numerical integration of model (1) for several values of the exponent β . Smooth variations of the stimulus intensity S were imitated using Exp. (2). Data points related to the decreasing and increasing fragments of $S(t)$ are shown in red and blue, respectively. The other model parameters were set $g = 2$, $\frac{\eta_c}{\eta_*} = 0.1$. The details of numerical integration are presented in the text

To clarify the identified characteristics of the η -dynamics, let us refer to Figs. 3 and 4. Figure 3 illustrates the structure of the dynamical trap region specified by model (1) where the contribution of random sources is mimicked by the condition that variations in the control variable η are constrained from below by the uncertainty η_c in the η -control, such that $\eta \geq \eta_c$. In the left plot of Fig. 3, the solid curves represent the corresponding nullclines, which are the points where the right-hand side of Eq. (1a) (excluding random sources) equals zero. In this case, as illustrated in Fig. 4, the dependence $\eta_\downarrow(S)$ has to undergo a sharp jump from about η_* to η_c . In turn, the dependence $\eta_\uparrow(S)$ has to show a step-wise increase when Line 2 intersects with the lower branch of the nullclines.

Figure 3 (right plot) exhibits the rate, $\frac{d\eta}{dt}$, of the control variable variations along Line 1 ($\frac{\eta}{\eta_*} = \frac{S}{S_c}$) for the stimulus intensity $0 < S < S_c$ and along Line 2 ($\eta = \eta_c$) for the stimulus intensity $S > S_c$. The system motion along Line 2 mimics random dynamics of the variable η when the subject's response to

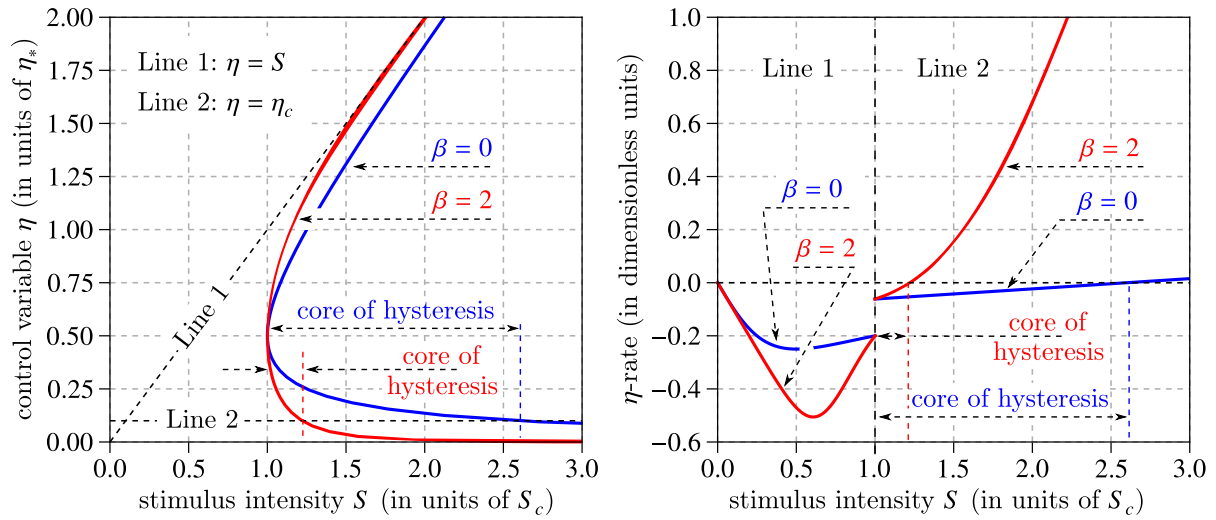


Figure 3. Characteristic structure of the dynamical trap region specified by model (1). On the left plot, the solid curves are nullclines of Eq. (1a) with the random sources ignored, shown for two values of the parameter β : 0 (blue) and 2 (red). On the right plot, these curves exhibit the corresponding regular rate of the η -variations along Line 1 for $0 \leq S \leq 1$ and along Line 2 for $1 \leq S \leq 3$. The values of the stimulus intensity S and the control variable η are given in units of S_c (uncertainty in the S -stimulus perception) and η_* (the value of the variable $\eta|_{S=S_c}$ when the dynamical trap effect is ignored), respectively. In plotting the η -variation rate, time t is measured in the units of τ (the subject's reaction time). The other system parameters were $g = 2$ and $\eta_c = 0.1\eta_*$ (uncertainty in the η -variable control)

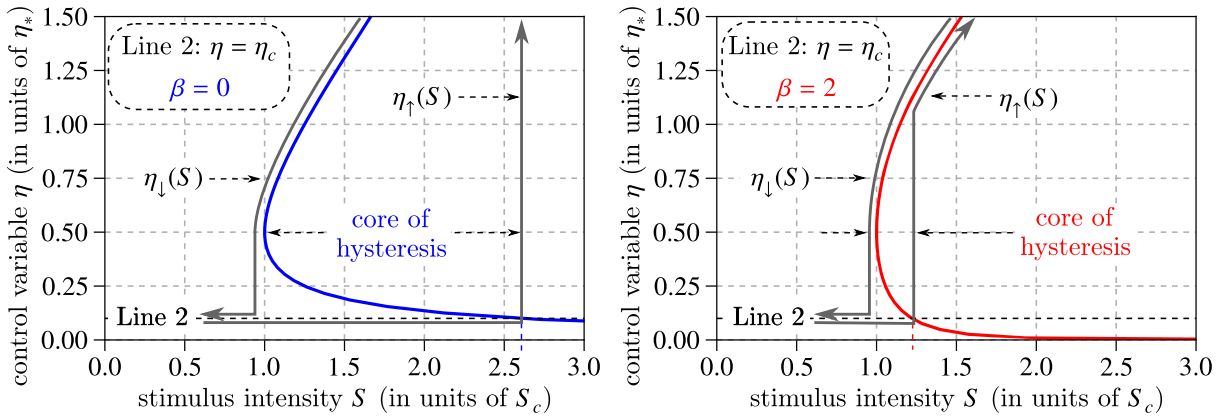


Figure 4. The dependence of the control variable η on the stimulus intensity S for its gradual decrease, $\eta_\downarrow(S)$, and increase, $\eta_\uparrow(S)$, meeting the structure of the dynamical trap region shown in Fig. 3. The cumulative dependence of the control variable η on the up-and-down variations in the stimulus intensity S forms the hysteresis in the suspension-activation of the subject's control. The explanations for the other notions can be found in the caption for Fig. 3

the stimulus S has been suspended, and the variable η is kept near zero within the uncertainty η_c of the subject's control. As can be seen, for $\beta = 2$, model (1) effectively captures the pronounced response of the subject to the stimulus S near the perception threshold. Specifically, the region of expected hysteresis, resulting from the up-and-down variations in stimulus intensity, turns out to be relatively narrow (refer also to Fig. 2). For $\beta = 0$, first, the hysteresis region, within the accepted regular-type approximation, is notably wide. Second, along Line 2, the rate $\frac{d\eta}{dt}$ of the control variable η is very low. Consequently, we can anticipate that mainly random fluctuations in the variable η govern the activation of the subject's control, which is justified by numerical simulations (see Fig. 2).

Conclusion

In the present paper, we have introduced a new model for the dynamical trap of the stimulus – response type that imitates human control over dynamic systems when the bounded capacity of human cognition plays a crucial role. To elucidate the gist of the model, we have focused on scenarios where the subject modulates a control variable η in response to a certain stimulus S with intensity S . In this case, the bounded capacity of human cognition manifests in the uncertainty of the stimulus perception quantified by a scale S_c as well as the uncertainty of the η -control quantified by a scale η_c .

When the stimulus intensity falls below the blurred threshold of S -perception, i. e., $S \lesssim S_c$, the subject is expected to suspend control and maintain the variable η near zero within the accuracy η_c accessible to their cognition. The subject's actions under suppressed control are commonly referred to as the passive mode of human control. As the stimulus intensity increases above the critical value S_c and becomes accessible to human cognition, the subject activates η -control, and the corresponding actions are typically termed the active mode of human control. As a result, the dynamics of such a system can be conceptualized as an alternating sequence of passive and active modes of human control with probabilistic transitions between them. Moreover, these transitions are expected to display hysteresis in response to up-and-down variations in stimulus intensity due to decision-making inertia — an inherent feature of human behavior.

The stimulus intensity, modeled as a predetermined function of time $S(t)$ with smooth up-and-down variations, enables us to analyze the suspension and activation of the subject's η -control on their own. Generally, the passive and active modes of human control are governed by different mechanisms, posing challenges in developing efficient algorithms for their description and numerical simulation.

The proposed model (1) has overcome this problem by introducing the dynamical trap of the stimulus-response type, which has a complex structure. The dynamical trap region on the $S\eta$ -plane consists of two subregions: the stagnation region of the subject's control suspension

$$\{S \lesssim S_c\} \times \{|\eta| \lesssim \eta_c\},$$

and the hysteresis region

$$\{S \lesssim S_c\} \times \{\eta_c \lesssim \eta \lesssim \eta_*\},$$

where the processes of “catching” the system by the trap and the system ‘escaping’ from the trap occur. The subject is assumed to have acquired proficiency in η -control, and thus, the uncertainty η_c in this control is significantly lower than the value η_* characterizing the boundary of the active mode when the stimulus intensity approaches its perception threshold, $S \sim S_c$. The proposed model deals with nonlinear stochastic dynamics of the control variable η , covering both the probabilistic transitions between passive and active modes as well as the internal dynamics of these modes within a unified framework. The model reproduces the control suspension and activation, probabilistic transitions between them with their hysteresis near the perception threshold.

Additionally, in the limiting case of $\beta = 0$, model (1) demonstrates the capability of mimicking similar behavior of the subject when the active mode represents an open-loop implementation of locally planned actions in controlling the system dynamics. This implementation is terminated when the variable η approaches the boundary η_c of its control. The subject's control is activated only when the stimulus intensity grows enough, $S \gg S_c$, and the risk of the subject losing the control over the system dynamics becomes essential.

The proposed concept of the dynamical trap of the stimulus-response type opens a gate towards developing sophisticated models of human control, wherein both the uncertainty of human perception and the conscious evaluation of active behavior necessity determine human actions.

References

- Lubashevsky I.* Dynamical traps caused by fuzzy rationality as a new emergence mechanism // *Advances in Complex Systems*. — 2012. — Vol. 15, No. 8. — P. 1250045. — 25 p.
- Lubashevsky I.* Human fuzzy rationality as a novel mechanism of emergent phenomena // *Handbook of applications of chaos theory* / C. H. Skiadas, C. Skiadas (ed.). — London: CRC Press, Taylor & Francis Group, 2016. — P. 827–878.
- Lubashevsky I.* *Physics of the human mind*. — Cham: Springer International Publishing AG, 2017.
- Lubashevsky I. A., Gafiychuk V. V., Demchuk A. V.* Anomalous relaxation oscillations due to dynamical traps // *Physica A: Statistical Mechanics and its Applications*. — 1998. — Vol. 255, No. 3–4. — P. 406–414.
- Lubashevsky I., Morimura K.* Physics of mind and car-following problem // *Complex dynamics of traffic management, encyclopedia of complexity and systems science series* / B. S. Kerner (ed.). — New York, NY: Springer Science + Business Media, LLC, 2019. — P. 559–592.
- Lubashevskiy V., Lubashevsky I.* Self-organized criticality and cognitive control reasoned by effort minimization // *Systems*. — 2023. — Vol. 11, No. 6. — P. 271.
- Rößler A.* Runge–Kutta methods for the strong approximation of solutions of stochastic differential equations // *SIAM Journal on Numerical Analysis*. — 2010. — Vol. 48, No. 3. — P. 922–952.
- Sautua S. I.* Does uncertainty cause inertia in decision making? An experimental study of the role of regret aversion and indecisiveness // *Journal of Economic Behavior & Organization*. — 2017. — Vol. 136. — P. 1–14.
- Stevens S. S.* On the psychophysical law // *Psychological Review*. — 1957. — Vol. 64, No. 3. — P. 153–181.
- Zaslavsky G. M.* Dynamical traps // *Physica D: Nonlinear Phenomena*. — 2002. — Vol. 168–169. — P. 292–304.
- Zaslavsky G. M.* From Hamiltonian chaos to Maxwell’s demon // *Chaos*. — 1995. — Vol. 5, No. 4. — P. 653–661.
- Zgonnikov A., Lubashevsky I., Kanemoto S. et al.* To react or not to react? Intrinsic stochasticity of human control in virtual stick balancing // *Journal of The Royal Society Interface*. — 2014. — Vol. 11, No. 99. — P. 20140636. — 13 p.