Ки&М

MODELING OF TRAFFIC IN INTELLIGENT TRANSPORTATION SYSTEMS

Methods for resolving the Braess paradox in the presence of autonomous vehicles

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Roads are a shared resource which can be used either by drivers and autonomous vehicles. Since the total number of vehicles increases annually, each considered vehicle spends more time in traffic jams, and thus the total travel time prolongs. The main purpose while planning the road system is to reduce the time spent on traveling. The optimization of transportation networks is a current goal, thus the formation of traffic flows by creating certain ligaments of the roads is of high importance. The Braess paradox states the existence of a network where the construction of a new edge leads to the increase of traveling time. The objective of this paper is to propose various solutions to the Braess paradox in the presence of autonomous vehicles. One of the methods of solving transportation topology problems is to introduce artificial restrictions on traffic. As an example of such restrictions, this article considers designated lanes which are available only for a certain type of vehicles. Designated lanes have their own location in the network and operating conditions. This article observes the most common two-roads traffic situations, analyzes them using analytical and numerical methods and presents the model of optimal traffic flow distribution, which considers different ways of lanes designation on isolated transportation networks. It was found that the modeling of designated lanes eliminates Braess' paradox and optimizes the total traveling time. The solutions were shown on artificial networks and on the real-life example. A modeling algorithm for Braess network was proposed and its correctness was verified using the real-life example.

Keywords: Braess paradox, mathematical modeling, autonomous vehicles

МОДЕЛИРОВАНИЕ ТРАФИКА В ИНТЕЛЛЕКТУАЛЬНЫХ ТРАНСПОРТНЫХ СИСТЕМАХ

Методы решения парадокса Браесса
на транспортной сети с автономным транспортом

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Дороги — ресурс, который может использоваться как водителями, так и автономными транспортными средствами. Ежегодно количество транспортных средств увеличивается, из-за чего каждое отдельно взятое транспортное средство тратит всё больше времени в пробках, тем самым увеличивая суммарные временные затраты. При планировании новой дороги ключевой задачей становится сокращение времени в пути. Оптимизация транспортных сетей в настоящее время часто происходит с помощью добавления новых связующих дорог между высоконагруженными частями трасс. Парадокс Браесса заключается в том, что построение нового ребра дорожной сети приводит к увеличению времени в пути для каждого транспортного средства в сети. Целью данной статьи является предложение различных разрешений парадокса Браесса при рассмотрении автономных транспортных средств в качестве участников дорожного движения. Один из вариантов топологического решения транспортной задачи — использование искусственных ограничителей трафика. Как пример таких ограничителей статья рассматривает введение выделенных полос, доступных только для определенных видов транспорта. Выделенные полосы занимают особое место в транспортной сети и могут обслуживать поток по-разному. В данной статье рассмотрены наиболее часто встречающиеся случаи распределения трафика на сети из двух дорог, приведены аналитический и численный методы оптимизации модели и представлена модель оптимального распределения трафика, которая рассматривает различные варианты выделения полос на изолированной транспортной сети. В результате проведенных исследований было обнаружено, что введение выделенных полос решает парадокс Браесса и приводит к уменьшению общего времени в пути. Решения приведены как для искусственно смоделированной сети, так и на реальных примерах. В статье представлен алгоритм моделирования трафика на браессовской сети и приведено обоснование его корректности на реальном примере.

Ключевые слова: парадокс Браесса, математическое моделирование, автономные транспортные средства

Introduction

The problems of transportation networks have recently become an urgent issue, since the number of vehicles increased and there is an objective of optimal planning of road systems. The problem is particularly acute in large cities with lots of cars and public transport. According to WHO, half of the modern world's population lives in cities. According to the UN forecast, by 2050, about 85% of the world's population will prefer an urban lifestyle [Thorns, 2017]. Since the total number of vehicles increases annually, the vehicle considered spends more time in traffic jams, and thus the total travel time prolongs. The main purpose while planning the route is to reduce the time spent on traveling. Since possible routes require different costs and time depending on the type of the vehicle, this article presents the model of optimal traffic flow distribution. This model considers different ways of lanes designation on isolated transportation networks and proposes an algorithm for implementing optimal traffic flow distribution over the transportation network.

Roads are a shared resource which can be used either by drivers and autonomous vehicles. The optimization of transportation networks is a current goal, thus the formation of traffic flows by creating certain ligaments of the roads is of high importance.

However, some problems occur in planning transportation systems. The main purpose is to reduce travel costs on transport networks. Therefore, it is important to consider and optimize the system as a whole and not by parts. Wardrop showed that the total traveling time on all routes used is not greater than the total traveling time of any vehicle on unused routes, since drivers tend to choose the route according to the total traveling time criterion [Wardrop, 1952]. Hence, the driver would not choose the route with less traffic if the total traveling time is greater than on a route with more traffic. The Nesterov – de Palma model [Nesterov, 2003] is based on the concepts of equilibrium proposed in [Wardrop, 1952]. Each driver tends to choose the most optimal trajectory, no matter what other drivers choose. This principle is similar to Nash equilibrium and is called Wardrop equilibrium [Wardrop, 1952]. However, this equilibrium is not necessarily Pareto-optimal [Wardrop, 1952]. Thus, the search for Wardrop equilibrium can be represented as a game model [Fisk, 1984]. This leads to the occurrence of the so-called Braess paradox on transport networks. This paradox states the existence of a network where the addition of the new edge leads to an increase in traveling time [Braess, 2005]. The search for such inefficient edges is an NP-hard problem [Roughgarden, 2006]. On the other hand, Miltaich indicated that the network of parallel edges is the only network in which Braess' paradox is impossible [Milchtaich, 2006]. Valiant and Roughgarden in 2006 showed that the Braess paradox appearance probability in random graphs with an infinite number of nodes converges to 1 [Valiant, 2010].

One of the methods of solving transportation topology problems is to introduce artificial restrictions on traffic. As an example of such restrictions consider designated lanes which are available only for a certain type of vehicles. Designated lanes have their own location in the network and operating conditions [Kieslingm 2006]. Nevertheless, designating lanes for autonomous vehicles may lead to an increased capacity of roads which are available for all types of vehicles and, as a consequence, to a growth in individual and total costs [Rezende, 2014]. It deals with the Downs – Thomson paradox demonstrated in 1962.

Problem Setting

In general, the real network is represented by a set of roads and road intersections. There are two main different representation methods for urban street networks, called the primal approach and the dual approach. The primal representation is a natural and intuitional approach, which takes just the road segments as edges, and intersections or the ends of roads as vertices for an urban street network.

We consider a road network as a weighted graph. In what follows, we assume that the travel time cost t_i on the road segment i can be represented by the linear function

$$t_i = \tau_i + b_i n_i,$$

where
τ_i is travel time at the highest possible speed and in the absence of other vehicles on the road segment i,
b_i is inversed coefficient of capacity of the road segment i. This coefficient shows that the more traffic is on the road, the longer is the travel time for each considered vehicle, and
n_i is the number of vehicles on the road segment i.

Generally, in Beckmann-based traffic models the dependence between the fleet-size and the total delay is not linear. However, the **NP**-complexity of general problem setting implies the need for heuristics, which can be developed using linear regression methods. In this article, we define two types of vehicles: autonomous vehicles and common ones. As far as the designated lanes are concerned, the travel costs for each of them may differ.

Let $t_{i_a} = \tau_{i_a} + b_{i_a} n_{i_a}$ be the travel time of autonomous vehicles, and $t_{i_c} = \tau_{i_c} + b_{i_c} n_{i_c}$ be the travel time of common vehicles.

After splitting the fleet of vehicles into autonomous and common ones, we get:

$$b_{i_a} = \frac{b_i}{\lambda_i},$$

$$b_{i_c} = \frac{b_i \chi}{1 - \lambda_i},$$

where
$\chi > 1$, χ is the efficiency (inefficiency) coefficient and
$\lambda_i \in [0, 1]$ is the quotient of the number of designated lanes to the total number of lanes.

The coefficient of efficiency shows the rate of change in the speed of autonomous vehicles from the fleet-size, which depends on the fleet homogeneity [Daganzo, 2002; Kerner, 2006].

Each member of the fleet of common vehicles chooses a path with the minimal travel time cost. It moves towards the user equilibrium in the traffic network. In what follows the total time cost in the network is referred as to TC. User equilibrium is not necessary Pareto-optimal. The problem of minimization of TC leads us to the system optimum [Wardrop, 1952; Beckmann, 1955; Valiant, 2010] in the network considered. In this article a part of a road network on which Braess's paradox occurs is considered. As a solution to the paradox the idea of creating designated lanes for certain types of transport (e.g. autonomous vehicles) is proposed.

Analytical solutions

Consider a simple transportation subnetwork of two parallel roads (Fig. 1). Let

$$t_{1_a} = \tau_1 + b_a m_1,$$
$$t_{2_a} = \tau_2 + d_a m_2,$$
$$t_{1_c} = \tau_1 + b_c n_1,$$
$$t_{2_c} = \tau_2 + d_c n_2,$$

where a denotes coefficients of autonomous vehicles and c denotes coefficients of common vehicles and $m_i > 0$ and $n_i > 0$ is the total number of vehicles on the road segment i, $i \in \{1, 2\}$.
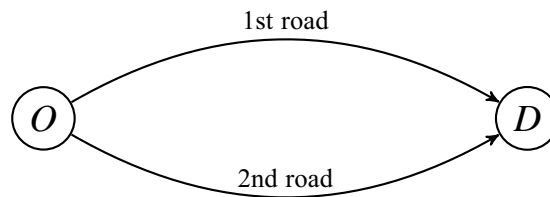
Figure 1. The main example of a two-roads network. O is a source and D is a sink

Let $m_1 + m_2 = m > 0$ be the total number of autonomous vehicles and $n_1 + n_2 = n > 0$ be the total number of common vehicles.

We aim to find the user equilibrium under the assumption that there exist no unused roads. This implies that there is a positive number of vehicles on each route ($m_1 + n_1 > 0$, $m_2 + n_2 > 0$). According to Wardrop equilibrium, the total traveling time on route 1 is equal to the total traveling time on route 2, then

$$\tau_1 + b_c n_1 = \tau_2 + d_c n_2.$$

Hence,

$$n_1 = \frac{\tau_2 - \tau_1 + d_c n}{d_c + b_c},$$

$$n_2 = \frac{\tau_1 - \tau_2 + b_c n}{d_c + b_c}.$$

Since $n_i \in (0, n)$, we get $-d_c n < \tau_2 - \tau_1 < b_c n$.
Hence,

$$TC_c = \frac{\tau_1 d_c + b_c \tau_2 + b_c d_c n}{d_c + b_c} n \qquad (1)$$

is the total time cost for common vehicles.

To find the optimal total time, the following must hold:

$$TC_a \xrightarrow[m_1, m_2]{} \min.$$

The minimum of this function is reached if

$$m_1^0 = \frac{2md_a + \tau_2 - \tau_1}{2(d_a + b_a)},$$

$$m_2^0 = \frac{2mb_a + \tau_1 - \tau_2}{2(d_a + b_a)}.$$

The solutions satisfy the conditions for equilibrium if $m_1^0 \in (0, m)$ and $m_2^0 \in (0, m)$.
Thus,

$$-d_a m < \frac{\tau_2 - \tau_1}{2} < b_a m.$$

The system optimum for autonomous vehicles is the solution of the optimization problem

$$TC_a^0 = \frac{2md_a + \tau_2 - \tau_1}{2(d_a + b_a)} \tau_1 + b_a \frac{2md_a + \tau_2 - \tau_1}{2(d_a + b_a)} + \frac{2mb_a + \tau_1 - \tau_2}{2(d_a + b_a)} \tau_2 + d_a \frac{2mb_a + \tau_1 - \tau_2}{2(d_a + b_a)}.$$

If $m_1^0 \leqslant 0$, then all autonomous vehicles should move along the second road, hence

$$TC_a^0 = \frac{2md_a - \tau_1 + \tau_2}{2(d_a + b_a)} \tau_2 + b_a \frac{2md_a - \tau_1 + \tau_2}{2(d_a + b_a)}.$$

If $m_2^0 \leqslant 0$, then all autonomous vehicles move along the first road, hence

$$TC_a^0 = \frac{2mb_a + \tau_1 - \tau_2}{2(d_a + b_a)}\tau_2 + d_a\frac{2mb_a - \tau_1 + \tau_2}{2(d_a + b_a)}.$$

Thus, it is possible to optimize the total time costs by setting the optimal vehicles distribution along the traffic flows. Consider four possible scenarios of arrangement of designated lanes on two roads:

1. There are designated lanes for autonomous lanes only on one road.

2. One road is available only for autonomous vehicles, while the second one has a designated lane and lanes for common vehicles.

3. One road has a fixed coefficient for a designated lane and the second road is proposed for modeling.

4. Both roads have designated lanes.

These options are the main cases of distribution of designated lanes over two roads.

Option 1

Suppose there exists a network (Fig. 2). If $\lambda_1 \in (0, 1)$, $\lambda_2 = 0$ (road 2 has no designated lanes), then all autonomous vehicles move along the road 1, thus

$$TC = TC_0 + TC_a = \frac{\tau_1 d_c + b_c\tau_2 + d_c b_c n}{d_c + b_c}n + (\tau_1 + b_a m)\,m,$$

$$d_c = d \quad (\text{since } \lambda_2 \in (0, 1)),$$

$$b_c = \frac{b\chi}{1 - \lambda_1}; \quad b_a = \frac{b}{\lambda_1} \quad (\text{since } \lambda_1 \in (0, 1)),$$

$$TC = (n + m)\,\tau_1 + \frac{bm^2}{\lambda_1} + \frac{bn\chi\,(\tau_2 + dn - \tau_1)}{d\,(1 - \lambda_1) + b\chi}. \tag{2}$$

The minimum is

$$\lambda_1 = \frac{m(d + b\chi)}{\sqrt{bn\chi(\tau_2 + dn - \tau_1)} + dm}. \tag{3}$$

Substitute (3) into (2):

$$TC = (n + m)\tau_1 + \frac{bm(\sqrt{bn\chi\,(\tau_2 + dn - \tau_1)} + dm)}{(d + b\chi)} + \frac{bn\chi\,(\tau_2 + dn - \tau_1)}{d\left(1 - \dfrac{m(d + b\chi)}{\sqrt{bn\chi\,(\tau_2 + dn - \tau_1)} + dm}\right) + b\chi} \tag{4}$$
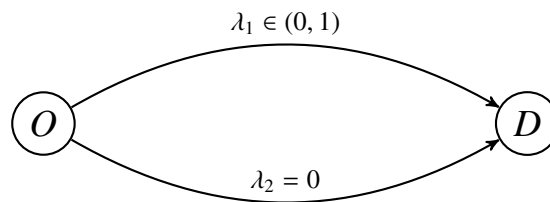
is the optimal value of TC.

Figure 2. A network of two lines where the first lane has a λ_1 autonomous vehicles capacity and the second one is closed for them

λ is the minimum of the function (1) if $B = bn\chi(\tau_2 + dn - \tau_1) > 0$ and $\lambda \in (0,1)$. If $\lambda > \dfrac{m(d + b\chi)}{\sqrt{bn\chi(\tau_2 + dn - \tau_1)} + dm}$, then TC increases and if $\lambda < \dfrac{m(d + b\chi)}{\sqrt{bn\chi(\tau_2 + dn - \tau_1)} + dm}$, TC decreases.

Option 2

Assume that the second road is available only for autonomous vehicles (Fig.3), so $\lambda_2 = 1$ and $\lambda_1 \in (0,1)$. This implies:

$$d_a = d; \quad b_c = \frac{b\chi}{1 - \lambda}; \quad b_a = \frac{b}{\lambda}.$$

Suppose $\tau_2 - \tau_1 = k$.

Thus, the minimum is achieved when

$$m_1^0 = \frac{2md_a + k}{2(d_1 + b_a)}; \quad m_2^0 = \frac{2mb_a - k}{2(d_1 + b_a)}.$$

So, the system optimum of TC for common vehicles is

$$TC = \frac{m(\tau_1 d\lambda + \tau_2 b + bdm)}{b + d\lambda} + \left(\tau_1 + \frac{b\chi}{1 - \lambda}n\right)n - \frac{k^2 \cdot \lambda}{4(b + d\lambda)}. \tag{5}$$

Hence,

$$\lambda = \frac{-2\sqrt{(\chi n^2)(b + d)^2(2dm + k)^2}}{-4d^2\chi n^2 + 4d^2 m^2 + 4dkm + k^2} + +\frac{4bdxn^2 + 4d^2 m^2 + 4dkm + k^2}{-4d^2\chi n^2 + 4d^2 m^2 + 4dkm + k^2}. \tag{6}$$

Substitute (6) into (5):

$$TC = \tau_1 n + \frac{b\chi n^2\left(-(k + 2dm)^2 + 4d^2\chi n^2\right)}{4d(b + d)xn^2 - 2\sqrt{(b + d)^2 x(k + 2dm)^2 n^2}} +$$
$$+ \frac{k^2\left(-(k + 2dm)^2 - 4bd\chi n^2\right)}{4(b + d)(k + 2dm)^2 - 8d\sqrt{(b + d)^2\chi(k + 2dm)^2 n^2}} +$$
$$+ \frac{2m(k + 2dm)^2(\tau_1 d + b(\tau_2 + dm))}{2(b + d)(k + 2dm)^2 - 4d\sqrt{(b + d)^2\chi(k + 2dm)^2 n^2}} +$$
$$+ \frac{-8bd^2\chi m(-\tau_1 + \tau_2 + dm)n^2}{2(b + d)(k + 2dm)^2 - 4d\sqrt{(b + d)^2\chi(k + 2dm)^2 n^2}} +$$
$$+ \frac{\left(k^2 - 4\tau_1 dm\right)\sqrt{(b + d)^2\chi(k + 2dm)^2 n^2}}{2(b + d)(k + 2dm)^2 - 4d\sqrt{(b + d)^2\chi(k + 2dm)^2 n^2}}. \tag{7}$$

Therefore, since the Pareto-optimal value of TC has been found, the total traveling time may be evaluated. The total optimal traveling time for this case may be evaluated using (7).
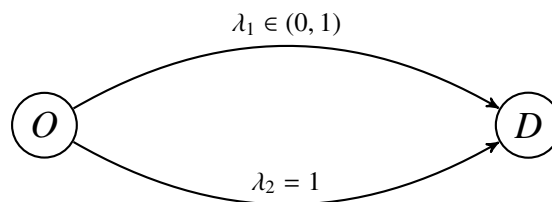

Figure 3. A network of two lines where the first lane has a λ_1 autonomous vehicles capacity and the second one is opened only for them

Option 3

Consider the case where the road 2 has a fixed coefficient of designated lanes and $\lambda_1 \in (0, 1)$ (Fig. 4). This implies

$$d_a = \frac{d}{\lambda_2}; \qquad b_c = \frac{b\chi}{1 - \lambda_1}; \qquad b_a = \frac{b}{\lambda_1}; \qquad d_c = \frac{b\chi}{1 - \lambda_2},$$

$$TC_c = \frac{\tau_1 d\chi}{1 - \lambda_2} + \frac{\tau_2 b\chi}{1 - \lambda_1} + \frac{\dfrac{bnd\chi^2}{(1 - \lambda_1)(1 - \lambda_2)n}}{\dfrac{d\chi}{1 - \lambda_2} + \dfrac{b\chi}{1 - \lambda_1}}n,$$

$$TC_a^0 = \frac{m}{\left(\dfrac{d}{\lambda_2} + \dfrac{b}{\lambda_1}\right)}\left[\tau_1\frac{d}{\lambda_2} + \tau_2\frac{b}{\lambda_1} + \frac{db}{\lambda_2\lambda_1}m\right] - \frac{(\tau_2 - \tau_1)^2}{4\left(\dfrac{b}{\lambda_1} + \dfrac{d}{\lambda_2}\right)}.$$

The minimum of the $TC = TC_c + TC_a$ is reached at the point

$$\lambda_1 = \frac{\sqrt{C}\left(\dfrac{b}{d}(1 - \lambda_2) + 1\right) - \sqrt{B}\dfrac{b\lambda_2}{d}}{\sqrt{B} + \sqrt{C}}, \qquad (8)$$

where

$$A = \left(\tau_1 n + \frac{m\left(4\tau_1 d - (\tau_2 - \tau_1)^2 \lambda_2\right)}{4d}\right),$$

$$B = n\left(\frac{b(1 - \lambda_2)}{d}(\tau_2 - \tau_1) + b\chi n\right),$$

$$C = \frac{mb}{d}\left((\tau_2\lambda_2 + dm) - \frac{\lambda_2\left(4\tau_1 d - (\tau_2 - \tau_1)^2 \lambda_2\right)}{4d}\right).$$

However, the constraints imposed on $\lambda \in (0, 1)$ can provide the boundary solutions described in Options 1 and 2.
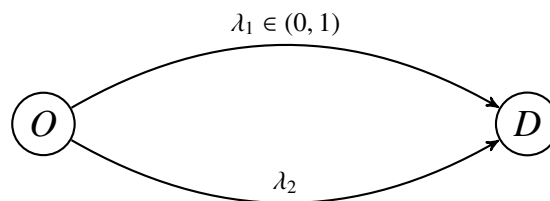

Figure 4. A network of two lines where the first lane has a λ_1 autonomous vehicles capacity and autonomous vehicles capacity coefficient for the second road λ_2 is fixed

Option 4

Consider the case where the autonomous vehicles can move along both roads and the coefficient for each road is not fixed, $\lambda_1 \in (0, 1)$, $\lambda_2 \in (0, 1)$ (Fig. 5). In this case we have to find the extrema of the function of two variables: λ_1, λ_2. This leads us to the search for two separate extrema as functions
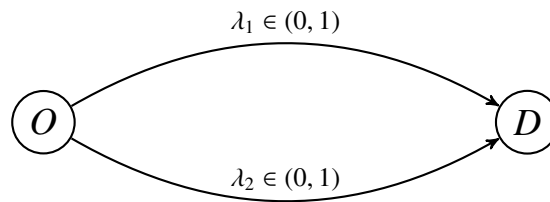
Figure 5. A network of two lines where λ_i is an ith lane's autonomous vehicles capacity coefficient

of λ_i from λ_j, $i, j = 1, 2$, based on the results obtained in Option 3. However, the boundary solutions have more complex structure in this case.

$$
\begin{aligned}
TC &= TC_c + TC_a^0 = \\
&= \frac{\tau_1 d\,(1 - \lambda_1) - b\tau_2\,(1 - \lambda_2) + bd\chi n}{b\,(1 - \lambda_2) + d\,(1 - \lambda_1)} n + \frac{m}{(d\lambda_1 + b\lambda_2)} \times \\
&\times \left([\tau_1 d\lambda_1 + \tau_2 b\lambda_2 + dbm] - \frac{1}{4}\,(\tau_1 - \tau_2)^2\,\lambda_1\lambda_2 \right).
\end{aligned} \tag{9}
$$

Suppose $k = \tau_2 - \tau_1$.

Thus,

$$
\begin{cases}
\lambda_1 = \dfrac{(b + d)\,(2mb + k\lambda_2)}{b\left(2mb + k\lambda_2 + 2\sqrt{\psi}\right)} - \dfrac{d\lambda_2}{b}, \\[4mm]
\lambda_2 = \dfrac{(b + d)\,(2mb + k\lambda_1)}{b\left(2mb + k\lambda_1 + 2\sqrt{\xi}\right)} - \dfrac{d\lambda_1}{b},
\end{cases} \tag{10}
$$

where $\psi = nb\,(k\,(1 - \lambda_2) + bn\chi)$, $\xi = nb\,(k\,(1 - \lambda_1) + bn\chi)$.

The solution of (10) can be found using a computer environment with further substitution of the resulting values into equation (9). Equation (9) implies that λ_1 and λ_2 exist only if:

$$
\begin{cases}
nd(k(1 - \lambda_2) + dn\chi) & \geqslant 0; \\[1mm]
nb(k(1 - \lambda_1) + bn\chi) & \geqslant 0; \\[1mm]
\dfrac{dn\chi}{k} - 1 & \geqslant \lambda_2; \\[2mm]
\dfrac{bn\chi}{k} - 1 & \geqslant \lambda_1; \\[2mm]
\lambda_1 \in (0, 1), \quad \lambda_2 \in (0, 1).
\end{cases}
$$

If (10) has no solutions on the interval $(0, 1)$, then the optimal values are not in the domain. Therefore, consider $\lambda_1, \lambda_2 \in \{0, 1\}$, where $\lambda_i = 0$ corresponds to the road without designated lanes and $\lambda_i = 1$ corresponds to the road available only for autonomous vehicle. Hereby, all special cases of designating lanes on two roads were considered.

Model description on Braess' network

Consider the simple Braess network (Fig. 6). Let N_a, N_c be the total number of autonomous and common vehicles, respectively.

Thus, the total number of autonomous and common vehicles on each of the five roads is equal to

$$
n^1 = N^1 + N^3, \quad n^2 = N^1, \quad n^3 = N^2, \quad n^4 = N^2 + N^3, \quad n^5 = N^3.
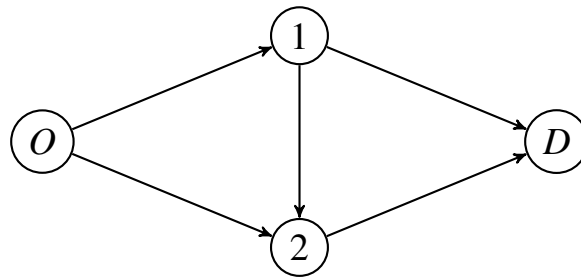$$

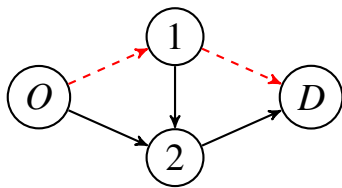Figure 6. An example of a simple Braess network

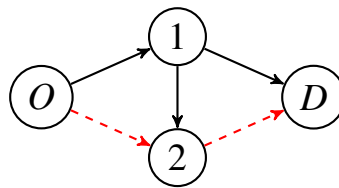Figure 7. 1st network traversal option
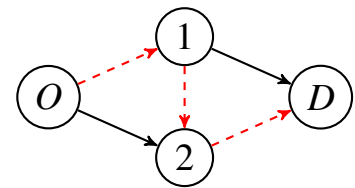
Figure 8. 2nd network traversal option

Figure 9. 3rd network traversal option

Using the results obtained from Options 1, 2, 3, and 4, TC for each route can be evaluated. At the same time computational modeling allows evaluation of possible delays in general form.

Algorithm

We propose the following algorithm:

1. Fix the set of lambda.

2. Look for the social optimum for the autonomous vehicles. It will reach the minimum TC_a on the set N_a on 3 trajectories.

$$TC_a = (t_a^1 + t_a^2)N_a^1 + (t_a^1 + t_a^4 + t_a^5)N_a^3 + (t_a^3 + t_a^4)N_a^2.$$

3. Look for the user equilibrium for the common vehicles. Choose the quant of size Q which is a small part of a virtual vehicle.

4. Set W as a number of common vehicles. There will be $W \cdot Q$ of quants in queue.

5. Extract the next quant from queue and find the best route for it. Repeat until the queue becomes empty. Add the quant size to route counters.

6. TC_c is calculated from the values of the route counters.

7. Find $TC = TC_a + TC_c$. Repeat step 1 to step 6 to find the minimum TC.

The algorithm (page 290) iterates through the possible combinations of edges and distribution of designated lanes on them, calculating the smallest value of TC and the corresponding value λ. According to the principle of greedy behavior, each driver chooses the optimal route in terms of the total traveling time. This forms Wardrop equilibrium and the system optimum is reached. The algorithm (page 290) was realized in *C*++ programming language.

Results of modeling

Only common vehicles

Assume that we have only common vehicles on all routes. At first compute TC and the equilibrium on traffic flows. Fix the parameters of the Braess paradox network which were proposed in [Steinberg, 1983] (Fig. 10) and increase the value of the lanes coefficient λ for autonomous vehicles on edge $A - B$.
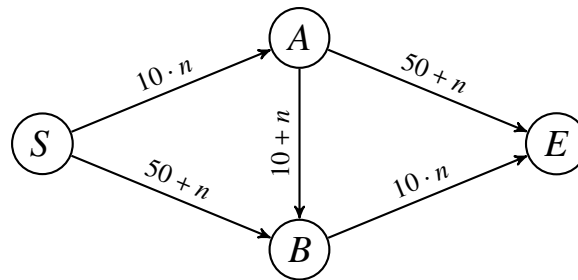

Figure 10. An example of Braess' network that was used for testing our numerical model

From the results shown in the chart $TC(\lambda)$ (Fig. 11) it can be concluded that the optimal solution is reached if the road A–B is available only for autonomous vehicles. The same results were obtained in [Steinberg, 1983], which proves the correctness of the developed program and the applied algorithm.
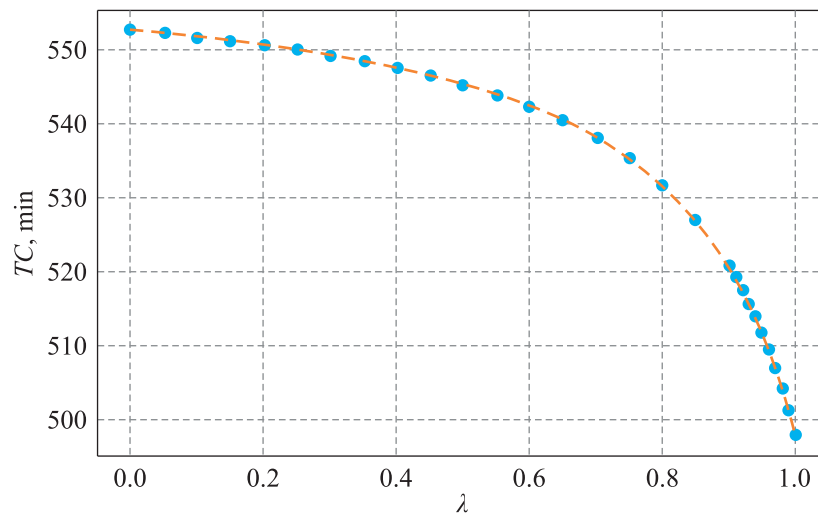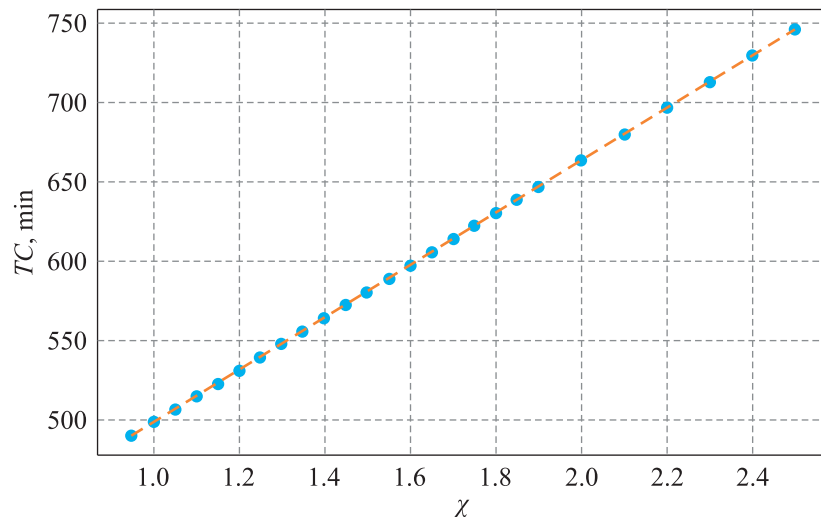

Figure 11. Relation between TC and λ graph

Autonomous and common vehicles

Consider the traffic flows in the same transportation network and explore the dependence of the optimal total time on the efficiency coefficient χ. If we fix the parameter λ and increase parameter χ, we get the results of the algorithm [Steinberg, 1983] (page 290) which are demonstrated on the following chart $TC(\chi)$.

This chart (Fig. 12) shows the linear dependence, which implies that there should be no common vehicles on route 3 (Fig. 9), nor on route A–B. In the case of Braess' paradox in the model with common vehicles, the only solution is to make the road A–B available only for autonomous vehicles.

Figure 12. Relation between TC and χ graph

Examples of real-life network modeling

It is not a rare scenario when Braess' paradox occurs after some time when the amount of traffic on the network reaches a certain value. The simplest way to deal with it is to remove the "problematic" edge from the network. In this article an alternative solution is proposed. This solution involves the traffic flow adjustment method, which is based on the principle of addition of designated lanes. Consider an example of an isolated network in one of Moscow districts (Fig. 13).

There are two possible ways to get to VDNKh when the vehicle moves along the Schyolkovskoye highway (Fig. 13). The first trajectory is along Third Ring Rd and then along Prospekt Mira (white trajectory) or along Rostokinsky proezd (red trajectory). The black route is faster, since it takes
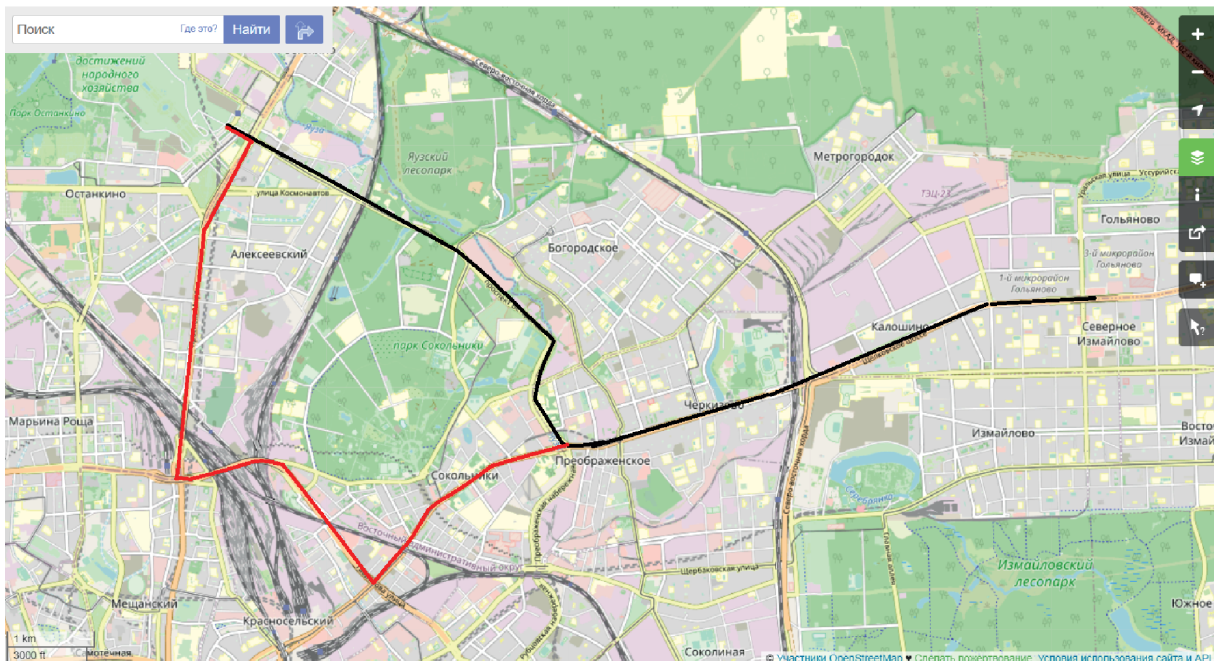

Figure 13. Map used for modeling, [OpenStreetMap, CODD, Otraslevoy...]

16 minutes to get from the starting point (red) to the end point (blue) without traffic jams. The white toute takes 23 minutes without traffic jams. However, the white route has greater capacity (4 lanes may serve up to 8300 vehicles per hour), while the black route has only 2 lanes and a capacity of 6700 vehicles per hour.

Since Rostokinsky proezd is located near Bogorodskoye and Metrogorodok districts, increasingly more drivers from these districts choose the black route. In order to save time, the driver would rather choose Rostokinsky proezd. When the number of vehicles on this route is large, the traffic will be backed up. This is the situation when Braess' paradox occurs. If there were no route Schyolkovskoye highway – Veteranov Avenue – Rostokinsky proezd, the traffic on this road system would be more optimal. Perform analysis for this network. Consider the graph of the network.

There are two possible trajectories. The capacity of the route through Metrogorodok is 6700 vehicles per hour and through TRR – 8300 vehicles per hour. If the traffic through Rostokinsky proezd is restricted for drivers from Schyolkovskoye highway, the total system load decreases. To solve the paradox, we can delete the Schyolkovskoye highway – Metrogorodok edge from the network. Consider an example. Assume that there are 7000 vehicles. 5000 vehicles move along the Schyolkovskoye highway and have to choose the trajectory and the remaining 2000 vehicles are in Metrogorodok. Suppose that 2000 vehicles choose Rostokinsky proezd and 3000 choose TRR (the data was obtained during observation from 7 to 8 am during rush hours).

Let the total time cost on the road segment i, where $i \in \{1, 2\}$ is the set of trajectories, be a linear function $t_i = \tau_i + b_i n_i$, where for routes 1 and 2 $b_1 = \dfrac{60}{6700} = 0.009$, $b_2 = \dfrac{60}{8300} = 0.007$, respectively. Thus, $T = t_1 + t_2 = \tau_1 + b_1 n_1 + \tau_2 + b_2 n_2 = 96$ min is the total traveling time through both trajectories. The average time is 48 min.

Now consider the case where there is no edge which connects the Schyolkovskoye highway and Metrogorodok. Then $n_1 = 2000$, $n_2 = 5000$. Thus $T = t_1 + t_2 = \tau_1 + b_1 n_1 + \tau_2 + b_2 n_2 = 92$ min. The average time is 46 min.

Therefore, this implies that the removal of this edge from the network optimizes the total network load.

Now consider the modeling on this transportation network. According to gucodd.ru, in Moscow, public transport accounts for 2% of the total amount of vehicles. Thus, if we have 7000 vehicles in total, 140 of them are public. According to the efficiency coefficient, $\chi = 0.7$, which implies that the public transport traffic is less effective. Since $\lambda_1 \in (0, 1), \lambda_2 = 0$ by definition, all public vehicles will move along the first road. Therefore,

$$TC = (n + m)\tau_1 + \frac{bm\left(\sqrt{bn\chi\,(\tau_2 + dn - \tau_1)} + dm\right)}{(d + b\chi)} + \frac{bn\chi\,(\tau_2 + dn - \tau_1)}{d\left(1 - \dfrac{m(d + b\chi)}{\sqrt{bn\chi\,(\tau_2 + dn - \tau_1)} + dm}\right) + b\chi}$$

is the optimal value of TC for this system (5).

Therefore, the total time cost is

$$TC = 112\,000 + \frac{62.72}{0.0133} + \frac{2377.9}{0.013} = 299\,631.$$

By evaluating the average traveling time for both trajectories in this network for a common vehicle, we get

$$t_{1,2} = \frac{299\,631}{7000} = 42.8 \text{ min.}$$

Hence, the proposed method allows to reduction of the network load.

Conclusion

In this article the problem of network system modeling has been solved using a set of examples. It was shown that Braess' paradox can significantly amplify the total network load. The model of designated lanes was proposed. Simple cases of organizing designated lanes were considered. Optimal strategies for common and autonomous vehicles were obtained. The model realization was illustrated by an example. It was found that the modeling of designated lanes eliminates Braess' paradox and optimizes the total traveling time. The algorithm which simulates drivers' behavior to determine the system equilibrium was proposed and realized in programming language.

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