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О моделях шины, учитывающих как деформированное состояние, так и эффекты сухого трения в области контакта

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Предложена новая приближенная модель качения деформируемого колеса с пневматиком, позволяющая учесть как усилия в пневматике, так и влияние сил сухого трения на устойчивость прямолинейного качения колеса при прогнозировании явления шимми. Модель основана на теории сухого трения с комбинированной кинематикой относительного движения соприкасающихся тел, т. е. при одновременном качении, скольжении и верчении при учете реальной формы области контакта и распределения контактного давления. Главный вектор и главный момент сил, возникающих при контактном взаимодействии с сухим трением, определяются путем интегрирования по области контакта. При этом контактное давление покоя при нулевых скоростях относительного поступательного движения и верчения и в отсутствие качения определяется из решения статической контактной задачи для пневматика с учетом его реальной структуры и физических свойств материалов. В работе использована конечно-элементная модель типового пневматика с продольным протектором. Расчет осуществлен при фиксированном внутреннем давлении наддува, заданной вертикальной силе и коэффициенте трения покоя, равном 0.5. Получены также решения задач о напряженно-деформированном состоянии пневматика при кинематическом нагружении в боковом направлении и при скручивании относительно вертикальной оси. Показано, что с достаточной степенью точности контактное взаимодействие пневматика с абсолютно жесткой опорной поверхностью можно представить в виде двух этапов — адгезии и проскальзывания, при этом, однако, форма пятна контакта остается близкой к круговой. Построены диаграммы, аппроксимирующие численные решения, для боковой силы и момента; на начальном участке взаимодействия зависимости линейны и соответствуют упругой деформации пневматика, на втором участке величины силы и момента постоянны и соответствуют силе сухого трения и моменту трения верчения. Для последних участков получены приближенные выражения для продольной и боковой силы трения, а также момента трения верчения в соответствии с теорией сухого трения с комбинированной кинематикой. Полученная модель может трактоваться как комбинация модели упруго деформируемого колеса по Келдышу, катящегося без проскальзывания, и жесткого колеса по Климову – Журавлёву, взаимодействующего с опорой посредством сил сухого трения.

Ключевые слова: трение сухое, кинематика комбинированная, шины пневматические, состояние деформированное

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MODELS IN PHYSICS AND TECHNOLOGY

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On tire models accounting for both deformed state and coupled dry friction in a contact spot

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A proposed approximate model of the rolling of a deforming wheel with a pneumatic tire allows one to account as well forces in tires as the effect of the dry friction on the stability of the rolling upon the shimmy phenomenon prognosis. The model os based on the theory of the dry friction with combined kinematics of relative motion of interacting bodies, i. e. under the condition of simultaneous rolling, sliding, and spinning with accounting for the real shape of a contact spot and contact pressure distribution. The resultant vector and couple of the forces generated by the contact interaction with dry friction are defined by integration over the contact area, whereas the static contact pressure under the conditions of vanishing velocity of sliding and angular velocity of spinning is computed after the finite-element solution for the statical contact of a pneumatic with a rigid road with accounting forreal internal structure and properties of a tire. The solid finite element model of a typical tire with longitudinal thread is used below as a background. Given constant boost pressure, vertical load and static friction factor 0.5 the numerical solution is constructed, as well as the appropriate solutions for lateral and torsional kinematic loading. It is shown that the contact interaction of a pneumatic tire and an absolutely rigid road could be represented without crucial loss of accuracy as two typical stages, the adhesion and the slip; the contact area shape remains nevertheless close to a circle. The approximate diagrams are constructed for both lateral force and friction torque; on the initial stage the diagrams are linear so that corresponds to the elastic deformation of a tire while on the second stage both force and torque values are constant and correspond to the dry friction force and torque. For the last stages the approximate formulae for the longitudinal and lateral friction force and the friction torque are constructed on the background of the theory of the dry friction with combined kinematics. The obtained model can be treated as a combination of the Keldysh model of elastic wheel with no slip and spin and the Klimov rigid wheel model interacting with a road by dry friction forces.

Keywords: dry friction, combined kinematics, pneumatic tires, deformed state

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Introduction

The dynamics of various vehicles equipped by pneumatic wheels becomes a result of many complex interactions of their structural components, but "...a major role is played by the pneumatic tyre" ([Paceika, 2002], p. v). H. B. Paceika noted that "the mechanical behavior of the tyre needs to be investigated ...in terms of its reaction to various inputs associated with wheel motions and road conditions" [Paceika, 2002]. The complex behavior of tires required appropriate simplified models that could be useable in the engineering practice; a wide range of so-called empirical and semi-empirical models can be mentioned, e. g. the well-known "Magic Formula Model" [Paceika, 2002; Bakker et al., 1987]. A more detailed description of the tire-road interaction can be offered by the Tire Brush Model [Paceika, 2002]; an almost full survey of existing models used for pneumatic wheels was presented recently in [Kièbrè, 2010]. Another way allowed by the powerful modern hard- and software consists in the direct finite element simulation of the rolling tires [Reza Ghoreishy, 2008]; such an approach can be interpreted as the must detailed one, but the three-dimensional finite element models of tires are too consumptive even in statics. This method deals primarily with final design stages and has almost nothing to do with preliminary ones where simple models with a few degrees of freedom are needed.

One of the problems of tire modeling that has intensively studied last years consists in the improved modeling of friction in the tire-road contact spot [van der Steen, 2007]. Most of the complex physical as well as empirical tire models accounts for the dry friction, including viscosity effects and variable boundary of sliding and adhesion zones (e. g. see [Kièbrè, 2010]). It is clear that the friction acts as a main cause of wear and outage for wheels; but this is not the only friction effect. It should be noted that the dry friction effects significantly on the dynamics of rolling wheels, moreover it may cause the well-known shimmy phenomenon as it was shown in [Zhuravlev, Klimov, 2009; Zhuravlev, Klimov, 2010]. This phenomenon was found theoretically only after the introduction of the so-called Multi-Component Dry Friction theory [Zhuravlev, 2003; Andronov, Zhuravlev, 2010] accounting for the strong coupling of dry friction forces and torque in the case of the combined sliding and spin that was observed by Th. Erismann [Erissmann, 1954] and P. Contensou [Contensou, 1953] (see also [Ivanov, 2018a; Ivanov, 2018b]). Indeed, the use of the traditional Amonton – Coulomb dry friction law for vanishing contact spots leads to the results being inconsistent with the reality [Andronov, Zhuravlev, 2010; Contensou, 1953]. At the same time its incremental representation for the point of the finite contact spot with the summary sliding velocity due to the simultaneous slip and spin and further computing of resulting dry friction vector and couple by means of integration over the contact spot results in qualitatively different models that are able to predict some known effects not described by uncoupled models [Zhuravlev, 2003; Andronov, Zhuravlev, 2010], e. g. the shimmy of quasi-rigid wheels with negligible elastic deformations [Zhuravlev, Klimov, 2009; Zhuravlev, Klimov, 2010]. Such an effect cannot be found using the traditional shimmy theories such as the ones of M. V. Keldysh [Keldysh, 1985] and others [Gozdek, 1969] based on the non-holonomic rolling condition and neglecting the slip and spinning. The rigid wheel model has shown its efficiency in the study of the shimmy oscillations of landing gears of aircrafts observed shortly after touchdowns [Zagordan, Zhavoronok, 2011]; its further improvement requires accounting for the elastic deformations of tires to be consistent with the classical approach [Paceika, 2002; Keldysh, 1985]. Accounting for the real distribution of the contact pressure at different vertical loads obtained from the finite element solution [Bogoslovskii, Kurdyumov, 2015] together with the simple approximations for resulting vectors and couple of friction [Zhuravlev, Kireenkov, 2005; Kireenkov, 2005; Ramodanov, Kireenkov, 2017] became the first improvement of the friction-based shimmy model [Kireenkov, Zhavoronok, 2017; Kireenkov, 2017; Kireenkov, 2018] whereas the accounting for the tire tread effect on the background of the anisotropic theory of the combined dry friction became the second one [Zhavoronok, Kireenkov, 2017; Kireenkov, 2018; Kireenkov et al., 2018]. Let us also note that further use of various higher-order theories of heterogeneous anisotropic shells [Amosov et al., 2004;

Zhavoronok et al., 2010; Zhavoronok, 2015] seems to be promising as well. Here the friction anisotropy and the complex distribution of the contact pressure as well as the lateral and torsional compliance of the tire obtained from the three-dimensional finite element simulations are introduced, and the first attempt to combine the approaches [Keldysh, 1985; Zhuravlev, Klimov, 2009] under an unified formulation is made.

On the anisotropic dry friction under the conditions of the combined kinematics

The simultaneous spinning, sliding, and rolling of a deforming body on a rigid plane with a finite spot of contact leads to the qualitative improvement of the dry friction law as it was shown in [Contensou, 1953; Erissmann, 1954; Zhuravlev, 2003; Andronov, Zhuravlev, 2010; Kireenkov, 2005]. Indeed, let us consider a solid G interacting with a rigid rough plane through the finite contact area S. The relative motion of G and S_0 could be defined by the summary tangent velocity vector $\mathbf{v}_\Sigma$ defined at each point $M \in S$ [Kireenkov, Zhavornok, 2020]:

$$\mathbf{v}_\Sigma = \mathbf{v}_0 - R\boldsymbol{\omega}_\tau \times \boldsymbol{\nu} + \boldsymbol{\omega}_\nu \times \mathbf{r}_\tau, \tag{1}$$

where $\mathbf{v}_0$ is the absolute longitudinal velocity, $\boldsymbol{\omega}_\nu$ is the angular velocity of spinning while $\boldsymbol{\omega}_\tau$ denotes the angular velocity of rolling; R is the radius of the rolling body G, $\mathbf{r}_\tau$ is the vector radius in the contact plane, and $\boldsymbol{\nu}$ is the normal unit vector. Thus, accordingly to the Amonton–Coulomb law, the tangent stress vector $\boldsymbol{\tau}$ at a point $M \in S$ could be defined as follows [Kireenkov, Zhavoronok, 2017; Zhavoronok, Kireenkov, 2017]:

$$\boldsymbol{\tau} = -p\mathbf{f} \cdot \frac{\mathbf{v}_\Sigma}{|\mathbf{v}_\Sigma|}, \quad |\mathbf{v}_\Sigma| \neq 0, \tag{2}$$

here the second rank tensor $\mathbf{f}$ is the coefficient of the anisotropic static dry friction, and $p \geq 0$ is the contact pressure accounting for the rolling effect:

$$p = p_0 \left[1 + \left(\mathbf{r}_\tau \times \mathbf{h} \cdot \frac{\boldsymbol{\omega}_\tau}{|\boldsymbol{\omega}_\tau|}\right) \cdot \boldsymbol{\nu}\right], \quad p_0 = p|_{\omega_\tau = 0}, \tag{3}$$

where the second rank tensor $\mathbf{h}$ corresponds to the rolling friction coefficient; it is assumed to be homogeneous and positively defined. Finally, taking into account (3) and (1), we obtain the tangent stress at a point $M \in S$:

$$\boldsymbol{\tau} = -p_0 \left[1 + \left(\mathbf{r}_\tau \times \mathbf{h} \cdot \frac{\boldsymbol{\omega}_\tau}{|\boldsymbol{\omega}_\tau|}\right) \cdot \boldsymbol{\nu}\right] \mathbf{f} \cdot \frac{\mathbf{v}_0 - R\boldsymbol{\omega}_\tau \times \boldsymbol{\nu} + \boldsymbol{\omega}_\nu \times \mathbf{r}_\tau}{|\mathbf{v}_0 - R\boldsymbol{\omega}_\tau \times \boldsymbol{\nu} + \boldsymbol{\omega}_\nu \times \mathbf{r}_\tau|}. \tag{4}$$

Let us note that the components of $\mathbf{f}$ and $\mathbf{h}$ and the pressure distribution p could be obtained as well after physical tests as after numerical simulations; the random distribution of the pressure could be also considered [Kireenkov, Ramodanov, 2019].

Let us define hence the resultant vector $\mathbf{F}$ and couple $\mathbf{M}$ of the forces in the contact spot: $\mathbf{F} = \mathbf{N} + \mathbf{T}$, $\mathbf{M} = \mathbf{M}_\nu + \mathbf{M}_\tau$, here $\mathbf{N}$ is the normal reaction, $\mathbf{T}$ is the dry friction force, $\mathbf{M}_\tau$ is the rolling couple, and $\mathbf{M}_\nu$ is the rolling torque. These quantities could be computed after integration over the contact spot S:

$$\mathbf{N} = \boldsymbol{\nu} \int_S p\, dS = \int_S p_0 \left(\boldsymbol{\nu} + \mathbf{r}_\tau \times \mathbf{h} \cdot \frac{\boldsymbol{\omega}_\tau}{|\boldsymbol{\omega}_\tau|}\right) dS, \tag{5}$$

$$\mathbf{T} = \int_S \boldsymbol{\tau}\, dS = \int_S p_0 \left[1 + \left(\mathbf{r}_\tau \times \mathbf{h} \cdot \frac{\boldsymbol{\omega}_\tau}{|\boldsymbol{\omega}_\tau|}\right) \cdot \boldsymbol{\nu}\right] \mathbf{f} \cdot \frac{\mathbf{v}_0 - R\boldsymbol{\omega}_\tau \times \boldsymbol{\nu} + \boldsymbol{\omega}_\nu \times \mathbf{r}_\tau}{|\mathbf{v}_0 - R\boldsymbol{\omega}_\tau \times \boldsymbol{\nu} + \boldsymbol{\omega}_\nu \times \mathbf{r}_\tau|}\, dS, \tag{6}$$

$$\mathbf{M}_\nu = \int_S \mathbf{r}_\tau \times \boldsymbol{\tau}\, dS = \int_S p_0 \left[1 + \left(\mathbf{r}_\tau \times \mathbf{h} \cdot \frac{\omega_\tau}{|\omega_\tau|}\right) \cdot \boldsymbol{\nu}\right] \frac{\mathbf{r}_\tau \times \mathbf{f} \cdot [\mathbf{v}_0 - R\omega_\tau \times \boldsymbol{\nu} + \omega_\nu \times \mathbf{r}_\tau]}{|\mathbf{v}_0 - R\omega_\tau \times \boldsymbol{\nu} + \omega_\nu \times \mathbf{r}_\tau|}\, dS, \tag{7}$$

$$\mathbf{M}_\tau = \int_S p\boldsymbol{\nu} \times \mathbf{r}_\tau\, dS = \int_S p_0 \left(\boldsymbol{\nu} + \mathbf{r}_\tau \times \mathbf{h} \cdot \frac{\omega_\tau}{|\omega_\tau|}\right) \times \mathbf{r}_\tau\, dS. \tag{8}$$

The relations (5)–(8) could be computed numerically using an appropriate frame attached to the contact spot. However, they are too complex to be evaluated analytically. The use of approximations gives nevertheless accurate values for the friction force and torque as it was shown, for instance, in [Kireenkov, 2005].

Numerical simulation of the tire-road interaction with dry friction

Let us consider hence a typical pneumatic tire with a longitudinally structured thread (Fig. 1). The outer tire diameter is equal to 0.61 m, the inner one is 0.38 m, while the thread depth is equal to 0.008 m. The breaker strip of the tire consists in two reinforcement layers of steel cord with angles ±65° and fiber cross-section area 2 mm^2. The elastic modulus of the cord is $E = 2.1 \times 10^5$ MPa. The polymeric cord consists in the layers of $0/90°$ structure of fiber reinforcement with the cross-section area of the fiber equal to 1 mm^2; the Marlow-type model of the hyperelastic material is based on the experimental strain diagram. Finally, the rubber materials are simulated on the background of the neo-Hookean model.

Figure 1. The geometry of a typical pneumatic tire

The finite-element model of the tire (Fig. 2) uses the 8-node solid elements for the thread and beads; the breaker strip is simulated by the reduced 4-node elements. The road surface is assumed to be absolutely rigid. The tire is loaded by the internal pressure 0.24 MPa and the steady vertical force of 3750 N. The local friction factor is equal to 0.5. The lateral sliding as well as the spinning of the tire are simulated under the conditions of the quasi-static problem statement; the lateral displacement of the center point of the tire axle is applied up to the value of 0.1 m, while the applied rotation angle is limited by the value of 0.4 rad.

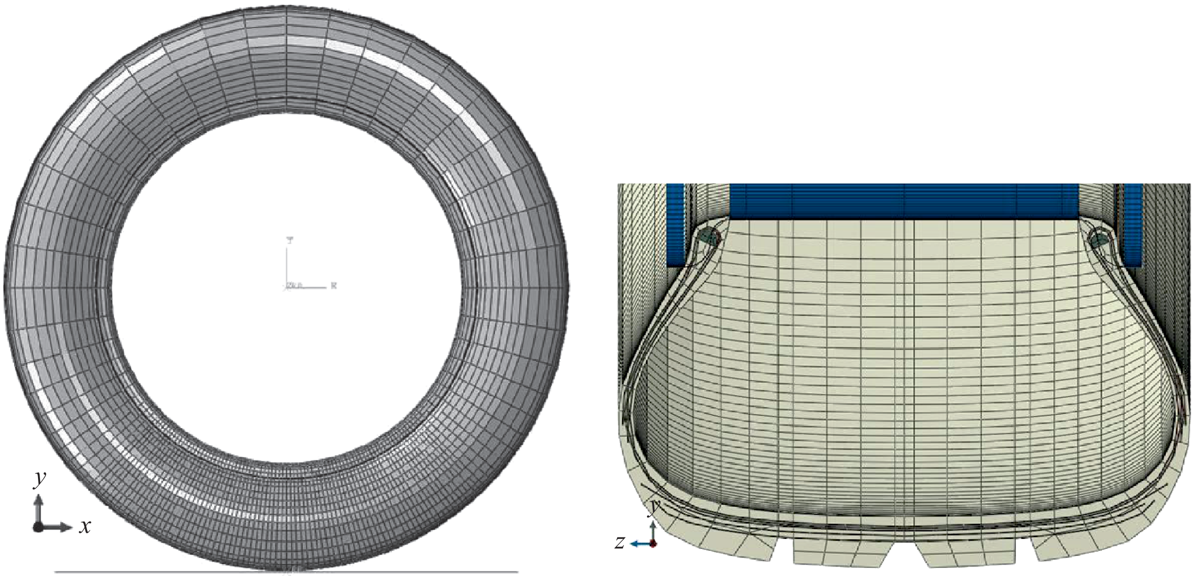

Figure 2. The solid finite element model of a tire used in modeling

Two main stages of the tire-road interaction were observed, the pure adhesion and the sliding (in the case of the lateral loading) and the spinning (if the rotation with respect to the wertical axis is considered). The stages of combined slipping/adhesion with sliding subdomains appearing in the contact spot are relatively small. The computed distribution of the contact pressure across the tire section is shown on the Fig. 3.

Thus, the significant effect of the deformed state on the pressure distribution under the lateral sliding conditions could be observed. On the other hand it could be seen that the contact spot shape remains close to the circular one as well for the lateral sliding (Fig. 4) as for the spinning of the tire (Fig. 5).

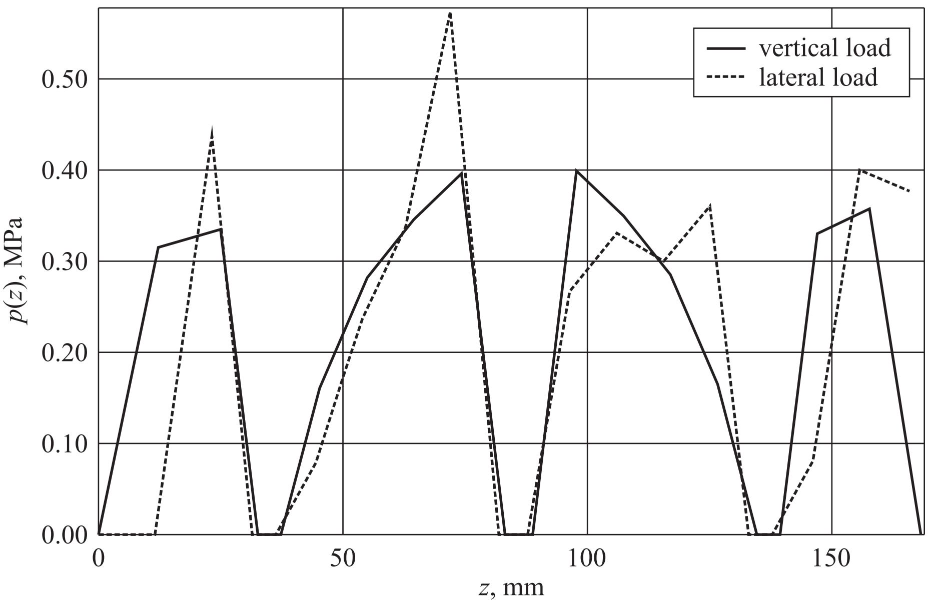

Figure 3. Distribution of the contact pressure $p(z)$, MPa across the contact spot: vertical loading with no lateral displacement (solid line), steady lateral sliding with dry friction force (dotted line)

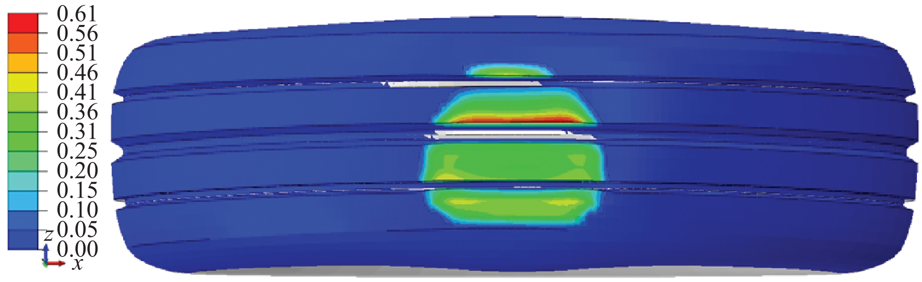

Figure 4. The shape of the contact spot under the condition of the sliding and the corresponding distribution of the contact pressure p, MPa

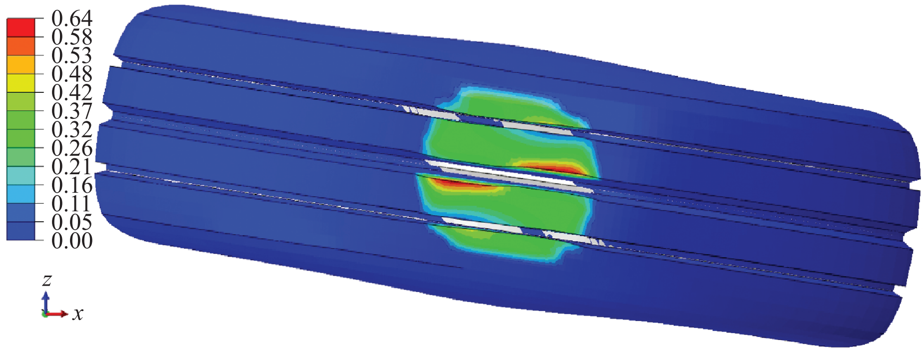

Figure 5. The shape of the contact spot under the condition of the spinning and the corresponding distribution of the contact pressure p, MPa

Approximated diagrams for the lateral force and torque

The dependence of the lateral force F_y on the lateral displacement λ is presented on the Fig. 6 while the analogous dependence of the torque on the Fig. 7. As it can be seen, these dependencies have almost linear initial curve pieces and almost constant second pieces. Thus, at the first stages the elastic strain exists with no sliding and spin whereas after reaching threshold displacement value λ_* and ϕ_* the sliding and spin appear, and the generalized forces dry friction forces.

On the other hand, the obtained dependencies could be approximated:

$$\begin{aligned} &F(\lambda) = C_F\lambda, \quad \lambda \in [0, \lambda_*], \qquad && M(\phi) = C_M\phi, \quad \phi \in [0, \phi_*], \\ &F(\phi) = F_0, \quad \lambda > \lambda_*, && M(\phi) = M_0, \quad \phi > \phi_*. \end{aligned}$$

The factors values as well as confidence bounds and relative mean-square errors of approximation are shown in Table 1.

Table 1. Piecewise-Linear approximation for the dependence of the lateral force F and torque M_z on the lateral displacement λ and spin angle ϕ

Constant	Conf. bounds	RMSE
$C_F = 8.274 \times 10^4$ N/m	[8.255 8.299]×10^4	4.534
$F_0 = 1.862 \times 10^3$ N	[1.861 1.864]×10^3	2.152
$C_M = 1.561 \times 10^6$ N	[1.467 1.655]×10^6	5302
$M_0 = 8.110 \times 10^4$ N×rad	[8.104 8.116]×10^4	431.3

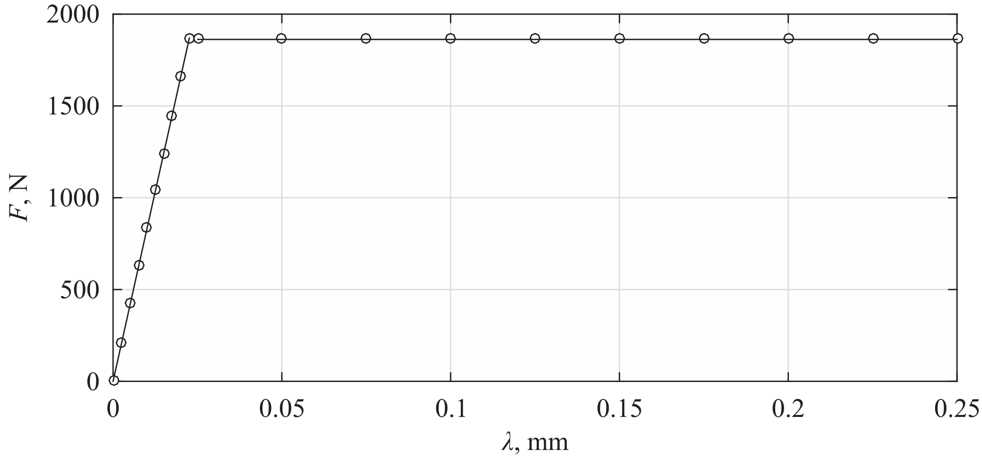

Figure 6. Dependency of the lateral force F, N on the lateral displacement λ, mm: finite element simulation (dots), linear approximation (line)

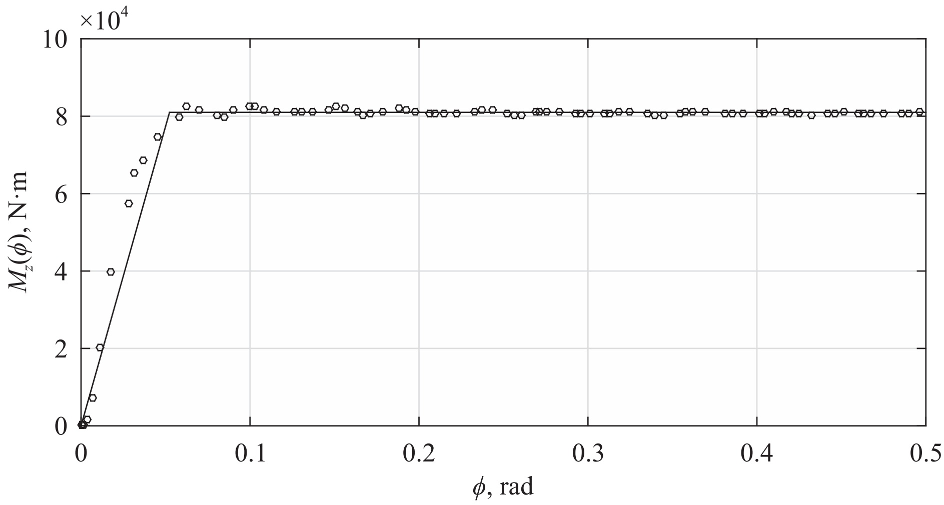

Figure 7. Dependency of the torque M_z, N·m on the spin angle ϕ, radians: finite element simulation (dots), linear approximation (line)

Use of numerical results in non-holonomic and coupled dry friction models of rolling tires

The introduced factors could be hence interpreted as follows. As the tire-road interaction remains very close to the pure adhesion with no significant slip or spin under small lateral forces or spinning torques, the classical non-holonomic models for the rolling wheel of Keldysh [Keldysh, 1985], Gozdek [Gozdek, 1969], etc. could be applied to model the dynamics. The corresponding coefficients C_F and C_M correspond to the lateral and torsional stiffness of the tire.

On the other hand, the road-tire interaction become correspond to the various friction-based models such as [Zhuravlev, Klimov, 2009; Zhuravlev, Klimov, 2010; Kireenkov, Zhavoronok, 2017; Kireenkov, 2017; Kireenkov, 2018; Kireenkov, Zhavoronok, 2018; Kireenkov et al., 2018] after reaching threshold values λ_* or ϕ_*. Therefore, considering the approximation for the model [Kireenkov, 2005] of the coupled dry friction under combined kinematics

$$T_{\|} = \frac{F_0 v}{\sqrt{v^2 + au^2}}, \quad T_{\perp} = \frac{\mu_0 k F_0 v u^2}{\sqrt{v^6 + bu^6}}, \quad M_v = \frac{M_0 u}{\sqrt{u^2 + mv^2}}$$

we could interpret the obtained values F_0 and M_0 as resting friction forces and resting friction torque correspondingly. Here $T_{\|}$ is the longitudinal friction force, $T_{\perp}$ is the lateral one, M_ν is the sinning torque, $u = \dot{\phi}R$ $(\phi > \phi_*)$, $v = \dot{\lambda}$ $(\lambda > \lambda_*)$, R denotes the medium radius of the contact spot that can be introduced (see Fig. 4 and 5), and a, b, and m are the coefficients of the model [Kireenkov, 2005] that could be computed numerically for various distributions of the contact pressure as it was shown in [Kireenkov, Zhavoronok, 2017] or obtained from the test diagrams for the longitudinal force $T_{\|}$, lateral force $T_{\perp}$, and torque M_ν [Kireenkov, Zhavoronok, 2019].

Conclusion

The obtained numerical simulation results for the quasi-static deformed state of a typical tire with a thread under the condition of internal pressurization, vertical loading and the loading by the lateral force/displacement or spinning torque/rotation have shown that:

- the contact interaction could be conditionally subdivided into two main stages, the pure adhesion and pure sliding (spinning) with negligibly small transition stage;
- the contact spot remains close to the circular one up to the stages of steady sliding and spinning;
- the obtained diagrams "lateral force – lateral displacement" and "spinning torque – spinning angle" are very close to the piecewise-linear ones with linear initial stages corresponding to the elastic deforming under the pure adhesion and constant final stages corresponding to the steady sliding (spinning) with dry friction.

Thus, the rheological model of the tire could be introduced as a combination of the classical model of rolling based on the non-holonomic constraint (adhesion stage) and the coupled dry friction models. Indeed, the piecewise-linear approximation of the numerically obtained diagrams gives the lateral and torsional elastic stiffness of tires as well as the resting forces and torques.

The developed theory could offer the possibility of comprehensive modeling of rolling wheels and analysis of rolling stability at initial stages of design of various perspective vehicles including landing gears of aircrafts, high-velocity automotive, etc.

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