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Cluster method of mathematical modeling of interval-stochastic thermal processes in electronic systems

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A cluster method of mathematical modeling of interval-stochastic thermal processes in complex electronic systems (ES), is developed. In the cluster method, the construction of a complex ES is represented in the form of a thermal model, which is a system of clusters, each of which contains a core that combines the heat-generating elements falling into a given cluster, the cluster shell and a medium flow through the cluster. The state of the thermal process in each cluster and every moment of time is characterized by three interval-stochastic state variables, namely, the temperatures of the core, shell, and medium flow. The elements of each cluster, namely, the core, shell, and medium flow, are in thermal interaction between themselves and elements of neighboring clusters. In contrast to existing methods, the cluster method allows you to simulate thermal processes in complex ESs, taking into account the uneven distribution of temperature in the medium flow pumped into the ES, the conjugate nature of heat exchange between the medium flow in the ES, core and shells of clusters, and the interval-stochastic nature of thermal processes in the ES, caused by statistical technological variation in the manufacture and installation of electronic elements in ES and random fluctuations in the thermal parameters of the environment. The mathematical model describing the state of thermal processes in a cluster thermal model is a system of interval-stochastic matrix-block equations with matrix and vector blocks corresponding to the clusters of the thermal model. The solution to the interval-stochastic equations are statistical measures of the state variables of thermal processes in clusters — mathematical expectations, covariances between state variables and variance. The methodology for applying the cluster method is shown on the example of a real ES.

Keywords: mathematical modeling, thermal model, cluster, electronic system, stochastic, thermal process, statistical measures, mathematical expectations, covariances, variances

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Кластерный метод математического моделирования интервально-стохастических тепловых процессов в электронных системах

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В работе разработан кластерный метод математического моделирования интервально-стохастических тепловых процессов в сложных технических, в частности электронных, системах (ЭС). В кластерном методе конструкция сложной ЭС представляется в виде тепловой модели, являющейся системой кластеров, каждый из которых содержит ядро, объединяющее в себе тепловыделяющие элементы, попадающие в данный кластер, оболочку кластера и поток среды, протекающий через кластер. Состояние теплового процесса в каждом кластере и в каждый момент времени характеризуется тремя интервально-стохастическими переменными состояния, а именно температурами ядра, оболочки и потока среды. При этом элементы каждого кластера, а именно ядро, оболочка и поток среды, находятся в тепловом взаимодействии между собой и элементами соседних кластеров. В отличие от существующих методов кластерный метод позволяет моделировать тепловые процессы в сложных ЭС с учетом неравномерного распределения температуры в потоке среды нагнетаемой в ЭС, сопряженного характера теплообмена между потоком среды в ЭС, ядрами и оболочками кластеров и интервально-стохастического характера тепловых процессов в ЭС, вызванного статистическим технологическим разбросом изготовления и монтажа электронных элементов в ЭС, и случайными флуктуациями тепловых параметров окружающей среды. Математическая модель, описывающая состояния тепловых процессов в кластерной тепловой модели, представляет собой систему интервально-стохастических матрично-блочных уравнений с матричными и векторными блоками, соответствующими кластерам тепловой модели. Решением интервально-стохастических уравнений являются статистические меры переменных состояния тепловых процессов в кластерах — математические ожидания, ковариации между переменными состояния и дисперсии. Методика применения кластерного метода показана на примере реальной ЭС.

Ключевые слова: математическое моделирование, тепловая модель, кластер, электронная система, стохастический, тепловой процесс, статистические меры, математические ожидания, ковариации, дисперсии

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1. Introduction

Thermal design of complex electronic systems (ES) may be adequate when methods of mathematical and computer modeling of thermal processes allow modeling [Sergeyev, Khadakov, 2012; Kuuse et al., 2005; Madera, Kandalov, 2016; Madera, 2018; Madera, 2019]:

- complex ES structures characterized by structural versatility and a large number of both electronic (active) heat-generating elements (processors, integrated circuits (IC), electric and radio elements (ERE)), and structural (passive) heat-dissipating elements (electrical connectors that do not consume ERE power, fixing elements, etc.);
- physical processes arising when the ES is operating, namely, thermal processes and the resulting thermal feedback, thermal stress and heat exchange in the fluid flow;
- impact of destabilizing (chemical, radiation, vibration and mechanical) and climatic factors,
- thermal and electric processes under the conditions of interval-stochastic uncertainty of the physical and structural factors shaping these processes.

A mathematical model describing thermal and associated physical processes in complex ES is a system of time-dependent, non-linear, interval-stochastic differential equations in partial derivatives, including equations of motion and energy in the cooling fluid flow both inside, and outside of the ES. Considering that the number of equations in the mathematical model is comparable with the number of elements in the ES, the solution to the model is extremely difficult in mathematical and computational aspects, even when modern supercomputers are used. The hierarchical method of modeling of thermal processes in complex ES developed in [Madera, 2019] allows overcoming the above-mentioned difficulties, simplifying the mathematical model significantly and reducing the number of equations, including computer computations and RAM memory consumed.

Thermal processes in the ES structure depend on the energy consumption by active (heat-generating) elements and heat exchange between active and passive elements in the cooling fluid (air or liquid) flow inside the ES. While mathematical modeling of liquid cooling in the ES by the liquid flow forced via the channels of the structure has been developed in sufficient details [Ellison, 2011; Dulnev, 1971; Spolding et al., 1990; Schlichting et al., 2017], mathematical modeling of the heat exchange between ES elements in the air flow forced through the ES case still requires to be solved and brought to the methodological ES design level. When an ES is designed, despite that heat dissipation power in the ES is uneven, it is usually assumed that the temperature of the fluid flow is distributed across the ES evenly or linearly. In the first case, the temperature of the fluid is considered to be isothermal and equal to a mean temperature determined as the arithmetic mean value between the inlet and outlet temperatures of the ES [Ellison, 2011; Dulnev, 1971]. In the second case, it is a priori assumed that the relationship between the air temperature and the distance passed by the fluid flow from the inlet to the outlet of the ES is linear [Madera, 2019]. At low flow rates and small ES dimensions, the assumption about mean fluid temperature or linear distribution of the temperature in the ES may in a number of cases give acceptable for the thermal design practice results, however, at high fluid flow rates (but with a Mach number < 1) and at larger ES dimensions, distribution of the temperature in the air flow may differ significantly from a linear, and even more, an isothermal one.

It should be noted that difficulties in mathematical modeling of heat exchange between the fluid flow forced through the ES and the active elements of the same are caused, firstly, by a an extremely complicated chaotic motion of the fluid flow via the network of channels formed by the structural and electronic elements of the ES, which channels have various shapes, directions and cross sections, secondly, by the interval-stochastic nature of the heat exchange in the fluid flow, and, thirdly, by the conjugate nature of the heat exchange, where the interaction between a heated element and the flow enthalpy is mutually conditioning, thus closing the feedback loop.

The air flowing via a network of channels between active heat-generating and passive elements in the ES accumulates heat, thus increasing its enthalpy (heat content), which is then transferred and conveyed to ES elements and results both in additional heating, and an increase in the flow enthalpy. Therefore, the distribution of temperature in the fluid flow going through ES elements from the inlet to the outlet of the ES will be significantly different from an isothermal or a linear one.

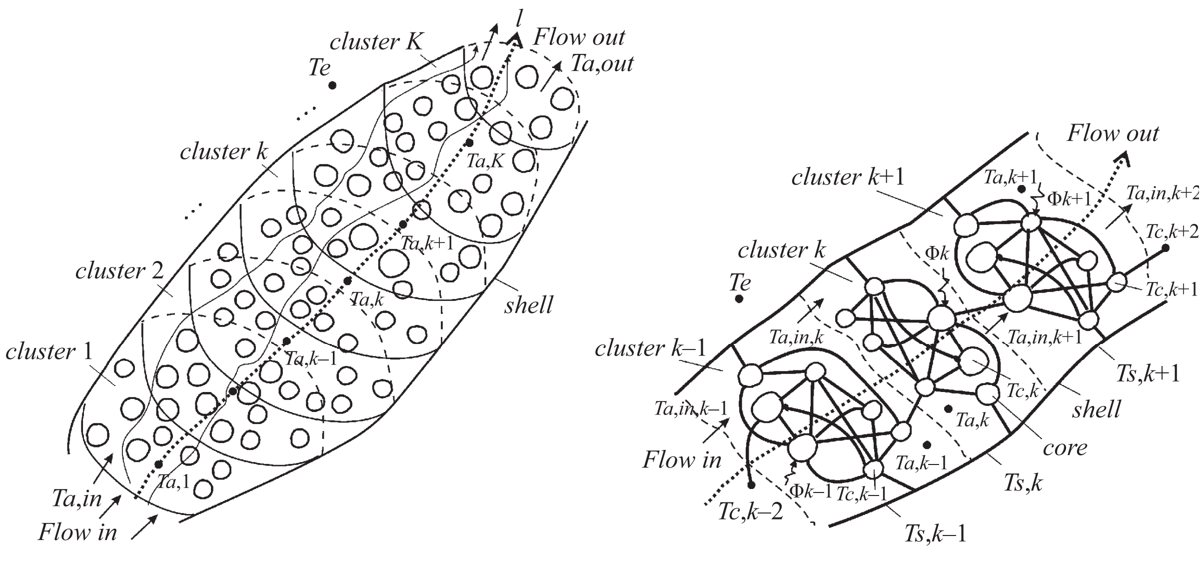
This paper describes a cluster method of mathematical modeling of conjugate interval-stochastic thermal processes in complex ES. The method is based on representation of a complex ES structure by a thermal model as a system of clusters, each of which contains a core that combines heat-generating elements falling into a given cluster, a cluster shell, and fluid flow through the cluster. The state of the thermal process in each cluster is characterized by three state variables, namely, temperatures of the core, shell, and fluid flow in the cluster. The cluster approach allows simulating distribution of the temperature in the air forced through the ES, temperature of active heat-generating and passive heat-dissipating elements of the ES, and distribution of the temperature in the ES case. Interval-stochastic thermal processes are analyzed using the author's method of obtaining equations for statistical measures of the thermal process — mathematical expectations, covariances and variances. Application of the method is shown through an example of modeling of an interval-stochastic thermal process in a real ES (computer system). The statistical measures produced using the method are easily programmed and embedded in automated ES thermal design systems [Madera, Reshetnikov, 2017].

2. Cluster thermal and mathematical models of interval-stochastic thermal processes in ES

The thermal model of the ES used in the analysis and modeling of thermal conditions in complex ES is a system of N isothermal bodies (Fig. 1, *a*) [Ellison, 2011; Dulnev, 1971; Madera, Kandalov, 2016], in which both solid-state active and passive elements of the ES structure, and the fluid inside the ES case, whose temperature is assumed to be equal to the arithmetic mean value of the inlet and outlet fluid flow temperatures in the ES, are isothermal. At low rates and small ES dimensions, such an assumption is perfectly reasonable for the engineering ES thermal design practice, however, at higher fluid flow rates (but with a Mach number < 1) or larger ES dimensions, distribution of the temperature in the air flow may be significantly different from an isothermal one.

In a cluster thermal model, a complex ES structure is divided into clusters $k = 1, 2, \ldots, K$ (Fig. 1, *b*), each of which contains a core that combines active elements falling into a given cluster, a cluster shell, and fluid surrounding the core and the shell. In real ES structures active and passive elements are in conductive thermal interaction with each other carried out through multiple solid-state connecting, fixing and mounting elements (printed circuit boards, electrical connectors, heat dissipators, etc.), and in convective heat exchange with the fluid flow forced through the cluster and radiant heat exchange between the elements and the fluid flow in the cluster, with additional convective and radiant heat exchange with the environment on the outer surface of the cluster shell. Thus, the temperatures of the elements and the environment rapidly mix and equalize in local ES structure volumes, therefore the volume and the shape of each individual cluster may be selected such that it may be assumed, with an accuracy sufficient for the engineering practice, that the temperatures of the core, shell and fluid within a single cluster are isothermal. A complex ES structure is divided into clusters, and the size of the clusters is selected by reference to specific features of the given ES structure, objectives, assumed accuracy of modeling and so that in the resulting clusters, the temperatures of the core, shell and fluid flow going through the cluster might, with an accuracy sufficient for the engineering practice, assumed to be isothermal.

The state of the thermal process in each cluster k, $k = 1, 2, \ldots, K$, at any specific time is fully determined by three state variables, namely, the temperature of the core, the temperature of the shell and the temperature of the fluid. Thermal processes in real ES, as shown in research papers [Madera, Kandalov, 2016; Madera, Kandalov, 2020], are interval-stochastic ones on account of statistical pro-



(a) (b)

Fig. 1. A cluster thermal model of an ES (a) and a fragment of the system consisting of k – 1, k, k + 1-th clusters (b). Legend: $T_{c,k}(t,\omega)$, $T_{s,k}(t,\omega)$, $T_{a,k}(t,\omega)$ are interval-stochastic isothermal temperatures of the core of the k-th cluster, shell of the cluster and fluid flow in the cluster; $T_{a,in,k}(t,\omega)$, $T_{a,in}(\omega)$, $T_e(\omega)$ are interval-stochastic fluid temperatures at the inlet of the k-th cluster, inlet of the ES and environment outside of the shell of the cluster; l is the direction of the fluid flow; $\Phi_k(\omega)$ is the total interval-stochastic heat generation power in the k-th cluster, $k = 1, 2, \ldots, K$

cess variation in the manufacture and installation of electronic elements in ES and random fluctuations in the thermal parameters of the environment. Therefore, isothermal temperatures of the core, shell and fluid in the k-th cluster are interval-stochastic and equal to $T_{c,k}(t,\omega)$, $T_{s,k}(t,\omega)$, $T_{a,k}(t,\omega)$ respectively, where ω is elementary events from the space of elementary events Ω in the probability space $\{\Omega, U, P\}$, U is σ-algebra of subsets Ω, P is probability in U [Adomian, 1983; Pugachev, 1962]. The fluid flows to the inlet of the k-th cluster and flows out of the cluster with interval-stochastic temperatures $T_{a,in,k}(t,\omega)$ and $T_{a,out,k}(t,\omega)$, respectively. Being a combination of active heat-generating elements, the core of each k-th cluster consumes total interval-stochastic power $\Phi_k(\omega)$ evenly distributed across the volume of the cluster's core.

The core, shell and fluid flow are in thermal interaction within a cluster not only between each other (Fig. 1, *b*), but also with the core, shell and fluid of the neighboring clusters and the environment outside of the shell having an interval-stochastic temperature $T_e(\omega)$.

Thermal interaction between the cores and between the ends of the contacting shells of the neighboring clusters k, $k-1$ and k, $k+1$ results from the conductive heat exchange carried out through solid-state links and connections, and convective and radiant heat exchange with the common fluid flow through the clusters. This is a consequence of interaction between the heated cores and shells and the fluid flow from through the cluster system when the enthalpy (heat content) of the fluid grows and is further transferred and conveyed to downstream clusters. This leads both to additional heating of the cores and shells of the clusters and increment in the enthalpy of the fluid flow, and correlation relationship between interval-stochastic thermal processes developing in the clusters of the system.

The mathematical model describing interval-stochastic processes in the cluster thermal model (Fig. 1) is based on the following conditions:

- the fluid inside the cluster shell is incompressible; the fluid flow rate changes along the flow direction (*l*, Fig. 1) and has constant cross section; the convective heat flow in the fluid significantly exceeds the heat flow of the thermal conductivity; the internal heat sources originating from the fluid viscosity are small to negligible in comparison with heat generation by active elements of the ES;
- the radiation between the clusters, cores, shells and fluid inside and outside of the clusters is insignificant and is not taken into account, as the maximum temperature of elements in real ES does not exceed 125 °C;
- the dependence of thermophysical properties of the materials of solid-state elements of the ES and fluid inside and outside of the clusters on the temperature in a real range of the operating temperatures of the ES (≤ 125 °C) is small to negligible and not taken into account;
- interval-stochastic values of heat generation power in cluster cores $\Phi_k(\omega)$, ambient temperature $T_e(\omega)$ and inlet temperature of the fluid flow in the ES $T_{a,in}(\omega)$ are statistically independent with mathematical expectations $\bar{\Phi}_k$, $\bar{T}_e$ and $\bar{T}_{a,in}$ and variances D_{Φ_k}, D_{T_e} and $D_{T_{a,in}}$ known from input data;
- interval-stochastic processes in clusters, whose state is determined by temperatures $T_{c,k}(t,\omega)$, $T_{s,k}(t,\omega)$, $T_{a,k}(t,\omega)$, are independent between each other for any clusters with different numbers k and i, $k \neq i$ $(k,i=1,2,\ldots,K)$ and any moments of time, while interval-stochastic temperatures $T_{c,k}(t,\omega)$, $T_{s,k}(t,\omega)$, $T_{a,k}(t,\omega)$, related to the cluster k are dependent.

In these conditions, the mathematical model of conjugate interval-stochastic thermal processes in the *k*-th cluster for each $\omega \in \Omega$, will be as follows [Madera, 2019]:

- for the core of the *k*-th cluster with isothermal temperature $T_{c,k}(t,\omega)$ in the state of conductive heat exchange with the shell of the cluster with the isothermal temperature $T_{s,k}(t,\omega)$, convective heat exchange with the fluid flow with the temperature $T_{a,k}(t,\omega)$ in the cluster, and conductive heat exchange between the cores and the shells of the neighboring clusters $k-1$ and $k+1$

$$\begin{gathered}
h_{c,k}\frac{dT_{c,k}(t,\omega)}{dt}+J_{c-s,k}\left(T_{c,k},T_{s,k},t,\omega\right)+J_{c-a,k}\left(T_{c,k},T_{a,k},t,\omega\right)- \\
-J_{c,k-1-c,k}\left(T_{c,k-1},T_{c,k},t,\omega\right)+J_{c,k-c,k+1}\left(T_{ck},T_{c,k+1},t,\omega\right)=\Phi_k(\omega), \\
T_{c,k}(t=0,\omega)=T_e(\omega),
\end{gathered} \tag{1}$$

where $h_{c,k}=\rho_{c,k}c_{c,k}V_{c,k}$ is the total heat capacity of the core of the *k*-th cluster with the density $\rho_{c,k}$, specific heat capacity $c_{c,k}$, volume $V_{c,k}$; $\Phi_k(\omega)$ is the aggregate power of the internal heat sources (power of consumption by active elements) in the core of the *k*-th cluster;

$$J_{c-s,k}\left(T_{c,k},T_{s,k},t,\omega\right)=g_{c-s,k}^{cond}\cdot\left(T_{c,k}(t,\omega)-T_{s,k}(t,\omega)\right)$$

— the conductive heat flow between the core and the shell of the k-th cluster transferred by the conductive heat transfer $g_{c-s,k}^{cond}$; in the absence of conductive contact between the core and the shell $g_{c-s,k}^{cond}=0$;

$$J_{c-a,k}\left(T_{c,k},T_{a,k},t,\omega\right)=g_{c-a,k}^{conv}\cdot\left(T_{c,k}\left(t,\omega\right)-T_{a,k}\left(t,\omega\right)\right)$$

— the convective heat flow between the core of the k-th cluster and the fluid flow forced н through the k-th cluster, $g_{c-a,k}^{conv}=\alpha_{c-a,k}S_{c-a,k}$ is the convective heat transfer, $\alpha_{c-a,k}$ is the heat-exchange coefficient [Ellison, 2011; Spolding et al., 1990], $S_{c-a,k}$ is the heat-release surface of the core of the k-th cluster;

$$J_{c,k-1-c,k}\left(T_{c,k-1},T_{c,k},t,\omega\right)=g_{c,k-1-c,k}^{cond}\cdot\left(T_{c,k-1}\left(t,\omega\right)-T_{c,k}\left(t,\omega\right)\right),$$

$$J_{c,k-c,k+1}\left(T_{ck},T_{c,k+1},t,\omega\right)=g_{c,k-c,k+1}^{cond}\cdot\left(T_{c,k}\left(t,\omega\right)-T_{c,k+1}\left(t,\omega\right)\right)$$

— the conductive heat flows between the cores of the $k-1$-th and k-the clusters and between the k-the and $k+1$-th clusters, $g_{c,k-1-c,k}^{cond}$ and $g_{c,k-c,k+1}^{cond}$ are the conductive heat transfers; in the absence of conductive thermal contact between the adjacent cores of two clusters the heat transfers $g_{c,k-1-c,k}^{cond}$ and $g_{c,k-c,k+1}^{cond}$ are equal to zero; in contrast, in case of an ideal thermal contact between the adjacent cores, the thermal contact resistances $R_{c,k-1-c,k}^{cond}=1/g_{c,k-1-c,k}^{cond}$ and $R_{c,k-c,k+1}^{cond}=1/g_{c,k-c,k+1}^{cond}$ are equal to zero;

- for the shell of the k-th cluster with isothermal temperature $T_{s,k}\left(t,\omega\right)$ in the state of conductive heat exchange with the core of the k-th cluster with mean temperature $T_{c,k}\left(t,\omega\right)$, convective heat exchange with the fluid flow inside the k-th cluster with isothermal temperature $T_{a,k}\left(t,\omega\right)$, convective heat exchange with the environment with $T_e(\omega)$, conductive heat exchange with the shells of the neighboring clusters $k-1$ and $k+1$

$$h_{s,k}\frac{dT_{s,k}\left(t,\omega\right)}{dt}-J_{c-s,k}\left(T_{s,k},T_{s,k},t,\omega\right)+J_{s-e,k}\left(T_{s,k},T_e,t,\omega\right)-$$
$$-J_{s-a,k}\left(T_{s,k},T_{a,k},t,\omega\right)-J_{s,k-1-s,k}\left(T_{s,k-1},T_{s,k},t,\omega\right)+J_{s,k-s,k+1}\left(T_{s,k},T_{s,k+1},t,\omega\right)=0, \tag{2}$$
$$T_{s,k}\left(t=0,\omega\right)=T_e(\omega),$$

where $h_{s,k}=\rho_{s,k}c_{s,k}V_{s,k}$ is the total heat capacity of the core of the k-the cluster with the density $\rho_{s,k}$, specific heat capacity $c_{s,k}$ and volume $V_{s,k}$;

$$J_{s-e,k}\left(T_{s,k},T_e,t,\omega\right)=g_{s-e,k}^{conv}\cdot\left(T_{s,k}\left(t,\omega\right)-T_e(\omega)\right),$$

$$J_{s-a,k}\left(T_{s,k},T_{a,k},t,\omega\right)=g_{s-a,k}^{conv}\cdot\left(T_{s,k}\left(t,\omega\right)-T_{a,k}\left(t,\omega\right)\right)$$

— the convective heat flows from the external shell surface to the ambient environment and from the internal shell surface to the fluid flow inside the k-th cluster, respectively; $g_{s-e,k}^{conv}=\alpha_{s-e,k}S_{s-e,k}$ and $g_{s-a,k}^{conv}=\alpha_{s-a,k}S_{s-a,k}$ are the convective heat transfers with heat-exchange coefficients $\alpha_{s-e,k}$ and $\alpha_{s-a,k}$ [Ellison, 2011; Spolding et al., 1990] and the outside $S_{s-e,k}$ and inside $S_{s-a,k}$ heat-release surfaces of the shell;

$$J_{s,k-1-s,k}\left(T_{s,k-1},T_{s,k},t,\omega\right)=g^{cond}_{s,k-1-s,k}\cdot\left(T_{s,k-1}\left(t,\omega\right)-T_{s,k}\left(t,\omega\right)\right),$$

$$J_{s,k-s,k+1}\left(T_{sk},T_{s,k+1},t,\omega\right)=g^{cond}_{s,k-s,k+1}\cdot\left(T_{s,k}\left(t,\omega\right)-T_{s,k+1}\left(t,\omega\right)\right)$$

— the conductive heat flows between the shells of the clusters $k-1$-th, *k*-th and *k*-th, $k+1$-th with conductive heat transfers $g^{cond}_{s,k-1-s,k}$ and $g^{cond}_{s,k-s,k+1}$; in the absence of the conductive thermal contact between the shells of the adjacent clusters with the heat transfers $g^{cond}_{s,k-1-s,k}$ and $g^{cond}_{s,k-s,k+1}$ are equal to zero, otherwise, in case of an ideal thermal contact between the shells of the adjacent clusters, the thermal contact resistances $R^{cond}_{s,k-1-s,k}=1/g^{cond}_{s,k-1-s,k}$ and $R^{cond}_{s,k-s,k+1}=1/g^{cond}_{s,k-s,k+1}$ are equal to zero;

- for the fluid flow isothermal temperature $T_{a,k}\left(t,\omega\right)$ in the *k*-th cluster in the state of convective heat exchange with the core and shell of the cluster with isothermal temperatures $T_{c,k}\left(t,\omega\right)$ and $T_{s,k}\left(t,\omega\right)$, respectively [Madera, Kandalov, 2016; Spolding et al., 1990]

$$h_{a,k}\frac{dT_{a,k}\left(t,\omega\right)}{dt}-J_{c-a,k}\left(T_{c,k},T_{a,k},t,\omega\right)+J_{s-a,k}\left(T_{s,k},T_{a,k},t,\omega\right)+$$
$$+J_{a,k}\left(T_{a,k,out},T_{a,k,in},t,\omega\right)=0,$$
$$T_{a,k}\left(t=0,\omega\right)=T_e(\omega), \tag{3}$$

where $h_{a,k}=\rho_{a,k}c_{a,k}V_{a,k}$ is the total heat capacity of the fluid flow through the *k*-th cluster with the density $\rho_{a,k}$, specific heat capacity $c_{a,k}$, volume $V_{a,k}$;

$$J_{a,k}\left(T_{a,k,out},T_{a,k,in},t,\omega\right)=c_{a,k}G_k\left(T_{a,k,out}\left(t,\omega\right)-T_{a,k,in}\left(t,\omega\right)\right)$$

— enthalpy flow of the fluid accumulating heat from the heat-generating elements of the ES in the given cluster; $G_k=\rho_{a,k,in}\upsilon_{a,k,in}S_{a,k,in}=\rho_{a,k,out}\upsilon_{a,k,out}S_{a,k,out}$ is the mass flow of the fluid flowing to the inlet of the *k*-th cluster through the opening with the area $S_{a,k,in}$ at the rate $\upsilon_{a,k,in}$ and with the temperature $T_{a,k,in}\left(t,\omega\right)$, and flowing out of the *k*-th cluster through the outlet with the area $S_{a,k,out}$ at the rate $\upsilon_{a,k,out}$ and with the temperature $T_{a,k,out}\left(t,\omega\right)$.

It should be noted that the constants of the time of thermal processes in the air (τ_a) and solid-state elements (τ_s) are interrelated to each other as $\tau_a \ll \tau_s$. Therefore, thermal processes in the air run at a significantly higher rate than in sold-state elements, and the temperature setting time in the air is significantly shorter that in sold-state elements. In view of this, it may be assumed, with an accuracy sufficient for the engineering practice, that the average temperature of the fluid flow within a single cluster is correlated to the temperatures of the flow at the input and the output of the cluster as $2T_{a,k}\left(t,\omega\right)=T_{a,k,out}\left(t,\omega\right)-T_{a,k,in}\left(t,\omega\right)$ [Ellison, 2011; Dulnev, 1971; Madera, Kandalov, 2016], therefore the flow $J_{a,k}\left(T_{a,k,out},T_{a,k,in},t,\omega\right)$ may be written as $J_{a,k}\left(T_{a,k,out},T_{a,k,in},t,\omega\right)=$ $=2c_{a,k}G_k\left(T_{a,k}\left(t,\omega\right)-T_{a,k,in}\left(t,\omega\right)\right)$.

By applying the expressions for the thermal processes to the equations (1), (2), (3), we will get a mathematical model of interval-stochastic processes in the *k*-th cluster:

- for the core of the *k*-th cluster

$$\begin{gathered} h_{c,k}\frac{dT_{c,k}(t,\omega)}{dt}+g_{c-s,k}^{cond}\cdot\left(T_{c,k}(t,\omega)-T_{s,k}(t,\omega)\right)+ \\ +g_{c-a,k}^{conv}\cdot\left(T_{c,k}(t,\omega)-T_{a,k}(t,\omega)\right)-g_{c,k-1-c,k}^{cond}\cdot\left(T_{c,k-1}(t,\omega)-T_{c,k}(t,\omega)\right)+ \\ +g_{c,k-c,k+1}^{cond}\cdot\left(T_{c,k}(t,\omega)-T_{c,k+1}(t,\omega)\right)=\Phi_k(\omega), \end{gathered} \tag{4}$$

- for the shell of the *k*-th cluster

$$\begin{gathered} h_{s,k}\frac{dT_{s,k}(t,\omega)}{dt}-g_{c-s,k}^{cond}\cdot\left(T_{c,k}(t,\omega)-T_{s,k}(t,\omega)\right)+g_{s-e,k}^{conv}\cdot\left(T_{s,k}(t,\omega)-T_e(\omega)\right)- \\ -g_{s-a,k}^{conv}\cdot\left(T_{s,k}(t,\omega)-T_{a,k}(t,\omega)\right)-g_{s,k-1-s,k}^{cond}\cdot\left(T_{s,k-1}(t,\omega)-T_{s,k}(t,\omega)\right)+ \\ +g_{s,k-s,k+1}^{cond}\cdot\left(T_{s,k}(t,\omega)-T_{s,k+1}(t,\omega)\right)=0, \end{gathered} \tag{5}$$

- for the fluid flow in the *k*-th cluster

$$\begin{gathered} h_{a,k}\frac{dT_{a,k}(t,\omega)}{dt}-g_{c-a,k}^{conv}\cdot\left(T_{c,k}(t,\omega)-T_{a,k}(t,\omega)\right)+g_{s-a,k}^{conv}\cdot\left(T_{s,k}(t,\omega)-T_{a,k}(t,\omega)\right)+ \\ +2c_{a,k}G_k\left(T_{a,k}(t,\omega)-T_{a,k,in}(t,\omega)\right)=0, \end{gathered} \tag{6}$$

or in matrix form

$$\begin{gathered} H_k\frac{dT_k(t,\omega)}{dt}+\mathcal{G}_k\cdot T_k(t,\omega)=P_k(t,\omega), \\ T_k(t=0,\omega)=T_e(\omega)I, \end{gathered} \tag{7}$$

where $T_k(t,\omega)=\left(T_{c,k}(t,\omega),T_{s,k}(t,\omega),T_{a,k}(t,\omega)\right)^T$ is the vector of the interval-stochastic temperatures of the core, shell and fluid flow in the *k*-th cluster; $I=(1,1,1)$ is the unit vector; $H_k=\mathrm{diag}\{h_{c,k},h_{s,k},h_{a,k}\}$ is the deterministic diagonal matrix of the total heat capacities of the core $h_{c,k}$, shell $h_{s,k}$ and fluid flow $h_{a,k}$ in the *k*-th cluster; $\mathcal{G}_k$ is the deterministic matrix of thermal heat transfers of the *k*-th cluster equal to

$$\mathcal{G}_k=\begin{pmatrix} g_k^{(1)} & -g_{c-s,k}^{cond} & -g_{c-a,k}^{conv} \\ -g_{c-s,k}^{cond} & g_k^{(2)} & g_{s-a,k}^{conv} \\ -g_{c-a,k}^{conv} & g_{s-a,k}^{conv} & g_k^{(3)} \end{pmatrix}, \tag{8}$$

$$g_k^{(1)}=g_{c-s,k}^{cond}+g_{c-a,k}^{conv}+g_{c,k-1-c,k}^{cond}+g_{c,k-c,k+1}^{cond},$$

$$g_k^{(2)}=g_{c-s,k}^{cond}+g_{s-e,k}^{conv}-g_{s-a,k}^{conv}+g_{s,k-1-s,k}^{cond}+g_{s,k-s,k+1}^{cond},$$

$$g_k^{(3)}=g_{c-a,k}^{conv}-g_{s-a,k}^{conv}+2c_{a,k}G_k;$$

$P_k(t,\omega)$ is the interval-stochastic vector of the right-hand side of the matrix equation (7)

$$P_k(t,\omega) = \begin{pmatrix} \Phi_k(\omega) + g^{cond}_{c,k-1-c,k} T_{c,k-1}(t,\omega) + g^{cond}_{c,k-c,k+1} T_{c,k+1}(t,\omega) \\ g^{conv}_{s-e,k} T_e(\omega) + g^{cond}_{s,k-1-s,k} T_{s,k-1}(t,\omega) + g^{cond}_{s,k-s,k+1} T_{s,k+1}(t,\omega) \\ 2c_{a,k} G_k T_{a,k,in}(t,\omega) \end{pmatrix}. \tag{9}$$

After a number of manipulations, and taking into account the recurrence relation $T_{a,k,in}(t,\omega) = 2T_{a,k}(t,\omega) - T_{a,k-1,in}(t,\omega)$ resulting from an obvious equation $T_{a,k,in}(t,\omega) = T_{a,k-1,out}(t,\omega)$, $k = 1,2,\ldots,K$, instead of the equation (7) we will obtain a matrix-block equation expressed through the a priori known inlet temperature $T_{a,in}(\omega)$ of the fluid flow in the ES

$$H \frac{dT(t,\omega)}{dt} + \mathcal{G} \cdot T(t,\omega) = Q(\omega), \quad T(t=0,\omega) = T_e(\omega) I, \tag{10}$$

where $T(t,\omega) = (T_1(t,\omega), T_2(t,\omega), \ldots, T_K(t,\omega))^T$ is a block vector of interval-stochastic temperatures of clusters, where each k-the vector block is equal to $T_k(t,\omega) = (T_{c,k}(t,\omega), T_{s,k}(t,\omega), T_{a,k}(t,\omega))^T$, $k = 1,2,\ldots,K$; $H = \operatorname{diag}\{H_1, H_2, \ldots, H_K\}$ is a deterministic block-diagonal matrix of total heat capacities of clusters, consisting of diagonal matrix blocks $H_k = \operatorname{diag}\{h_{c,k}, h_{s,k}, h_{a,k}\}$, $k = 1,2,\ldots,K$; $Q(\omega) = (Q_1(\omega), Q_2(\omega), \ldots, Q_K(\omega))^T$ is an interval-stochastic block vector containing a priori known cluster power $\Phi_k(\omega)$, temperatures of the ambient environment $T_e(\omega)$ and fluid flow at the inlet of the ES $T_{a,in}(\omega)$, where the k-th vector block is equal to $Q_k(t,\omega) = \left(\Phi_k(\omega), g^{conv}_{s-e,k} T_e(\omega), 2(-1)^{k-1} c_{a,k} G_k T_{a,in}(\omega)\right)^T$; $\mathcal{G}$ is a deterministic block $3K \times 3K$-matrix of the heat transfers of the clusters with the structure

$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} & 0 & \cdots & 0 \\ \mathcal{G}_{21} & \mathcal{G}_{22} & \mathcal{G}_{23} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \mathcal{G}_{K,1} & \mathcal{G}_{K,2} & \mathcal{G}_{K,3} & \cdots & \mathcal{G}_{KK} \end{pmatrix} \tag{11}$$

with diagonal matrix blocks $\mathcal{G}_{kk}$, $k = 1,2,\ldots,K$, equal to matrixes $\mathcal{G}_k$, as shown in (8), and diagonal matrix blocks equal to:

$$\mathcal{G}_{i,i-1} = \operatorname{diag}\{g^{cond}_{c,i-1-c,i},\ g^{cond}_{s,k-1-s,i},\ 4c_{a,i} G_i\}, \quad i = 2,3,\ldots,K,$$

$$\mathcal{G}_{i,i+1} = \operatorname{diag}\{g^{cond}_{c,i-c,i+1},\ g^{cond}_{s,i-s,i+1},\ 4c_{a,i} G_i\}, \quad i = 1,2,\ldots,K-1,$$

$$\mathcal{G}_{ij} = (-1)^{i-j} \operatorname{diag}\{0, 0,\ 4c_{a,i} G_i\}, \quad i = 3,4,\ldots,K, \ j = 1,2,\ldots,K-2.$$

3. Determination of equations for interval statistical measures of the state variables of interval-stochastic thermal processes in clusters

The states of interval-stochastic measures of thermal processes in the k-th cluster are determined by the temperatures of the core $T_{c,k}(t,\omega)$, shell $T_{s,k}(t,\omega)$ and fluid flow $T_{a,k}(t,\omega)$, which are fully characterized by their statistical measures [Madera, 2020; Madera, Kandalov, 2016], namely:

- mathematical expectations of the temperatures

$$\overline{T}_{c,k}(t) = E\{\overset{0}{T}_{c,k}(t,\omega)\}, \quad \overset{0}{T}_{c,k}(t,\omega) = T_{c,k}(t,\omega) - \overline{T}_{c,k}(t),$$
$$\overline{T}_{s,k}(t) = E\{\overset{0}{T}_{s,k}(t,\omega)\}, \quad \overset{0}{T}_{s,k}(t,\omega) = T_{s,k}(t,\omega) - \overline{T}_{s,k}(t),$$
$$\overline{T}_{a,k}(t) = E\{\overset{0}{T}_{a,k}(t,\omega)\}, \quad \overset{0}{T}_{a,k}(t,\omega) = T_{a,k}(t,\omega) - \overline{T}_{a,k}(t),$$

  where $E\{\cdot\}$ is the mathematical expectation operator;
- covariance matrix $K_{TT}(t) = E\{\overset{0}{T}_k(t,\omega)\cdot(\overset{0}{T}_k(t,\omega))^T\}$ of the vector of state temperatures of the k-th cluster $T_k(t,\omega) = (T_{c,k}(t,\omega), T_{s,k}(t,\omega), T_{a,k}(t,\omega))^T$;
- variances $D_{T_{c,k}}$, $D_{T_{s,k}}$, $D_{T_{a,k}}$ equal to diagonal matrix elements $K_{TT}(t)$, and standard deviations $\sigma_{T_{c,k}}$, $\sigma_{T_{s,k}}$, $\sigma_{T_{a,k}}$.

The resulting statistical measures $\overline{T}$ and σ_T of the interval stochastic temperature $T(t,\omega)$ are used to find the intervals $[T_{bot}(t), T_{up}(t)]$, which will contain real values of the temperatures $T(t,\omega)$ of various clusters in the thermal model of the ES. The lower and the upper limits of the intervals $T_{bot}(t)$ and $T_{up}(t)$ are

$$T_{bot}(t) = \overline{T}(t) - \epsilon\cdot\sigma_T(t) \text{ and } T_{up}(t) = \overline{T} + \epsilon\cdot\sigma_T(t),$$

where ϵ is a coefficient determined by the confidence level and Chebyshev's inequality [Pugachev, 1962; Madera, Kandalov, 2016].

To find the statistical measures of the interval-stochastic temperatures $T_{c,k}(t,\omega)$, $T_{s,k}(t,\omega)$ and $T_{a,k}(t,\omega)$, we will use the method [Madera, Kandalov, 2016; Madera, 2019; Madera, 2020] in all clusters of the thermal model, and obtain equations for the statistical measures of the interval-stochastic block vector $T(t,\omega) = (T_1(t,\omega), T_2(t,\omega), \ldots, T_K(t,\omega))^T$ in all clusters, namely, the vector of mathematical expectations $\overline{T}(t) = E\{T(t,\omega)\}$ and covariance matrix $K_{TT}(t) = E\{\overset{0}{T}(t,\omega)\cdot(\overset{0}{T}(t,\omega))^T\}$ of the centered interval-stochastic vector of temperatures $\overset{0}{T}(t,\omega) = T(t,\omega) - \overline{T}(t)$.

The equation for the vector of mathematical expectations of the temperatures $\overline{T}(t)$ will be obtained right after the mathematical expectation operator has been applied to the equation (10)

$$H\frac{d\overline{T}(t)}{dt} + \mathcal{G}\cdot\overline{T}(t) = \overline{Q}, \quad \overline{T}(t=0) = \overline{T}_e I, \tag{12}$$

where $\overline{T}(t)=(\overline{T}_1(t),\overline{T}_2(t),\ldots,\overline{T}_K(t))^T$ is a block vector of mathematical expectations of the temperatures of the clusters (core, shell, fluid), where each vector block is equal to $\overline{T}_k(t)=(\overline{T}_{c,k}(t),\overline{T}_{s,k}(t),\overline{T}_{a,k}(t))^T$; $\overline{Q}=(\overline{Q}_1,\overline{Q}_2,\ldots,\overline{Q}_K)^T$ is block vector of mathematical expectations, where each vector block is equal to $\overline{Q}_k=(\overline{\Phi}_k,\ g_{s-e,k}^{conv}\,\overline{T}_e,\ 2(-1)^{k-1}c_{a,k}G_k\overline{T}_{a,in})^T$.

The equation for the covariance matrix $K_{TT}(t)$ will be obtained by subtracting the equation (12) from the equation (10):

$$H\frac{d\overset{0}{T}(t,\omega)}{dt}+\mathcal{G}\cdot\overset{0}{T}(t,\omega)=\overset{0}{Q}(\omega),\quad \overset{0}{T}_k(t=0,\omega)=\overset{0}{T}_e(\omega)I,$$

and applying the method [Madera, Kandalov, 2016; Madera, 2019; Madera, 2020] to the same. This will result in the equation for the covariance matrix $K_{TT}(t)$:

$$H\frac{dK_{TT}(t)}{dt}H+\mathcal{G}\cdot K_{TT}(t)H+HK_{TT}(t)\cdot\mathcal{G}^T=HK_{TQ}(t)+K_{TQ}^T(t)H,$$
$$K_{TT}(t=0)=D_{T_e}I\cdot I^T, \tag{13}$$

which is solved along with the matrix-block equation for the covariance matrix $K_{TQ}(t)=$ $=E\{\overset{0}{T}(t,\omega)\cdot(\overset{0}{Q}(t,\omega))^T\}$

$$H\frac{dK_{TQ}(t)}{dt}+\mathcal{G}K_{TQ}(t)=K_{QQ},\quad K_{TQ}(t=0)=D_{T_e}\mathcal{Q}, \tag{14}$$

where $K_{QQ}(t)=E\{\overset{0}{Q}(\omega)(\overset{0}{Q}(\omega))^T\}$ is square block covariance matrix with diagonal matrix blocks equal to

$$K_{Q,ij}=\operatorname{diag}\{0,g_{s-e,i}^{conv}\,g_{s-e,j}^{conv}\,D_{T_e},\ 4(-1)^{i+j}\,c_{a,i}c_{a,j}G_iG_j\,D_{T_{a,in}}\},$$
$$K_{Q,ii}=\operatorname{diag}\{D_{\Phi_i},g_{s-e,i}^{conv}g_{s-e,j}^{conv}\,D_{T_e},\ 4(-1)^{i+j}\,c_{a,i}c_{a,j}G_iG_jD_{T_{a,in}}\};$$

$\mathcal{Q}$ is rectangular block $3K\times K$ -matrix with rectangular 1×3 matrix blocks $\mathcal{Q}_{ij}=(0,g_{s-e,j}^{conv}\,D_{T_e},0)$.

To find the interval statistical measures for the equations (12), (13), (14), first we need to solve the matrix-block equation (12) for mathematical expectations of the temperatures $\overline{T}(t)$ in all clusters, then the matrix-block equation (14) for the covariance matrix $K_{TQ}(t)$, which will then be applied to the equation (13) and solved for the sought correlation matrix $K_{TT}(t)$.

The sets of equations (12), (13), (14) are matrix-block linear differential equations in ordinary first-order derivatives, which are solved using known numerical methods.

4. Application of the cluster method

Let us consider the application of the cluster method through an example of an ES, which is a device in a flat case (*laptop*) and contains four electronic modules (EM) cooled by the air flow forced from the environment to the inlet of the ES (Fig. 2). Due to statistical process variation in the manufacture of the electronic elements installed in the ES, and, consequently, heat generation power in EM, as well as the stochastic nature of the ambient temperature, thermal processes in the ES are interval-stochastic ones. The thermal model of the ES contains five clusters with the fourth EM being divided into two clusters — the fourth and the fifth ones (Fig. 2). The statistical measures (mathematical expectation (ME), variation interval (VI), standard deviation (SD)) of the interval-stochastic input data, which are entries for modeling, are provided in Table 1.

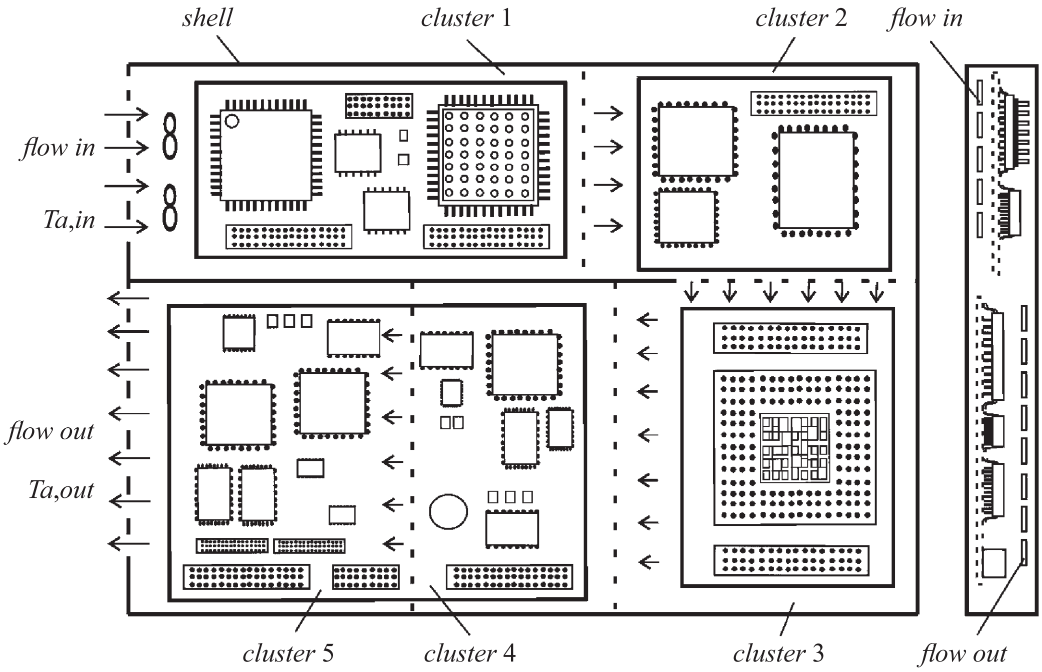


Fig. 2. Electronic system and cluster thermal model. The dashed line shows division into clusters

Table 1. Statistical measures (ME, VI, SD) of the interval-stochastic input data for modeling of thermal processes in ES

Statistical measures	**Temperature of the ambient environment and fluid flow at the inlet of the ES, °C**		**Heat generation power of the ES, W**				
	$T_e(\omega)$	$T_{a,in}(\omega)$	$\Phi_1(\omega)$	$\Phi_2(\omega)$	$\Phi_3(\omega)$	$\Phi_4(\omega)$	$\Phi_5(\omega)$
ME	23	23	22	10	15	8	14
VI	19.7÷23.3	19.7÷23.3	17.5÷26.5	7.6÷12.4	12÷18	6.2÷9.8	10.4÷17.6
SD	1.1	1.1	1.5	0.8	1	0.6	1.2

Interval-stochastic thermal processes in the ES have been modeled for stationary (steady-state) conditions, described by a stationary mathematical model following from the equations (12), (13), (14), namely:

$$\mathcal{G}\cdot\bar{T}=\bar{Q},$$
$$\mathcal{G}\cdot K_{TT}H+HK_{TT}\cdot\mathcal{G}^T=HK_{TQ}+K_{TQ}^T H,$$
$$\mathcal{G}K_{TQ}=K_{QQ}. \tag{15}$$

The first equation in (15) describes 3×5-block vector of stationary mathematical expectations in five clusters $\overline{T}(t)=(\overline{T}_1,\overline{T}_2,\overline{T}_3,\overline{T}_4,\overline{T}_5)^T$ with vector blocks $\overline{T}_k=(\overline{T}_{c,k},\overline{T}_{s,k},\overline{T}_{a,k})^T$ containing mathematical expectations of the temperatures of the core, shell and fluid in the k-th cluster, $k=1,2,3,4,5$. The first part of the equation is a 3×5 block vector $\overline{Q}=(\overline{Q}_1,\overline{Q}_2,\overline{Q}_3,\overline{Q}_4,\overline{Q}_5)^T$ with vector blocks $\overline{Q}_k=(\overline{\Phi}_k,\, g^{conv}_{s-e,k}\,\overline{T}_e,\, 2(-1)^{k-1}c_{a,k}G_k\overline{T}_{a,in})^T$, $k=1,2,3,4,5$. The second equation of the set (15) determines a covariance 15×15 matrix K_{TT} containing covariances between the temperatures of the core and fluid flow in all five clusters. The equation is solved along with the third equation in the set (15), which determines a matrix of covariances between the temperatures of the core, shell and fluid in all clusters, as well as between the temperatures of the core, shell and fluid in all clusters with known reference temperatures of the ambient environment and fluid flow at the inlet of the ES.

Solutions to the mathematical model equations (15) represented by statistical measures, namely, mathematical expectations (ME), and minimum (MIN) and maximum (MAX) temperature interval values of the core, shell, and fluid in all five clusters are provided in Table 2.

Table 2. Modeling results for the temperatures of the core $T_{c,k}$, shell $T_{s,k}$, fluid $T_{a,k}$ (°C) in clusters $k=1-5$

Statistical measures	**Cluster 1**			**Cluster 2**			**Cluster 3**			**Cluster 4**			**Cluster 5**		
	$T_{c,1}$	$T_{s,1}$	$T_{a,1}$	$T_{c,2}$	$T_{s,2}$	$T_{a,2}$	$T_{c,3}$	$T_{s,3}$	$T_{a,3}$	$T_{c,4}$	$T_{s,4}$	$T_{a,4}$	$T_{c,5}$	$T_{s,5}$	$T_{a,5}$
ME	76.9	27.4	25.9	64.7	33.4	30	77.3	37.5	32.7	66.7	40.9	35	57.1	42.8	36.2
MIN	62.5	23.2	22	51.6	27.9	25.2	63	31.1	27.3	53.7	33.8	29.1	45.5	35.2	30 2
MAX	91.2	31.6	29.8	77.8	38.9	34.7	91.6	43.9	38	79.7	48	40.8	68.6	50.5	42.4

Fig. 3 shows the calculated distributions of the mathematical expectation (center line), minimum (bottom line) and maximum (top line) of the temperature values of the core and fluid flow in clusters along the entire flow path from the inlet to the outlet of the ES. The results obtained are indicative of a significant variation of the temperature values of the core, and a little lower variation of the temperature in the fluid flow. Thus, maximum variation in the temperature of the core (cluster 1) is 28.6 °C, while variation in the temperature of the fluid flow (in cluster 5) is 12.4 °C. The reason is that the core of a cluster consisting of multiple active elements is heat-generating, while the fluid flow accumulates heat from the core. The results also show that the distribution of the temperature in the fluid flow along the path to the ES is different from a linear one, therefore the assumption that the distribution of temperature in the fluid flow to the ES is of linear nature may not be considered adequate. Interval temperature values in the clusters (core, shell, fluid flow) obtained from modeling suggest that any temperature from the variation intervals may be observed in operation of real ES of the same type. Considering significant dependence of the electrical parameters of ES on temperature, variation of the temperature of active elements (processors, integrated circuits) causes variation of electrical parameters. Thus, for a number of processors, an increase in the temperature of the core by 1 grade leads to a drop in performance by 3.5%, and considering that in the ES under examination the maximum interval variation in the temperature of the processor (core of the cluster) is almost 30 degrees, the drop in performance will be 35%.

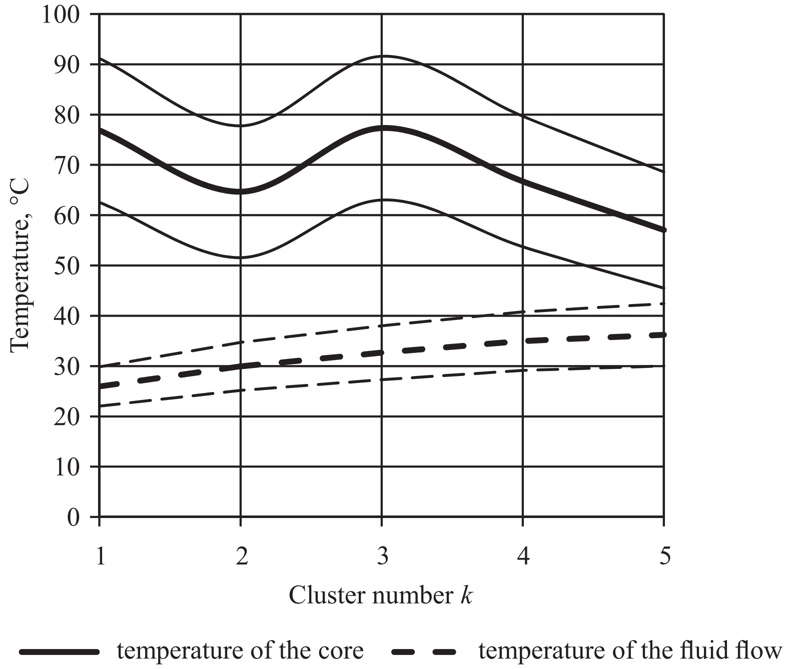


Fig. 3. Distribution of mathematical expectation (center line), minimum (bottom line) and maximum (top line) of the temperature values of the core and fluid flow by clusters k in the thermal model

5. Conclusion

The existing methods of modeling of thermal processes in complex ES assume that the distribution of temperature in the fluid flow forced through the ES is even. However, thermal interaction between the fluid flow and heat-generating elements in the ES in practice leads to an uneven distribution of temperature in the fluid flow. Indeed, the fluid flow inside the ES accumulates heat from heated elements, thus increasing its enthalpy, which is further transferred and conveyed to heat-generating elements upstream of the flow and causes their additional heating and a growth of the enthalpy of the flow resulting in the growth of the fluid temperature at the outlet of the ES over the inlet fluid temperature. Modeling of the heat exchange between the fluid and heat-generating elements in the ES should be also be considered in conjugate settings, where the interaction between the fluid and heated elements leads to a growth of the enthalpy of the flow, which in turn causes heating of elements, thus closing the feedback loop. In addition, due to unavoidable process variation in the manufacture and installation of elements in the ES, and random fluctuations in the thermal parameters of the environment, thermal processes in the ES are interval-stochastic ones. Ignoring such factors as uneven distribution of the fluid temperature in the ES, conjugate nature of the heat exchange between the fluid flow and heat-generating elements, and interval-stochastic nature of the thermal processes in ES, leads to inadequate modeling and significant errors in the ES design, and, eventually, creation of uncompetitive hardware.

The cluster method of modeling described in this paper allows determination of the distribution of temperature in heat-generating elements, ES case and fluid flow considering uneven temperature distributions, conjugate nature of the heat exchange and interval stochastic nature of the thermal processes in the ES. The cluster thermal model of the ES structure is a system of clusters, in each of which the state of a thermal process is characterized by three interval-stochastic and isothermal temperatures, namely, temperature of the core of the cluster heat-generating elements falling into the given cluster, temperature of the cluster shell, and temperature of the fluid flow within the volume of the cluster. All elements of an individual cluster (core, shell, fluid) and interacting elements of the neighboring clusters are in the state of conjugate heat exchange. The cluster mathematical model is based on the cluster thermal model being a system of interval-stochastic matrix-block equations with matrix and vector blocks corresponding to various clusters of the thermal model. These equations are used to obtain matrix-block equations for statistical measures of the state of stochastic thermal processes in clusters — mathematical expectations, covariances between state variables, and variances. Application of the described method is shown through an example of a real ES being a computer system containing several EM, which are in the state of heat exchange with the cooling fluid flow inside the ES case and the ambient environment.

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