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MODELS IN PHYSICS AND TECHNOLOGY

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A modified model of the effect of stress concentration near a broken fiber on the tensile strength of high-strength composites (MLLS-6)

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The article proposes a model for assessing the potential strength of a composite material based on modern fibers with brittle fracture.

Materials consisting of parallel cylindrical fibers that are quasi-statically stretched in one direction are simulated. It is assumed that the sample is not less than 100 pieces, which corresponds to almost significant cases. It is known that the fibers have a distribution of ultimate deformation in the sample and are not destroyed at the same moment. Usually the distribution of their properties is described by the Weibull–Gnedenko statistical distribution. To simulate the strength of the composite, a model of fiber breaks accumulation is used. It is assumed that the fibers united by the polymer matrix are crushed to twice the inefficient length — the distance at which the stresses increase from the end of the broken fiber to the middle one. However, this model greatly overestimates the strength of composites with brittle fibers. For example, carbon and glass fibers are destroyed in this way.

In some cases, earlier attempts were made to take into account the stress concentration near the broken fiber (Hedgepest model, Ermolenko model, shear analysis), but such models either required a lot of initial data or did not coincide with the experiment. In addition, such models idealize the packing of fibers in the composite to the regular hexagonal packing.

The model combines the shear analysis approach to stress distribution near the destroyed fiber and the statistical approach of fiber strength based on the Weibull–Gnedenko distribution, while introducing a number of assumptions that simplify the calculation without loss of accuracy.

It is assumed that the stress concentration on the adjacent fiber increases the probability of its destruction in accordance with the Weibull distribution, and the number of such fibers with an increased probability of destruction is directly related to the number already destroyed before. All initial data can be obtained from simple experiments. It is shown that accounting for redistribution only for the nearest fibers gives an accurate forecast.

This allowed a complete calculation of the strength of the composite. The experimental data we obtained on carbon fibers, glass fibers and model composites based on them (CFRP, GFRP), confirm some of the conclusions of the model.

Keywords: carbon fibers, modulus of elasticity, tensile deformation, speed of sound

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Ки&М

МОДЕЛИ В ФИЗИКЕ И ТЕХНОЛОГИИ

УДК: 519.6

Модифицированная модель влияния концентрации напряжений вблизи разорванного волокна на прочность высокопрочных композитов при растяжении (MLLS-6)

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В статье предложена модель для оценки потенциальной прочности композиционного материала на основе современных волокон, разрушающихся хрупко.

Моделируются материалы, состоящие из параллельных цилиндрических волокон, которые квазистатически растягиваются в одном направлении. Предполагается, что в выборке не меньше 100 штук, что соответствует практически значимым случаям. Известно, что волокна имеют разброс предельной деформации в выборке и разрушаются не одновременно. Обычно разброс их свойств описывается распределением Вейбулла–Гнеденко. Для моделирования прочности композита используется модель накопления разрывов волокон. Предполагается, что волокна, объединенные матрицей, дробятся до удвоенной неэффективной длины — расстояния, на котором возрастают напряжения от торца разорванного волокна до среднего. Однако такая модель сильно завышает прогноз прочности композитов с хрупкими волокнами. Например, так разрушаются углеродные и стеклянные волокна.

В ряде случаев ранее делались попытки учесть концентрацию напряжений около разорванного волокна (модель Хеджепеста, модель Ермоленко, сдвиговой анализ), однако такие модели требовали или очень много исходных данных или не совпадали с экспериментом. Кроме того, такие модели идеализировали упаковку волокон в композите до регулярной гексагональной упаковки.

В модели объединены подход сдвигового анализа к распределению напряжений около разрушенного волокна и статистический подход прочности волокон на основе распределения Вейбулла–Гнеденко, при этом введен ряд предположений, упрощающих расчет без потери точности.

Предполагается, что перенапряжение на соседнем волокне увеличивает вероятность его разрушения в соответствии с распределением Вейбулла и число таких волокон с повышенной вероятностью разрушения прямо связано с числом уже разрушенных до этого. Все исходные данные могут быть получены из простых экспериментов. Показано, что учет перераспределения только на ближайшие волокна дает точный прогноз.

Это позволило провести полный расчет прочности композита. Экспериментальные данные, полученные нами на углеродных волокнах, стеклянных волокнах и модельных композитах на их основе, качественно подтверждают выводы модели.

Ключевые слова: углеродные волокна, модуль упругости, деформация при растяжении, скорость звука

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1. Introduction

Recently, the production of high-strength fibers and composite materials based on them has sharply increased. The area of their application is expanding, and such materials are being used in new industries. Wind power and construction have been added to the traditional aerospace applications of composites. New fibers are being developed, including nanosized ones, and the properties of materials cannot always be predicted on the basis of conventional standards; therefore, the study of micromechanisms of deformation and fracture of composites is very important [Mikhailin, 2013].

In this paper, we investigate fibrous materials from parallel high-strength and high-modular fibers, united by a polymer matrix. In such composite materials, in contrast to ceramic or metal composites, the mechanical properties of fibers and matrices differ greatly, and to achieve the properties, it is necessary to ensure joint work. The ultimate strength of the fibers ranges from 2 to 7 GPa, the modulus of elasticity is 70 to 500 GPa, the ultimate deformation is 1 to 5 %, the majority of high-strength fibers are deformed almost linearly to failure. The tensile strength of the matrix that joins the fibers together ranges from 20 to 90 MPa, the modulus of elasticity is 1 to 5 GPa, the deformation diagram usually has a plastic section after 3–5 %.

For practical applications, materials with multidirectional layers or braided materials are important. They are made by laying out prepreg, infusion or winding, but even in this case, the layer with the 0º direction plays a significant role in the rigidity and strength, therefore, the prediction of the strength of such a layer is very important [Matthews, Rollings, 2004; Gorynin, Nemirovskii, Vlasko, 2017].

To predict the properties of unidirectional fiber composites, several alternative approaches are used, for example, calculations are performed according to the mixture rule without taking into account statistical properties, when the mechanical properties of fibers and the matrix are averaged. A variant of the calculation is possible when the properties of the matrix are not taken into account at all due to the huge difference in the values of the properties. A case is possible where the composite is considered to be an anisotropic solid [Vasiliev, Morozov, 2013].

However, this approach is insufficient, and models have emerged that describe the layered or fibrous structure of materials. Layered models are important because most real-world materials are formed from layers. In addition, layered models often provide good quality predictions. In particular, it is possible to analytically calculate the stress field in a layered structure, but most layers have up to 20 fiber diameters, and in this case the ultimate properties are determined by the fibers and it is necessary to know the micromechanism of destruction. In addition, most modern high tenacity fibers are produced as filaments rather than individual filaments [Daniels, 1945].

For quite a long time, the strength of composites has been described using the concept of accumulation of fiber breaks and the concept of an ineffective length, one at which stresses are transmitted from the break to adjacent fibers. These are calculations based on the Rosen model (elastic calculations) [Argon, 1978] or calculations based on the Kelly model (calculations for the case of plasticity) [Kelly, Tyson, 1965]:

$$\delta = R\frac{E_f}{\tau}\varepsilon, \tag{1}$$

where R is the radius of the fiber, E_f is the elastic modulus of the fiber, ε is the ultimate deformation, and τ is the yield stress of the matrix.

The statistical properties of fibers from bundles are described based on the Daniels model and the Weibull–Gnedenko distribution [Orlov, 2014]. This model assumes that the fiber is a chain, the breakage probability of which depends on the fiber length and on the defectiveness parameter m. The probability of fiber breakage $f(x)$ is described by the formula

$$f(x) = 1 - \exp\left(-L\left(\frac{x}{x_0}\right)^m\right), \tag{2}$$

where L is the bundle's length, x is the deformation of bundle, and m and x_0 are the strength distribution parameters of the fibers.

In this case, the load carried by the fiber bundle, $P(x)$, is described by the expression

$$P(x) = E_f \varepsilon \left[\exp\left(-L \left(\frac{\varepsilon}{\varepsilon_0} \right)^m \right) \right], \quad (3)$$

where L is the bundle length, E_f is the elastic modulus of the fiber, ε is deformation, and m and ε_0 are the parameters of the fiber strength distribution.

Regardless of the type of distribution, the expression for the load looks like

$$P(x) = E \cdot x \cdot (1 - f(x)), \quad (4)$$

where E_f is the elastic modulus of the fiber and $f(x)$ is the probability of destruction in the range from 0 to x.

In the case of distribution (3), the maximum strength of the bundle will be described by the expression (5) and the average strength is described by the expression (6) [Nemez, Strelyaev]:

$$P_m(x) = E\varepsilon_0 \left(\frac{1}{mL} \right)^{1/m} \exp\left(-\frac{1}{m} \right), \quad (5)$$

where E_f is the elastic modulus of the fiber, and m and ε_0 are the strength distribution parameters of the fibers:

$$Kf(m) = \frac{(2.71m)^{-1/m}}{\Gamma\left(\left(1 + \frac{1}{m} \right) \right)}, \quad (6)$$

where K is the ratio of the strength of the bundle to the average strength of the fibers and m is the strength distribution parameter of the fibers.

Formula (6) makes it possible to relate the starting strength of the fibers and the average strength of the fibers in the Weibull distribution.

These models are united by the assumption that the strength of the composite is the same as that of a bundle of reinforcing fibers at a doubled ineffective length (Fig. 1).

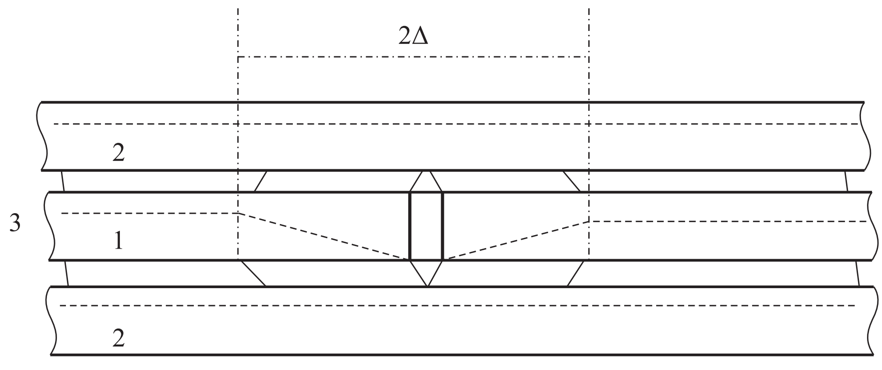

Fig. 1. Scheme of ineffective length. 1 — torn fiber, 2 — adjacent fibers, 3 — distribution of deformations in the matrix; Δ — ineffective length

Different models solve the problem of calculating the ineffective length and taking into account the redistribution of stresses in the region of a broken fiber or a group of fibers in different ways. A separate issue is forecast, that is, experimental determination of the ineffective length and determination of the parameters of the Weibull distribution for fibers.

A number of models assume that, after a fiber breaks, the stresses are evenly distributed to all other fibers; such models are called GLS. These models take into account the likelihood of multiple breaks and evolve to the concept of Critical number of breaks (CNB) or critical defect and are calculated by the Monte Carlo method. Different models interpret the size of the critical defect (CNB) differently, GLS models only give an overestimated strength limit. In the case of using the CNB concept, such models are close to the macromodels of a crack in an anisotropic solid [Vanegas-Jaramillo et al., 2018].

The exact calculation of the stress field in a fiber composite is difficult, since it requires knowledge of the elastic parameters of the fiber and matrix. Although such attempts have been made, shear analysis (SLA) models have been widely developed, which assume that tensile loads are carried only by fibers, and shear loads are carried only by a matrix, and this approach neglects Poisson's ratios of both. The model was proposed by Hedgepeth [Hedgepeth, 1961]. Within the framework of this model and its development, various versions of the calculation of shear fields have been proposed [Argon, 1978; Smith et al., 1983; McClintock, 1969; Kopiev, Ovchinsky, 1974]. These models are called LLS.

In the mid-1990s, LLS models began to take into account fracture statistics [Smith et al., 1983].

Methods for determining the ineffective length by studying a single fiber in a composite [Curtin, 2000] and a method for determining the properties of a matrix when pulling out individual fibers [Kopyev, Ovchinsky, Pompe, 1976] were developed separately. In this case, the experiment on a single fiber, in principle, cannot take into account the interaction of neighboring fibers. Figure 2 shows the arrangement of adjacent fibers in the computational models.

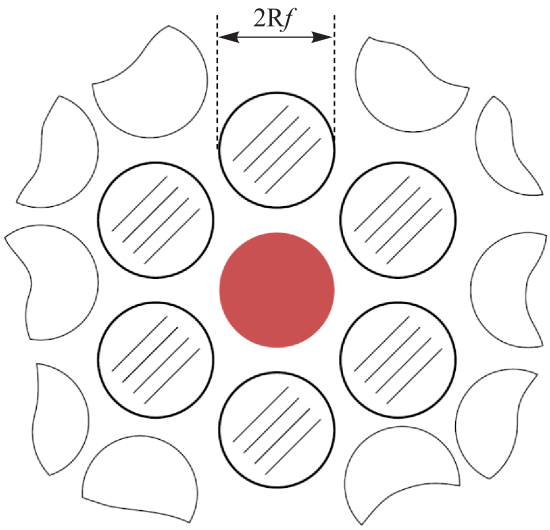

Fig. 2. Arrangement of adjacent fibers in computational models

An overview of the current state of the models is given in the article by Mishlaevsky [Mishnaevsky, Brondsted, 2009].

It follows from this review that most of the articles refer to binders and fibers developed before 2000, models based on mathematical modeling and Monte Carlo methods require more input data, and the result is weakly tied to experiment, the work of Soviet and Russian authors completely dropped out of the review.

Smith's model [Swolfs et al., 2013] calculates the size of a fiber cluster, at which its growth becomes irreversible, based on statistics and an assumption about the stress on the nearest fibers. In our opinion, this approach does not take into account both shear deformations in the cluster itself and the redistribution of stresses beyond the nearest fibers in the composite. As shown by McClintock [McClintock, 1969], in the case of plasticity in the layer, the stress distribution is strongly softened.

Ermolenko's model [Ermolenko, 1985] combines the concepts of shear analysis near the fiber and the stress distribution in an anisotropic body with averaged characteristics.

In a number of papers, the average fiber strength is taken for comparison with experiment, but not the strength of the bundle; this assumption introduces a systemic error in the calculations, since the strength of the fiber bundle is lower than the average strength of the fibers. Note that all modern fibers that are important for use are produced in the form of complex filaments, and not separate monofilaments. Therefore, the use of the properties of individual fibers to predict the properties of a composite can contain system errors both in terms of determining the elastic modulus and in terms of determining the strength of monofilaments, since they are damaged when they are removed from a complex yarn [Zhou, Wagner, 1999; Ermolenko, 1985]. In addition, it was noted that the distribution maximum shifts upward, as the most fragile fibers disintegrate [Kopiev, Ovchinsky, 1974].

The question of determining the parameters of strength distribution on fibers extracted from a complex yarn remains ambiguous, since in this case the fibers can be damaged, the number of fibers is limited, and the distribution approximation is used to determine m.

In addition, although in structural materials the fibers occupy a volume fraction of more than 50 % and are packed quite tightly, the distance between them varies randomly from 0 to 2 diameters, which affects the stress distribution and significantly reduces the value of beautiful theoretical models with a regular lattice [Ma et al., 2016].

In the present work, we are trying to combine the modified LLS (local stress distribution) model, taking into account the probability of the Weibull strength distribution of fibers, and we can obtain data on the distribution parameters from data on testing multifilament yarns from 100 to 1000 fibers. Figure 3 shows the cleavage of a sample from one complex glass filament after destruction. A photo taken with SEM (scanning electron microscope). After tensile fracture, the samples were examined using a JEOL JSM-35 SEM. To eliminate electrization under the action of an electron beam, gold was deposited on microcomposite samples (microplastics) using a SMARTLAB-R1 laboratory vacuum system. It can be seen that this is a real composite material, but the accumulation of fiber breaks is not noticeable despite the fact that the sample failed at a deformation of 3.5 %.

Fig. 3. Chipping of fiberglass VMN after fracture and under tension (photo from the archive of P. V. Mikheev, taken while working on a dissertation at the Institute of Chemical Physics of the Academy of Sciences of the USSR)

It should be noted that, when describing the mechanisms of destruction of high-strength fiber, we exclude from consideration fibers that have an internal structure — aramid and SVM, since, in our opinion, the mechanism of their destruction in the composite is completely different [Mikheev, Mostovoy, Konyushenkov, 2019]. This approach does not contradict the generally accepted one, since the strength of composites based on such fibers with an internal structure is significantly higher than the strength of untreated threads.

Our modified LLS model, let's call it MLLS, only describes high-strength homogeneous fibers that form a sharp end when broken

2. Statement of the problem and basic notation

The discrete structure of the composite makes it possible to assume that the stress from the broken fiber is distributed over 6 nearest fibers (Fig. 2), the concentration on which will be 1.167. In our model, this means an increased likelihood of destruction. On the other hand, the Weibull model gives us the number of fibers broken at a given deformation, so it is possible in a differential form to estimate the increment in the number of fibers associated with the concentration

$$f_k' = f'(x) + 6 f_k(x) f'(1.167x). \tag{7}$$

The damage accumulation model is based on two assumptions about power-law growth (formula (2)) and the idea that a long thread is a chain. The power function describes the probability of destruction of the final link. This probability is presented in formula (8).

$$F(x) = \begin{cases} 0 \text{ if } x < 0, \\ x^m \text{ if } 0 < x < 1, \\ 1 \text{ if } x > 1. \end{cases} \tag{8}$$

The strength of the beam without taking into account the concentration is given by the expression

$$P_m(x_m) = \left(1 - \frac{1}{2m}\right)\left(\frac{1}{2m}\right)^{\frac{1}{m}}. \tag{9}$$

This allows us to assume that the element of the chain is precisely the ineffective length, and formula (2) can be used to calculate the number of broken fibers at the ineffective length if we use *m*. In this simplified case, for the probability of fracture, taking into account the concentration on neighboring fibers, one can obtain the expression (10).

$$F_{k_1}(x) = x^m + 3 \cdot 1.155^{m-1} \cdot x^{2m}. \tag{10}$$

If we substitute the expression for the probability in formula (4), then we can numerically calculate the beam deformation diagram for different m.

2.1. Solution of the differential equation for different values of m

However, the solution of the differential equation (6) is more accurate for calculating strength. We assume that the modulus of elasticity is 1 and the length does not change and is equal to the ineffective one.

The drop in strength according to our model will be compared with the numerical solution of the load distribution on the whole fiber at the same time (GLS).

Consider the Cauchy problem for an ordinary differential equation (ODE) in the case where $f(x)$ is a power function.

$$\begin{cases} f_k' = f'(x) + 6 f_k(x) f'(1.167x), \\ f(x) = x^m, \\ f_k(x) = 0. \end{cases}$$

The equation has an analytical solution. The problem can be solved in terms of quadratures if the exponent $m = 2$.

In this case, the Cauchy problem has the form

$$\begin{cases} f_k' = 2x + 16.338 x f_k, \\ f_k(0) = 0. \end{cases}$$

The solution to this problem is

$$f_k(x) = \frac{1}{8.169}\left(e^{8.169x^2} - 1\right). \quad (11)$$

However, for practically important cases, the value of m ranges from 5 to 20.

Obviously, this function grows rapidly with increasing x. Substituting this function into the equation for $P(x)$, we obtain the following equation:

$$P(x) = Ex(1 - f_k(x)) = Ex\left(1 - \frac{1}{8.169}\left(e^{8.169x^2} - 1\right)\right). \quad (12)$$

The function $P(x)$ has a maximum that can be found by equating the derivative of this function to zero, but in this case an equation is obtained containing simultaneously terms with exponential and exponential functions, which cannot be solved analytically. Therefore, we will immediately proceed to numerical calculations on a computer, solving the Cauchy problem for values of m ranging from 2 to 25, and also looking for the maximum of expression (3) also for different values of m.

Let's write a program for calculations using R-Studio. The last operator in this program code is responsible for displaying all values in the form of a table.

The final data are presented in Table 1.

Table 1. Influence of concentration at different values of m

	m	5	6	7	8	9	10	11	12	13	14	15	16
1	average fiber strength	0.83	0.86	0.88	0.89	0.9	0.91	0.92	0.92	0.93	0.93	0.94	0.94
2	Beam deformation without regard to concentration	0.70	0.72	0.74	0.76	0.77	0.79	0.80	0.81	0.82	0.82	0.83	0.84
3	Beam strength without concentration	0.58	0.62	0.65	0.67	0.70	0.71	0.73	0.74	0.76	0.77	0.78	0.79
4	Percentage of broken fibers, %	0.17	0.14	0.12	0.11	0.1	0.09	0.08	0.08	0.07	0.07	0.06	0.06
5	Deformation of the beam, taking into account the concentration for the next 6 fibers (non-differential form)	0.62	0.65	0.68	0.70	0.72	0.73	0.75	0.76	0.77	0.78	0.79	0.79
6	Beam strength, taking into account the concentration for the next 6 fibers (non-differential form)	0.53	0.58	0.61	0.64	0.66	0.68	0.70	0.72	0.73	0.74	0.75	0.76
7	Deformation of the beam taking into account the concentration for the next 6 fibers (differential view)	0.6	0.63	0.66	0.69	0.71	0.72	0.74	0.75	0.76	0.77	0.78	0.79

Table 1 (ending)

	m	5	6	7	8	9	10	11	12	13	14	15	16
8	Beam strength, taking into account the concentration for the next 6 fibers (differential view)	0.53	0.57	0.61	0.63	0.66	0.68	0.7	0.71	0.73	0.74	0.75	0.76
9	Percentage of broken fibers, %	7.7	6.2	5.4	5.1	4.6	3.7	3.6	3.2	2.8	2.6	2.4	2.3

Since the integration results in rapidly growing functions with an increase in the value of the exponent *m*, the value of *x* in numerical calculations cannot exceed 1.

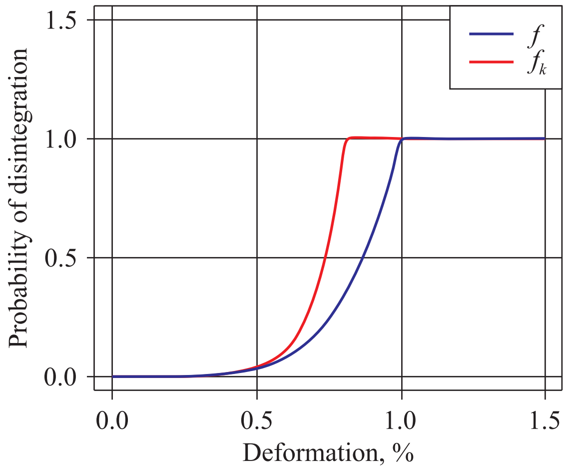

Fig. 4. The probability of failure for $m = 5$, calculated by formulas (8) and (10)

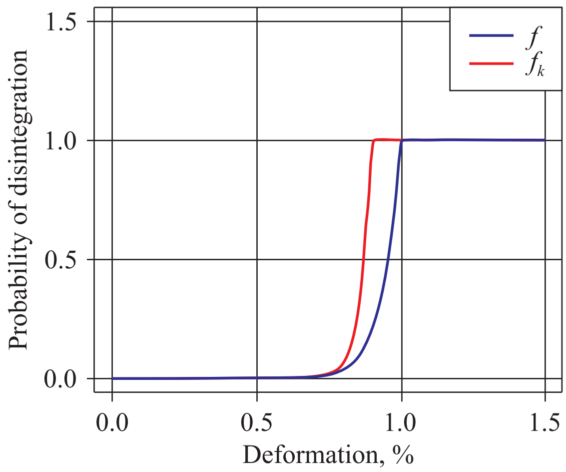

Fig. 5. The probability of failure at $m = 15$, calculated by formulas (8) and (10)

3. Results of numerical modeling of options

Figure 7 shows the dependence of the strength of the composite on *m* without taking into account the stress concentration, and according to the models without and with the differential equation.

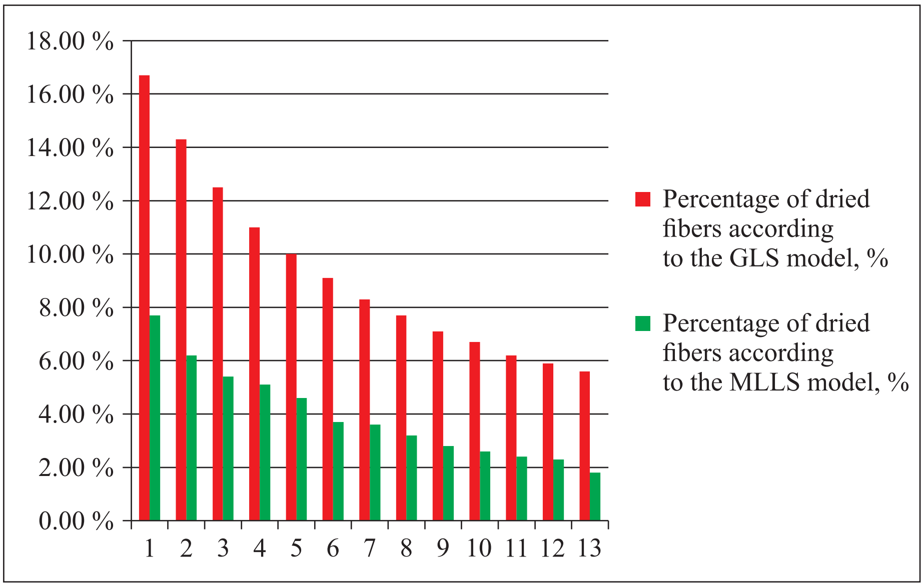

Fig. 6. The proportion of fibers destroyed when the maximum load is reached, calculated using formulas (8) and (10)

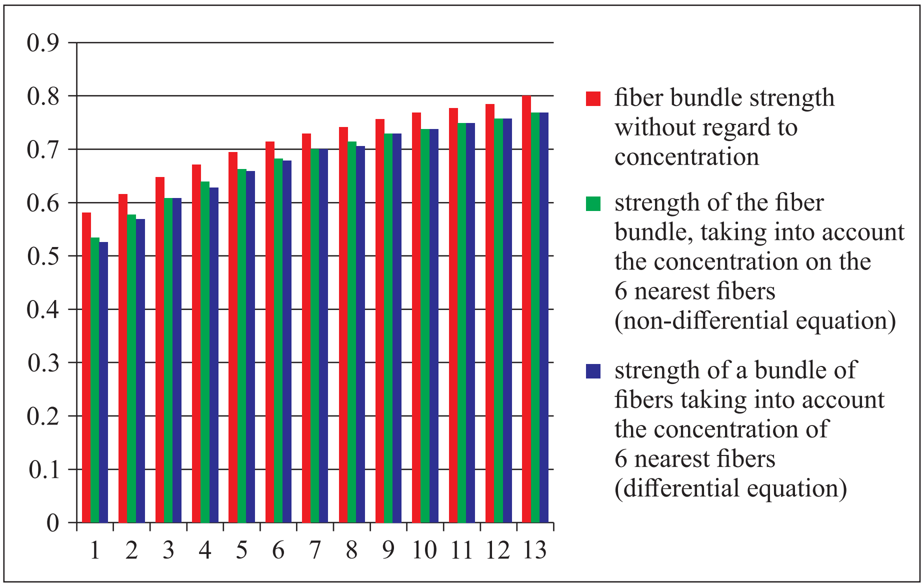

Fig. 7. Dependence on *m* of the bundle strength without taking into account the concentration and taking into account the stress concentration for two calculation methods

4. Comparison of model results with experiment

The prediction of the strength of composites according to MLLS-6 was compared with our data based on the results of the study of glass fibers and carbon fibers. Also, the forecast for basalt fibers was compared with the literature data from the works.

An investigation was made of carbon fiber UKN-5000 (410-tex) (5000 monofilaments) and glass threads VMN (67 tex) (200 monofilaments). The strength was determined under static stretching at a speed of 5 mm per minute at various lengths from 10 to 500 mm. The tests were carried out using a ZDM-250 and Zwick/RoellZ010 testing machine at room temperature. The dependence of strength on thread length was plotted in double logarithmic coordinates (logarithm of strength versus logarithm of thread length), since, according to the Weibull model, the dependence in such coordinates is a straight line [Fudzy, Dzako, 1982].

Glass and carbon filaments consisting of 5000 monofilaments (UKN-5000) were impregnated with EDT-10 epoxy binder (Dian resin with amine hardener) and cured according to the recommended mode [Nemez, Strelyaev, 1970]. Thus, a unidirectional model composite — microplastic was produced. The length of the working zone was 50 mm; to protect the samples from destruction in the clamps, the clamping part was glued into glass cloth. Then its tensile strength was determined at a speed of 5 mm per minute on a ZDM-250 tensile testing machine. This test is similar to ASTM D4018.

In order to detect them, a full cycle of mechanical studies and studies of the fracture surface by scanning electron microscopy (SEM) was carried out [Swolfs et al., 2013; Curtin, 2000].

Figures 8 and 9 show calculations of the deformation diagram for basalt plastic at $m = 5$ and $m = 15$, calculated using the GLS (Pr) model and MLLS-6 models without diffraction (Pr1) and with diffraction (Pr2).

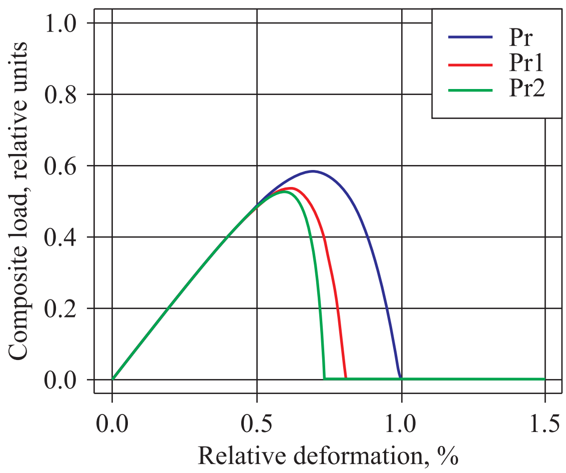

Fig. 8. Diagrams of deformation of basalt plastic ($m = 5$), calculated using the GLS (Pr) model and MLLS-6 models without diffraction (Pr1) and with diffraction (Pr2)

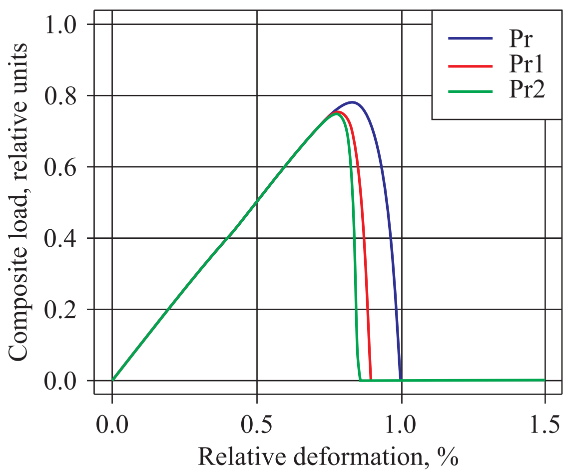

Fig. 9. Prediction of the deformation diagram of CFRP ($m = 15$), calculated using the GLS (Pr) model and MLLS-6 models without diffraction (Pr1) and with diffraction (Pr2)

Figures 10–12 show how the forecast according to our model corresponds to reality.

Figure 10 describes carbon fibers. From the graph, one can determine *m* and calculate the prediction of the strength of the fiber bundle to the ineffective length. The MLLS-6 model provides a more realistic description of the strength of the composite, but still overestimates it.

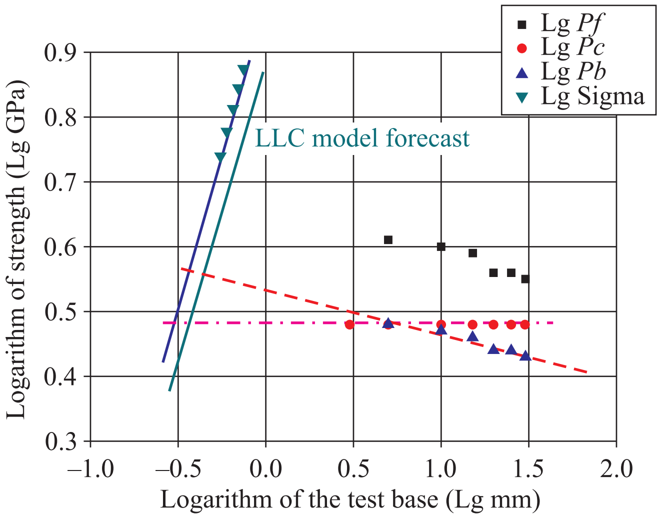

Fig. 10. Carbon fibers: *Pf* — fiber strength, *Pb* — bundle strength, *Pc* — experimental strength values, Sigma — ineffective length dependence, LLC Forecast — correction for 6 nearest fibers

Figure 11 describes glass fibers. From the graph, one can determine *m* and calculate the prediction of the strength of the fiber bundle to the ineffective length. The MLLS-6 model provides a more realistic description of the strength of the composite, but still overestimates it.

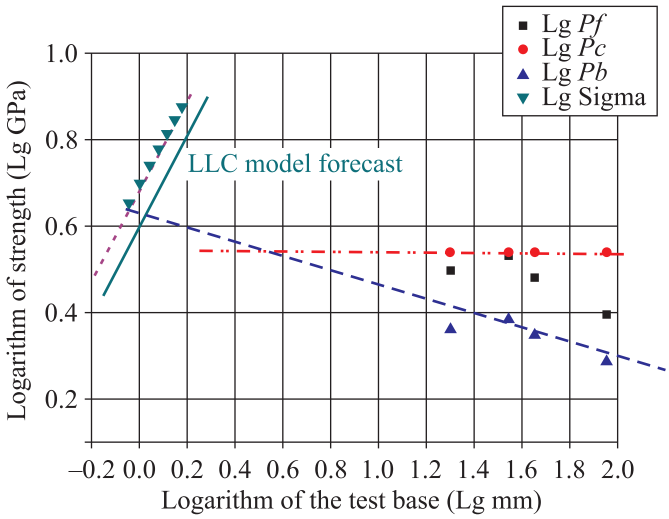

Fig. 11. High-strength glass VMN: *Pf* — fiber strength, *Pb* — bundle strength, *Pc* — experimental strength values, Sigma — ineffective length dependence, LLC Forecast — correction for 6 nearest fibers

Figure 12 describes basalt fibers based on data from [Dalinkevich et al., 2009]. From the graph, one can determine *m* and calculate the prediction of the strength of the fiber bundle to the ineffective length. The MLLS-6 model provides a more realistic description of the strength of the composite, but still overestimates it.

Table 2 shows the result of simulations for a number of modern carbon fibers: Rovilon ($m = 16$), HC-2430 ($m = 5$), UKN-5000 ($m = 11$), Basalt ($m = 15$), glass VMN ($m = 8$).

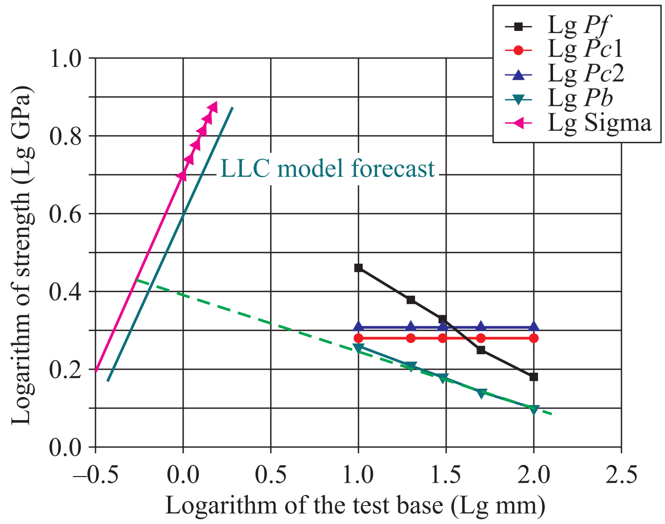

Fig. 12. Dependence for basalt fiber: *Pf* — fiber strength, *Pb* — bundle strength, *Pc*1 and *Pc*2 — experimental strength values, Sigma — ineffective length dependence, LLC Forecast — correction for 6 nearest fibers

Table 2. Forecast strength for a number of industrial fibers

	Composite characteristic	UV-2430	Glass VMN	UKN-5000	Basalt	Rovilon
1	*m*	5	8	11	15	16
2	Average fiber strength	0.83	0.89	0.92	0.94	0.94
3	Beam deformation without regard to concentration	0.699	0.76	0.798	0.831	0.838
4	Beam strength without regard to concentration	0.582	0.675	0.731	0.779	0.788
5	Percentage of destroyed fibers, %	16.70 %	11.00 %	8.30 %	6.2 %	5.9 %
6	Deformation of the beam taking into account the concentration for the next 6 fibers (Mikheev)	0.616	0.701	0.749	0.788	0.795
7	Strength of the bundle, taking into account the concentration for the next 6 fibers (Mikheev)	0.535	0.641	0.701	0.752	0.761
8	Deformation of the beam taking into account the concentration for the next 6 fibers (Borisova)	0.6	0.69	0.74	0.78	0.79
9	Strength of the bundle, taking into account the concentration for the next 6 fibers (Borisova)	0.527	0.63	0.7	0.75	0.76
10	Percentage of destroyed fibers, %	7.70 %	5.10 %	3.6 %	2.4 %	2.3 %
11	Forecast of strength loss	90.5 %	93.3 %	95.8 %	96.3 %	96.4 %

5. Conclusion

In this paper, a modified load distribution model (MLLS-6) is proposed to describe the fracture mechanism of high-strength composites under tension. The model is based on the combined model of Hedgepest and Phoenix (statistical approach to fiber strength). Taking into account the stress concentration only on the nearest fibers allows makes it possible to obtain analytical solutions to the problem and allows a comparison of the predicted strength with experiment. In this case, the properties of the binder in the model are expressed only in terms of the value of the ineffective length.

The forecast is made using the maximum load of the bundle of fibers (complex yarn) and not the average strength of the fibers, as it is done in some papers.

According to our data, the strength provided by the MLLS-6 model is lower than that provided by the GLS model, for carbon plastic by 10 %, for fiberglass plastic by 7 %, and for basalt plastic by 5 %.

The strength values have approached the experimental ones, but still remain higher. This result is unexpected, since we expected to obtain the lower limit of strength.

We explain this by the fact that the shear yield stress used in calculating the ineffective length of 30 MPa is taken as an average value, but, like the strength of the fibers, it has a spread, and in some cases pores and, consequently, an ineffective length is not a constant value either, and increases where the shear strength limit is less, and, in our opinion, these places become a source of fracture, which further occurs according to the proposed MLLS-6 model and almost without damage accumulation in the case of brittle high-strength fibers.

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