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Numerical Simulation, Parallel Algorithms and Software for Performance Forecast of the System “Fractured-Porous Reservoir – Producing Well” During its Commissioning Into Operation

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The mathematical model, finite-difference schemes and algorithms for computation of transient thermo- and hydrodynamic processes involved in commissioning the unified system including the oil producing well, electrical submersible pump and fractured-porous reservoir with bottom water are developed. These models are implemented in the computer package to simulate transient processes with simultaneous visualization of their results along with computations. An important feature of the package Oil-RWP is its interaction with the special external program GCS which simulates the work of the surface electric control station and data exchange between these two programs. The package Oil-RWP sends telemetry data and current parameters of the operating submersible unit to the program module GCS (direct coupling). The station controller analyzes incoming data and generates the required control parameters for the submersible pump. These parameters are sent to Oil-RWP (feedback). Such an approach allows us to consider the developed software as the “Intellectual Well System”.

Some principal results of the simulations can be briefly presented as follows. The transient time between inaction and quasi-steady operation of the producing well depends on the well stream watering, filtration and capacitive parameters of oil reservoir, physical-chemical properties of phases and technical characteristics of the submersible unit. For the large time solution of the nonstationary equations governing the nonsteady processes is practically identical to the inverse quasi-stationary problem solution with the same initial data. The developed software package is an effective tool for analysis, forecast and optimization of the exploiting parameters of the unified oil-producing complex during its commissioning into the operating regime.

Keywords: computer simulation, numerical methods, parallel algorithms, software packages, unsteady processes, heat and mass transfer, multi-phase flows, electric submersible pump, producing Well, two-phase filtration, fractured-porous oil reservoir

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1. Introduction

Exploitation of the oil producing wells equipped with electrical submersible pumping systems (ESPs) is attended by the interconnected thermo- and hydrodynamic processes in the multi-phase flows moving in the porous medium of the oil reservoir, tubes of well and channels of ESP. These processes become significantly nonsteady during the commissioning into operation of the wells after underground equipment repair. In this case the problem of computer forecast of the interconnected heat and mass transfer in the unified oil-producing system "reservoir – well – ESP" and the task of correct pump selection for the wells are quite actual.

Earlier a computer model was proposed to study such processes for cases of water-oil displacement [Konyukhov et al., 2013] in the nonuniform layered oil reservoir of porous structure. In this article, we continue our research and generalize the mathematical model [Konyukhov et al., 2013] of the transient processes in this unified system for a more complicated case of two-phase filtration in the porous and fractured reservoir in the presence of active bottom water. To describe this process, we will use the model [Chekalin et al., 2009; Diyashev et al., 2012] developed taking into account the gravitational forces arising due to the difference in density of oil and water, the compressibility of liquid phases and reservoir rocks, the dependence of the absolute permeability of fractures on the reservoir pressure and the non-Newtonian behavior of the oil phase viscosity. Its magnitude depends on the filtration rate modulus and significantly decreases in those more permeable zones of the reservoir where the filtration rate exceeds the limiting rate required for destruction of the initial structure of the oil phase. In this case, the filtration in the oil reservoir and the production well flow rate are determined by the interaction of the two effects. On the one hand, with a lower flow rate of the well, the elevation of the water cone rising up from the aquifer will be lower, so that the water content of the well debit will also be less and, consequently, the oil recovery of the reservoir will be higher. On the other hand, higher flow rate results in a significant increase in the filtration area with a destroyed oil structure and significantly lower viscosity (compared to oil with an undisturbed structure). This leads to an increase in oil inflow into the producing well and a decrease in the water content in the well debit.

The unsteady heat and mass transfer processes in three-phase gas-water-oil flows in the wells and working channels of the ESP are complicated by phase transitions during oil degassing in pipes and gas dissolution in pump channels, compressibility of phases, friction, gravity force, restructuring of gas-liquid flow, inversion of phases, drift motion of disperse components, heat exchange between flow and rock formation around the well, etc., see [Bratland, 2010; Hasan, Kabir, 1988; Salamatin, 1987]. Performance characteristics of the submersible unit essentially depend on the properties of the pumped mixtures [Lyapkov, 1979]. Important features of transient processes are the origination, movement and disappearance of boundaries between gas-liquid mixture and water, and between water-in-oil emulsion and oil-water-gas flow in the tubes of well and in the channels of ESP. In addition, at present the control for the current working modes of ESP is often realized with ground-based control stations (GCS) to analyze telemetering data (direct coupling) and to generate the actions (back coupling) required for improving the work conditions of submersible equipment, up to its shutoff in emergency situations.

Moreover, an increase in the efficiency of development of the fractured-porous reservoirs can be achieved only by the use of periodic impacts on the formation, which allow one to intensify the mass transfer between the blocks and fractures. Optimal parameters of such action can be effectively found and estimated on the basis of computational experiments.

As a result, the computation of these processes and optimization of oil production should be done taking into account all above-mentioned factors. These problems are very complex and can be effectively solved on the basis of mathematical and numerical modeling using parallel computing technologies.

2. Mathematical model

The mathematical model of transient heat and mass transfer during the commissioning into the operating regime of the oil producing system was developed in [Konyukhov et al., 2013] on the basis of [Barenblatt et al., 1984; Ertekin et al., 2001; Chekalin et al., 2009; Diyashev et al., 2012] for the case of a porous layered-nonuniform reservoir. Generalization of this unified model for the case of water flooding in the fractured-porous reservoir results in the following three groups of conjugated nonlinear differential equations. Firstly, the equations governs two-phase filtration in the vertical cross-section $D_r = \{0 < r < L_r,\ 0 < z < H_r\}$ of the plane-radial reservoir:

$$\bar{\alpha}_T \frac{\partial P}{\partial \tau} + \frac{1}{r}\frac{\partial\left(r\bar{V}_r\right)}{\partial r} + \frac{\partial \bar{V}_z}{\partial z} = -Q_\Sigma; \quad \bar{m}\frac{\partial \bar{S}}{\partial \tau} + \bar{\alpha}^*_{T3}\bar{S}\frac{\partial P}{\partial \tau} + \frac{1}{r}\frac{\partial\left(r\bar{V}_{3,r}\right)}{\partial r} + \frac{\partial \bar{V}_{3,z}}{\partial z} = -\lambda Q_\Sigma; \quad (1)$$

$$\alpha_T \frac{\partial P}{\partial \tau} + \frac{1}{r}\frac{\partial\left(rV_r\right)}{\partial r} + \frac{\partial V_z}{\partial z} = Q_\Sigma; \quad m\frac{\partial S}{\partial \tau} + \alpha^*_{T3}S\frac{\partial P}{\partial \tau} + \frac{1}{r}\frac{\partial\left(rV_{3,r}\right)}{\partial r} + \frac{\partial V_{3,z}}{\partial z} = \lambda Q_\Sigma; \quad (2)$$

$$\bar{V}_r = -\bar{K}\bar{K}^*\frac{\partial P}{\partial r},\ \ \bar{V}_z = -\bar{K}\bar{K}^*(\frac{\partial P}{\partial z} + g\rho); \quad V_r = -KK^*\frac{\partial P}{\partial r},\ \ V_z = -KK^*(\frac{\partial P}{\partial z} + g\rho); \quad (3)$$

$$\bar{\mathbf{V}}_3 = \bar{f}\bar{\mathbf{V}} - \bar{K}\rho_{3-1}\bar{\Psi}\mathbf{g}; \quad \mathbf{V}_3 = f\mathbf{V} - K\rho_{3-1}\Psi\mathbf{g}; \quad \bar{K}^* = \bar{K}^*_1 + \mu\bar{K}^*_3; \quad K^* = K^*_1 + \mu K^*_3; \quad (4)$$

$$\lambda = \begin{cases} f\left(S\right), & Q_\Sigma < 0, \\ \bar{f}\left(\bar{S}\right), & Q_\Sigma > 0; \end{cases} \quad f = \frac{K^*_3}{K^*}; \quad \bar{f} = \frac{\bar{K}^*_3}{\bar{K}^*}; \quad \bar{\Psi} = \frac{\bar{f}\bar{K}^*_1}{\mu_1}; \quad \Psi = \frac{fK^*_1}{\mu_1}. \quad (5)$$

Here τ is the time; r and z are spatial coordinates of the filtration region D_r; P is the pressure; S is the water saturation; the characteristics of the oil, gas and water are numbered by the subindex $i = 1, 2, 3$; the parameters of the blocks and fractures are denoted with and without the overline, respectively; $K^*_3(S)$ and $K^*_1(S)$ are the functions of relative permeability of water and oil phases (they are linear functions for the fractures and cubic ones for porous blocks, see [Chekalin et al., 2009]); V_r, V_z, $V_{3,r}$, $V_{3,z}$ are the projections of the filtration velocity vectors $\mathbf{V}$, $\mathbf{V}_3$ of the water-oil mixture and water phase to axes Or and Oz; $K(r, z)$ and $m(r, z)$ are the absolute permeability and the dynamic porosity of the porous medium; $f(S)$ is the fraction of water in the total two-phase flux (the Bacley – Leverett function); Q_Σ is the total two-phase overflow between blocks and fractures; ρ_3, ρ_1 and μ_3, $\mu_1 = \mu_1(|\mathbf{V}|)$ are the density and the coefficients of dynamic viscosity of water and oil phase, respectively; the function $\mu_1(|\mathbf{V}|)$ is determined by the empirical dependence [Chekalin et al., 2009]; $\mu = \mu_3/\mu_1$; $\rho = \rho_1 + \rho_{3-1}f$; $\rho_{3-1} = \rho_3 - \rho_1$; L_r and H_r are the length and the thickness of the reservoir; α_{TC} and $\alpha^*_{T\,i}$ are the coefficients of elastic capacity of the porous medium and the medium saturated with the ith phase; $\alpha_{T\,1}$ and $\alpha_{T\,3}$ are the similar parameters of oil and water; $\alpha_T = \alpha^*_{T\,3}S + \alpha^*_{T\,1}\left(1 - S\right)$; $\alpha^*_{T1} = \alpha_{TC} + m\alpha_{T1}$; $\alpha^*_{T3} = \alpha_{TC} + m\alpha_{T3}$; $\bar{\alpha}_T = \bar{\alpha}^*_{T\,3}\bar{S} + \bar{\alpha}^*_{T\,1}\left(1 - \bar{S}\right)$; $\bar{\alpha}^*_{T1} = \bar{\alpha}_{TC} + \bar{m}\bar{\alpha}_{T1}$; $\bar{\alpha}^*_{T3} = \bar{\alpha}_{TC} + \bar{m}\bar{\alpha}_{T3}$.

The second group is represented by equations of the unsteady dispersed water-oil-gas flow in the producing well $D_w = \{0 < z \le H_w\}$:

$$\frac{\partial\left(\rho_i\varphi_i\right)}{\partial \tau} + \frac{\partial\left(\rho_i\varphi_i\bar{w}_i\right)}{\partial z} = \chi_i, \quad i = 1, 2, 3; \quad \chi_2 = -\chi_1 = \frac{\rho_1\varphi_1}{1 - C_sF}\frac{d_1}{d\tau}\left(C_sF\right); \quad \chi_3 = 0; \quad (6)$$

$$\sum_{i=1}^{3}\rho_i\varphi_i\frac{d_i\bar{w}_i}{d\tau} = -\frac{\partial P}{\partial z} - \frac{2\tau_r}{r} + F_{1-2} + \rho g; \quad \sum_{i=1}^{3}\rho_i\varphi_iC_{Pi}\frac{d_iT}{d\tau} = T\sum_{i=1}^{3}\alpha_{Pi}\varphi_i\frac{d_iP}{d\tau} + Q_\Sigma. \quad (7)$$

These equations are obtained in the framework of the Zuber – Findlay model (see, e.g., [Bratland, 2010; Konyukhov, 1990; Salamatin, 1987; Wallis, 1969; Zuber, Findley, 1965]) for the case of three-phase mixture including such discrete components as gas bubbles or drops of water (or oil) inside the continuous (oil or water) phase.

In Eqs. (6) and (7), Oz is the vertical coordinate axis directed up the well from its beginning on the reservoir roof; P and T are the pressure and the temperature, identical for all phases; ρ is the mean multiphase mixture density; w is the overall mixture flow velocity; ρ_i, $\bar{w}_i$ and φ_i are the density, the actual velocity, the volumetric concentration of the ith phase, averaged over the well pipe cross-section f of radius r, $i = 1, 2, 3$; $F_{1-2} = \chi_2\left(\bar{w}_1 - \bar{w}_2\right)$; $F(P, T)$ is the relative gas factor which is defined as the ratio of the mass of gas released from the oil phase under certain (P, T)-conditions to the total amount of the initially dissolved gas; C_s is the corresponding mass concentration of gas in the oil phase at $P > P_s$, where P_s is saturation pressure; $Q_\Sigma = 2\left(\tau_r w - q_r\right)/r + Q_{\text{v}} - \chi_2 L$; L is the latent heat of gas dissolution into oil; τ_r and q_r are the hydraulic friction and the heat flux density at the internal surface of the producing well; Q_{v} is the intensity of the external heat source distributed along the producing well; α_{pi} and C_{pi} are the coefficients of volumetric thermal expansion and volumetric elasticity of the ith phase; g is the gravity acceleration; $d_i/d\tau = \partial/\partial\tau + \bar{w}_i \cdot \partial/\partial z$.

The third group consists of the equations governing the thermal and hydrodynamic processes in the channels of the multistage pump $D_e = \{0 < \xi \le L_e\}$:

$$\frac{\partial\left(\rho_i\varphi_i\right)}{\partial\tau} + \frac{\partial\left(\rho_i\varphi_i\upsilon\right)}{\partial\xi} = \chi_i, \quad i = 1, 2, 3; \quad \chi_2 = -\chi_1 = \frac{\rho_1\varphi_1}{1 - C_s F}\frac{d}{d\tau}\left(C_s F\right); \quad \chi_3 = 0; \tag{8}$$

$$l_s\frac{\partial P}{\partial\xi} = g\rho\,\Delta H; \quad \sum_{i=1}^{3}\rho_i\varphi_i C_{Pi}\frac{dT}{d\tau} = T\sum_{i=1}^{3}\alpha_{Pi}\varphi_i\frac{dP}{d\tau} + \frac{Q}{f_s}\frac{1-\eta}{\eta}\frac{\partial P}{\partial\xi} - \chi_2 L. \tag{9}$$

Equations (8) and (9) were developed in [Konyukhov, Konyukhov, 2012] under the assumption that all the phases become highly-dispersed in the pump stages and move without slippage as a result of the enormous rotation speed of their blades, i.e., $\bar{w}_i = \upsilon$.

In these equations ξ is the vertical coordinate axis directed up the pump from its first stage; l_s and f_s are the length and an effective cross-section of the pump stage, respectively; L_e is the total length of ESP; $d/d\tau = \partial/\partial\tau + \upsilon\cdot\partial/\partial\xi$; H, $\eta = g\rho HQ/N$ and N are the head, the efficiency factor and the power consumption of the pump stage. These characteristics depend on the volumetric flow rate $Q = G/\rho$ and the effective viscosity μ of the three-phase mixture which can significantly decrease as a result of compression of phases and gas dissolution in oil as the flow moves along the pump, where G is the overall debit of the three-phase mixture.

We would like to note that this paper provides only some relationships which define the basic operating parameters of the pump stages and characteristics of multiphase flows in the pipes and in the porous medium of the oil reservoirs. The set of special constitutive relations to close the equations is too large and can be found in our publications (see, e.g., [Chekalin et al., 2009; Konyukhov, 1990; Konyukhov, Konyukhov, 2012; Konyukhov et al., 2013]). Formulation of the boundary, initial and conjugation conditions for the system of differential equations is also discussed in detail in these papers. We present here only the boundary condition at the wellhead of the producing well and relationships simulating the control actions on the operation modes of the submersible pump and its electric motor by varying the frequency ω of the electric current using the ground-based equipment [Konyukhov et al., 2013]:

$$P|_{z=H_w} = P_{lin} + 0.5\varsigma_{dr}\left(d_{dr}\right)\rho w^2\big|_{z=H_w}; \tag{10}$$

$$Q_3 = Q_3^*\omega/\omega^*; \quad H_3 = H_3^*(\omega/\omega^*)^2; \quad N_3 = N_3^*(\omega/\omega^*)^3; \quad N_M = N_M^*\omega/\omega^*. \tag{11}$$

Here ς_{dr} is the local resistance coefficient of the regulating throttle valve which is a function of its variable diameter d_{dr}; P_{lin} is the line pressure behind the throttle valve, which can be assumed to be a constant; Q_3^*, H_3^* and N_3^* are the volumetric flow rate, head and useful capacity of a certain pump stage during its operation on water under the nominal conditions at ω^*=50 Hz; N_M^* is the nominal consumed power of the motor at $\omega = \omega^*$; Q_3, H_3, N_3 and N_M are the similar characteristics of the stage and motor at $\omega \ne \omega^*$.

3. Numerical model and software

The filtration problem (1)–(5) is solved by the finite-difference method taking into account a priori information about the features and properties of the solution of two-phase filtration equations [Chekalin et al., 2009; Diyashev et al., 2012]. We briefly describe here the main points of the numerical model and the computational algorithm.

The pressure is calculated by the parabolic equation

$$\mu_3 \left(\alpha_T + \bar{\alpha}_T\right) \frac{\partial P}{\partial t} = \operatorname{div}\left(\left[KK^* + \bar{K}\bar{K}^*\right](\operatorname{grad} P + \rho \mathbf{g})\right),$$

obtained by combining the corresponding equations (1)–(3). This equation is approximated by the implicit difference scheme of the second order in which the oil viscosity $\mu_1(|\mathbf{V}|)$ and the relative permeability $\bar{K}^*(\bar{S})$ of the fractures are taken from a previous point of time. The singularity of function $P(r, z, \tau)$ in a neighborhood of the bottom-hole under the plane-radial filtration is corrected by special factors for the phase flows, see [Chekalin et al., 2009; Diyashev et al., 2012].

The total two-phase overflow Q_Σ between blocks and fractures is computed by a discrete equation approximating the first differential mass balance equation (2). The overflow of the ith phase is determined by its saturation value taken at the previous point of time. The water fraction λ in the total overflow is determined from the relationship (5) by the Buckley – Leverett function $\bar{f}(\bar{S})$ if the two-phase mixture flows from fractures into blocks, otherwise by the function $f(S)$.

Functions S and $\bar{S}$ are calculated by mass-transfer equations (1) and (2), respectively, which are approximated by second-order finite-difference equations using the integral values of water saturation in the elementary grid cells. In these equations, the members $\bar{\alpha}^*_{T3}\bar{S}\partial P/\partial\tau$ and $\alpha^*_{T3}S\partial P/\partial\tau$, caused by the elastic forces are approximated taking into account the “direction” of the process: at compression, when $\partial P/\partial t > 0$), the water saturation is taken at the point of time considered, and otherwise at the previous one. In this case, the numerical methods for computing the functions S and $\bar{S}$ are not identical, since both phases move along blocks much slower and have spatial distributions significantly different from fractures. This fact is caused by the greater porosity and permeability of the fractures and the linear dependence of the functions $\bar{K}^*_1$, $\bar{K}^*_3$ on S.

We also note such an important feature of problem solution as the possibility of gravitational separation of oil and water phases because of a significant difference in their densities. Since the values of the flows $\mathbf{V}_3$ and $\bar{\mathbf{V}}_3$ depend on water saturation, their calculation across the boundaries of elementary cells in the direction of the vertical axis Or requires a correct assignment of the corresponding values S and $\bar{S}$. It is well known that the transfer equations with the convective members must be approximated by the upwind finite-difference scheme, see [Roache, 1976]. However, in the problem under consideration, the phase velocities $\mathbf{V}_i$ and $\bar{\mathbf{V}}_i$ can be multidirectional due to the density difference, and therefore the direction of the total filtration velocities $\mathbf{V}$ and $\bar{\mathbf{V}}$ will not coincide with one of them. As a result, there is uncertainty in the choice of the direction of velocity in this scheme, prescribing to take values of water saturation in those cells from which the fluid outflows. Moreover, in some situations, the water flow across the boundary of the elementary cell is determined not by the values of water saturation in the neighboring grid cells, but its value providing the maximum of the flow rate. In addition, due to gravity, the functions S and $\bar{S}$ have jumps at the boundaries of the discontinuity of the absolute permeability. To take into account the aforesaid specified features of the problem solution, the vertical components of the flows are computed by a special numerical method [Chekalin et al., 2009; Diyashev et al., 2012].

The solution of the filtration problem allows one to determine the flow rate of the mixture and its water content at the bottom-hole of the production well. These values allow one to form the boundary conditions for the equations of three phase flows in pipes (6), (7) and to start solving them. Then the characteristics of thermal and hydrodynamic processes in the ESP channels and in the oil-well tubing are computed by equations (8), (9) and (6), (7), respectively. Numerical methods for solving both of these systems are also based on the implicit upwind finite-difference schemes [Konyukhov, 1990; Konyukhov et al., 2013]. As a result of this calculation, we have the distribution of all characteristics of the water-oil-gas flow along the well and the pumping unit, including the pressure at the wellhead.

Finally, an iterative refinement of the unified system solution (1)–(9) is repeated until the boundary condition (10) is satisfied with the given accuracy of computations.

The developed unified numerical model is implemented in the computer simulator “OilRWP” using the C# programming language [Schildt, 2010], Task Parallel Library of the NET Framework platform and sets of libraries NVIDIA CUDA and OpenCL oriented to parallel computing technologies. The package Oil-RWP makes it possible to simulate transient processes with simultaneous visualization of results along with computations.

An important feature of this package is its interaction with the special external program “GCS” which simulates the operation of the surface electric control station. In particular, the dependencies (11) allow an improvement of the operation parameters of the electric motor and the pumping unit by varying the frequency ω during the computation of the transient processes.

Data exchange between two programs is based on WinAPI common memory technologies. The package Oil-RWP sends telemetry data and current parameters of the operating submersible unit to the program module GCS (direct coupling). Station controller analyzes incoming data and generates the required control parameters for the submersible motor and pump. These parameters are sent to Oil-RWP (feedback).

The developed software package for the simulation of the transient processes is an effective tool for analysis, forecast and optimization of the exploiting parameters of the system “oil reservoir – well – ESP” during its commissioning into the operating regime. Such an approach allows us to consider the developed software as the “Intellectual Well System”.

4. Results of computations and conclusions

The computational experiments and their analysis are carried out for a concrete oil-extracting system, see [Konyukhov, Konyukhov, 2012; Konyukhov et al., 2013]. Some theoretical and practical results can be briefly presented as follows.

1) Parallelization increases the performance of calculations several times in comparison with sequential calculations in computation of two-phase filtration characteristics in the fractured-porous reservoir and their visualization on the display.

2) The transient time between inaction and quasi-steady operation of the producing well depends on the well stream watering, filtration-capacitive parameters of the oil reservoir, physical-chemical properties of phases and technical characteristics of the submersible pumping unit.

3) A count of the ESP shutoffs by the surface control station during the producing well going into operation depends on the parameters of the oil-developed system.

4) For large times the solution of the equations governing the nonsteady processes is practically identical to the inverse quasi-stationary problem solution with the same initial data.

5) The mathematical model and the program package allow realization of optimal control for the operating modes of the producing well, its underground equipment and, as a result, for the development of the oil reservoir.

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