

UDC: 519.8

Modelling of trends in the volume and structure of accumulated credit indebtedness in the banking system

A. A. Pekhterev^a, D. V. Domaschenko^b, I. A. Guseva^c

Plekhanov Russian University of Economics,
36 Stremianny per., Moscow, 115093, Russia

E-mail: ^a lexpekhterev@gmail.com, ^b dendv@rambler.ru, ^c i.sorochinskaya@gmail.com

Received 27.03.2019, after completion – 18.08.2019.

Accepted for publication 17.09.2019.

The volume and structure of accumulated credit debt to the banking system depends on many factors, the most important of which is the level of interest rates. The correct assessment of borrowers' reaction to the changes in the monetary policy allows to develop econometric models, representing the structure of the credit portfolio in the banking system by terms of lending. These models help to calculate indicators characterizing the level of interest rate risk in the whole system. In the study, we carried out the identification of four types of models: discrete linear model based on transfer functions; the state-space model; the classical econometric model ARMAX, and a nonlinear Hammerstein–Wiener model. To describe them, we employed the formal language of automatic control theory; to identify the model, we used the MATLAB software pack-age. The study revealed that the discrete linear state-space model is most suitable for short-term forecasting of both the volume and the structure of credit debt, which in turn allows to predict trends in the structure of accumulated credit debt on the forecasting horizon of 1 year. The model based on the real data has shown a high sensitivity of the structure of credit debt by pay back periods reaction to the changes in the Central Bank monetary policy. Thus, a sharp increase in interest rates in response to external market shocks leads to shortening of credit terms by borrowers, at the same time the overall level of debt rises, primarily due to the increasing revaluation of nominal debt. During the stable falling trend of interest rates, the structure shifts toward long-term debts.

Keywords: credit debt, dynamic modeling, forecasting, interest rate, state-space model

Citation: *Computer Research and Modeling*, 2019, vol. 11, no. 5, pp. e965–e978 (Russian).

УДК: 519.8

Моделирование трендов динамики объема и структуры накопленной кредитной задолженности в банковской системе

А. А. Пехтерев^а, Д. В. Домашенко^б, И. А. Гусева^с

Российский экономический университет имени Г. В. Плеханова,
Россия, 115093, г. Москва, Стремянный пер., д. 36

E-mail: ^а lexpekhterev@gmail.com, ^б dendv@rambler.ru, ^с i.sorochinskaya@gmail.com

*Получено 27.03.2019, после доработки — 18.08.2019.
Принято к публикации 17.09.2019.*

Объем и структура накопленной кредитной задолженности перед банковской системой зависят от множества факторов, важнейшим из которых является текущий и ожидаемый уровень процентных ставок. Изменения в поведении заемщиков в ответ на сигналы денежно-кредитной политики позволяют разрабатывать эконометрические модели, представляющие динамику структуры кредитного портфеля банковской системы по срокам размещения средств. Эти модели помогают рассчитать показатели, характеризующие влияние регулирующих действий со стороны центрального банка на уровень процентного риска в целом. В работе проводилась идентификация четырех видов моделей: дискретной линейной модели, основанной на передаточных функциях, модели в пространстве состояний, классической эконометрической модели ARMAX и нелинейной модели типа Гаммерштейна–Винера. Для их описания использовался формальный язык теории автоматического управления, а для идентификации — программный пакет MATLAB. В ходе исследования было выявлено, что для краткосрочного прогнозирования объема и структуры кредитной задолженности больше всего подходит дискретная линейная модель в пространстве состояний, позволяющая прогнозировать тренды по структуре накопленной кредитной задолженности на прогнозном горизонте в 1 год. На примере реальных данных по российской банковской системе модель показывает высокую чувствительность реакции на изменения в денежно-кредитной политике, проводимой центральным банком РФ, структуры кредитной задолженности по срокам ее погашения. Так, при резком повышении процентных ставок в ответ на внешние рыночные шоки заемщики предпочитают сокращать сроки кредитования, при этом общий уровень задолженности повышается прежде всего за счет возрастающей переоценки номинального долга. При формировании устойчивого тренда снижения процентных ставок структура задолженности смещается в сторону долгосрочных кредитов.

Ключевые слова: кредитная задолженность, процентная ставка, динамическое моделирование, модель в пространстве состояний, прогнозирование

Introduction

The impact of interest rate dynamics is a determining factor in the borrowers' decision-making on the feasibility of Bank lending. However, it is impossible to determine accurately the extent of this influence due to many other factors, for example, due to the peculiarities of developing certain sectors of the economy, or due to the fiscal policy, or the level of credit risks and the like. Nevertheless, the factor of the interest rate may be evaluated by selecting effective econometric models obtained on the basis of real data, such as the term structure of the portfolio of credit indebtedness accumulated by organizations to the banking system, and the dynamics of weighted average interest rates on credit terms.

The correctly chosen method of modeling will allow to estimate not only the dynamics of accumulated credit debt under different scenarios of monetary policy of the Central Bank, but also the value of interest rate risk in the banking system based on the analysis of potential shifts in the term structure of credit debt.

Literature review

To solve the proposed issue, the authors considered three types of models: state-space model based on difference equations, Autoregressive–moving-average model with exogenous inputs (ARMAX) and nonlinear Hammerstein – Wiener model.

It was decided to apply methodology and mathematical tools that are commonly used in control engineering based on automatic control theory to build suitable models applicable for forecasting economic processes, occurring in dynamic non-linear economic systems. This approach is similar to quite young sciences of economic cybernetics and econophysics, emerged in the mid-1990s.

Dynamic modeling allows us to study system response response to the external influences, and the future states of the system, in this case, are a source of forecast. Concerning the application to the sphere of economics, many researchers who offered their own approach to building models studied this issue [Petrov et al., 1996; Krasnoshchyokov, Petrov, 2000; Krass, 1976; Romanovskij, Romanovskij, 2007]. For example, Stian Bu Solgård [Solgård, 2009] made an attempt to build models of macroeconomic systems in the form of systems of linear differential equations by using the software package of Simulink Matlab. According to his study, models that can describe the national economy and include its individual subsystems, describing meso-level, allow to analyze and design controls of parameters, influencing the possibility of a debt crisis. In his research, he tried to establish the effectiveness of several approaches to government regulation of the state economy, analyzing the dynamics of macroeconomic indicators. His study suggests that the modeling of economic systems was carried out “from the very beginning” in order to study the regulatory possibilities, while there is no link to real practical data.

The application of ideas and methods of cybernetics to economic systems was considered in the works of Kugaenko A. [Kugaenko, 2005, 2015]. He designed models in the form of linear differential equations for various socioeconomic systems and relationships. The developed models were presented in the form of structural diagrams, clearly describing all the connections in the differential equation system. All these relations were derived analytically by proceeding from well-known formulas as well as by the author's knowledge and practical experience. The main disadvantage of this research is the fact that the structure of most models is not supported by testing their compliance with a real practical data. Unlike the Kugaenko's studies, we applied system identification method for dynamic analysis of the models instead of analytic derivation of the equations. This approach allows to evaluate the parameters of the model, relying on the relationship between the already available input-output time series. System identification is a sub-domain of automatic control, which includes theoretical studies

together with software applications. Matlab System Identification Toolbox is one of those that designed to work with the experimental data measured by sensors in technical systems. This software add-on allows to choose both the type of the identified model and various identification methods, the basic principles of which are described in [Ljung, 1999, 2008].

Initially originated to solve engineering problems, linear dynamical models, namely the state-space models, were used by the founder of cybernetics N. Wiener and R. Kalman. Kalman subsequently developed the legendary Kalman filter, which is widely used both in engineering and in econometric applications. The use of such models for the time series analysis was considered by Durbin and Koopman, Künsch and Migon, Giovanni Petris [Durbin, Koopman, 2001; Migon et al., 2005; Künsch, 2005; Petris et al., 2009]. In descriptions of these models, time series are often represented as a set of three components: trend, seasonal and error. The interest in its usage is explained by the fact that the state-space mathematical models have a clear structure that can be implemented programmatically without any eminent problems and by the fact that they can be widely applied for solving various kinds of tasks. Varshavsky E. in his work on the market structure indicators dynamics and dynamic systems modeling successfully applied automatic control methods, state-space models and, in linear dynamic games, an approach based on the use of operational calculus (Z-transform) [Warsawsky, 2012, 2018].

ARMAX model is a subset of vector autoregressive (AR) model type that was proposed for the first time by S. Sims in 1980. Autoregressive models have found wide application for the time series analysis and forecasting in the field of econometrics. An essential advantage of these models, in comparison with models based on differential or difference equations, is the simplicity of implementation and its greater flexibility, which makes it possible to exclude dependencies on some variables. The disadvantage of such model is that it can catch insignificant or noisy information as a basis for forecasting. The fact that such models can give accurate predictions and are quite simple to implement is shown in [Zhang, Frey, 2015; Corrêa et al., 2016].

Despite the fact that there is an equivalence between the state-space models and a classical stationary autoregressive-moving average model, it is sometimes easier to work with one form rather than with another. The main advantage of the state-space models compared to AR is its possibility to build more effective structures with fewer parameters while describing multidimensional systems and also its ability to be used for modeling and forecasting in the cases of both one-dimensional and multidimensional systems [Shumway, Stoffer, 2011]. When constructing a model, its structure can be designed strictly fixed as well as dynamically changing.

Since in most cases the interrelationships between economic time series have nonlinear nature, it is appropriate to use nonlinear models for their description. The consideration of the nonlinear model can be justified if we take a look at the approach of system dynamics in the economy, where the behavior of economic systems is considered in the presence of feedback loops, reaction delays, environmental influences and others. Thus, it is assumed that the identification of a nonlinear model can be effective if there are deviations from the trend movement in the series. In our study, Hammerstein – Wiener models could satisfy these criteria. In classification, it belongs to block-oriented models and structurally represents the combination of dynamic linear systems and nonlinear memoryless blocks. Hence, such models are extremely effective if analyzed time series can be decomposed into linear and nonlinear components. The methods for identifying such models are described in [Billings, 1980; Haber, Keviczky, 1999; Yu et al., 2014; Zhang, Frey, 2015; Ma, Lium 2015]. But the phenomenon of overfitting is the main disadvantage of nonlinear models, when the model output corresponds too closely to the training set of data, but it is unsuitable for forecasting.

The closest by the subject to predicting credit debt structure by the yield curve are the studies on modeling the impact of the monetary policy of the regulator on macroeconomic parameters [Malyugin et al., 2009; Makarov et al., 2011; De Fiore, Tristani, 2013]. Authors consider large macroeconomic

models, including the dynamic stochastic general equilibrium model (DGSE). The main disadvantage of such model is their bulkiness, as well as the fact that the practicability of each individual differential equation is doubtful without separate verification. For such models, parameters are often evaluated “by eye”. Models of credit debt, or related parameters, such as the M2 money supply, are usually the part of such large models. Moreover, these works consider the main refinancing rate as an aggregated parameter affecting credit indebtedness.

Methodology

Transfer functions are commonly used in automatic control when it comes to technical description of the dynamic systems. A transfer function is a differential operator, which sets the equation between the inputs and the output of a linear stationary system.

A discrete transfer function (1) can be written as [Dorf, Bishop, 2001]

$$W(z) = \frac{Y(z)}{U(z)}, \quad (1)$$

$u(k)$ – input signal represented by discrete-time function, defined at distinct points in time;
 $y(k)$ – output signal represented by discrete-time function, defined at distinct points in time;
 $U(z)$ and $Y(z)$ – Z -transform of the input and output signals, which can be considered as a discrete-time equivalent of the Laplace transform.

In our case input signal is an interest rates vector:

$$IR(k) = (IR_1(k) \ IR_2(k) \ IR_3(k) \ IR_4(k) \ IR_5(k) \ IR_6(k)),$$

$IR_1(k)$ – interest rate values on loans for the period up to 30 days;
 $IR_2(k)$ – interest rate values on loans for a period of 30 to 90 days;
 $IR_3(k)$ – interest rate values on loans for a period of 90 to 180 days;
 $IR_4(k)$ – interest rate values on loans for a period of 180 days to 1 year;
 $IR_5(k)$ – interest rate values on loans for a period of 1 to 3 years;
 $IR_6(k)$ – interest rate value on loans for a period of over than 3 years.

And output signal is a credit debt vector:

$$L(k) = (L_1(k) \ L_2(k)),$$

$L_1(k)$ – total credit debt for a period of less than 1 year;
 $L_2(k)$ – total credit debt for a period of over than 1 year.

At the model identification stage, grid-search was made for the discrete transfer function form for each element of the input vector. The structural diagram of the model is shown in Figure 1. The general form transfer function can be written as follows:

$$W_{ij}(z) = \frac{(b_{ij}^0 + b_{ij}^1 \times z^{-1}) - b_{ij}^2 \times z^{-2}}{(a_{ij}^0 - a_{ij}^1 \times z^{-1}) + a_{ij}^2 \times z^{-2}} = \frac{L_j(z)}{IR_i(z)}, \quad i = 1, 2, \dots, 6, \quad j = 1, 2. \quad (2)$$

We made a transition from transfer functions to state-space description to conduct modelling in the software package (hereinafter referred to as Transfer Functions State-space model, TF SSM).

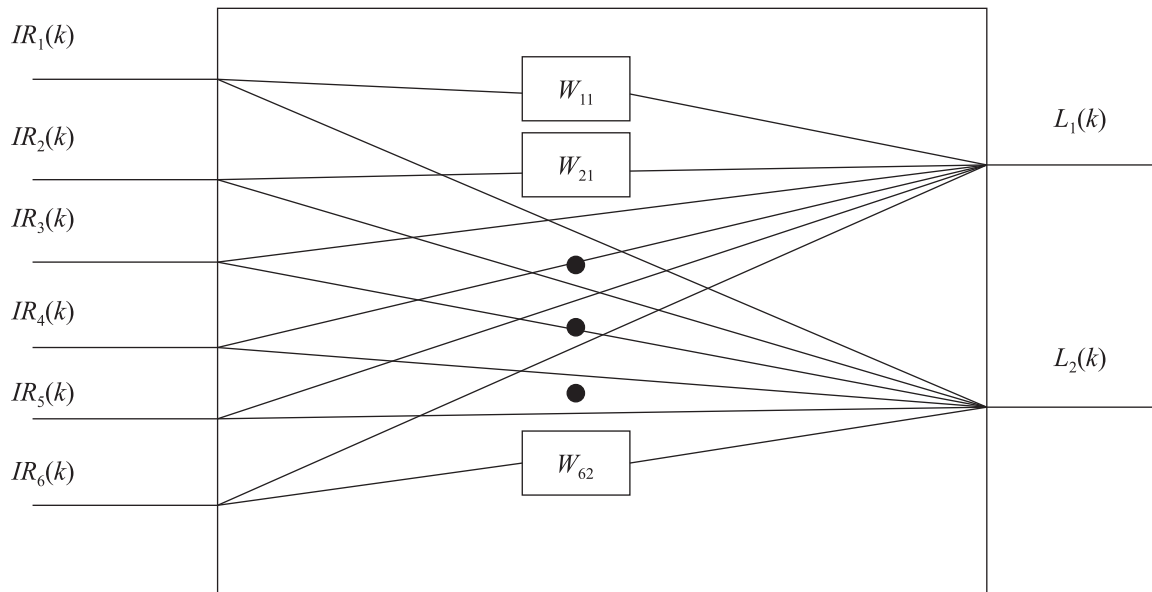


Figure 1. The structure of credit debts

Model represents a system of first order difference equations, which has interconnections between each other. In our case, system can be written as:

$$\begin{cases} x(nT_s + T_s) = A \times x(nT_s) + B \times IR(nT_s), \\ L(nT_s) = C \times x(nT_s) + D \times IR(nT_s), \end{cases} \quad (3)$$

$x(nT_s)$ – state vector;

A – state (system) matrix, 24×24 ;

B – input matrix, 24×6 ;

C – output matrix, 2×24 ;

D – feedthrough matrix, 2×6 ;

T_s – sample time.

For directly identified state-space models (hereinafter – State-space model, SSM), these matrices dimensions depends on the order of the system. We used grid-search to choose the order, while the estimation method for identification was chosen automatically by the toolbox.

Econometric ARMAX model in general can be described by the following equation:

$$\begin{aligned} y(t) + a_1 \times y(t-1) + \dots + a_{n_a} \times y(t-n_a) = & b_1 \times u(t-n_k) + \dots + \\ + b_{n_b} \times u(t-n_k-n_b+1) + c_1 \times e(t-1) + \dots + & c_{n_c} \times e(t-n_c) + e(t). \end{aligned} \quad (4)$$

Alternatively, in a more compact form:

$$A(q) \times y(t) = B(q) \times u(t-n_k) + C(q) \times e(t), \quad (5)$$

$y(t)$ – output signal;

n_a – number of poles;

n_b – number of zeros plus one;

n_c – number of C-coefficients;

n_k – number of points in the input signal that do not affect the output (dead time);

$y(t-1) \dots y(t-n_a)$ – previous outputs on which the current one depends;
 $u(t-n_k) \dots u(t-n_k-n_b+1)$ – previous outputs and delayed inputs on which the current output depends;
 $e(t-1) \dots e(t-n_c)$ – white noise.

Parameters n_a , n_b , and n_c are the ARMAX model orders. Identified nonlinear Hammerstein–Wiener model has the structure shown in Figure 2, where

f – input non-linearity, which converts input signal $u(t)$ into $w(t) = f(u(t))$;

h – output non-linearity, which converts signal $x(t)$ into $y(t) = h(x(t))$;

W_{ir} – linear discrete transfer function for $w(t)$ and $x(t)$.

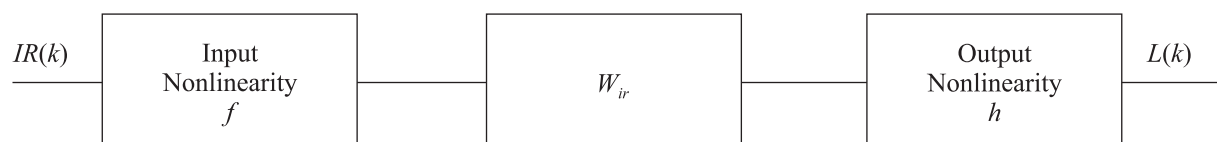


Figure 2. Hammerstein–Wiener model

Application of this type of models is usually based on the assumption that if the output signal of the system depends non-linearly on the input signal(s), then it is appropriate to separate this dependence into two or more components. In this case, system dynamics is described by linear transfer functions, similar to those described earlier, while the nonlinear properties of the output time series are determined by nonlinear blocks.

Model Identification

Model identification was conducted in Matlab software package using System Identification Toolbox. This tool is usually applied for designing models of dynamic systems based on experimental input and output data measured by sensors in the technical/mechanical systems.

In our case, we used time series of interest rates on loans for the periods from 2014 to 2019 as an input, and time series of credit debts for the periods of less and over than 1 year as an output. The source data are presented in the official “Bank of Russia Statistical Bulletin”.

To estimate the initial states of the models we used Instrumental Variables (IV) method, and to minimize the loss function, we opted for Sequential quadratic programming (SQP). As the nonlinear blocks of Hammerstein–Wiener model, we chose the piecewise linear functions. The accuracy of model parameters identification in the System Identification Toolbox was estimated by using the normalized root-mean-square deviation (NRMSD) for the output signals (Fig. 3).

$$fit(i) = 100 \times \left(1 - \frac{\|xref(:, i) - x(:, i)\|}{\|xref(:, i) - mean(xref(:, i))\|} \right), \quad (6)$$

$xref$ – reference data;

x – test data;

fit – goodness of fit between test and reference data.

The comparison of the identification accuracy is given in Table 1, where Hammerstein–Wiener model gives the best fit value; the other models are quite close in this indicator of accuracy.

In the course of research, it was found out that the identified nonlinear Hammerstein–Wiener model presents the best results in fitting the given output function but it is absolutely unsuitable for constructing forecasts due to the overfitting (Fig. 4). Non-linear blocks of the model are too “fitted” to the output signal. Consequently, this model was not included in the resulting table of forecast values.

The stability of the remaining models was evaluated by the graphs of step response for each of the input–output pairs (Fig. 5).

Table 1. The comparison of the identification accuracy

Model name	Identification fit on NRMSE, %		Validation fit on NRMSE, %		Max. error, $\times 10^5$	
	< 1 year	> 1 year	< 1 year	> 1 year	< 1 year	> 1 year
TF SSM	78.6	88.35	70.84	86.51	3.4	3.7
SSM	72.22	77.75	60.33	69.93	3.9	5.9
ARMAX	78.53	77.31	58.67	55.78	3.3	5.4
Hammerstein – Wiener	85.91	90.37	< 0	< 0	3.4	6

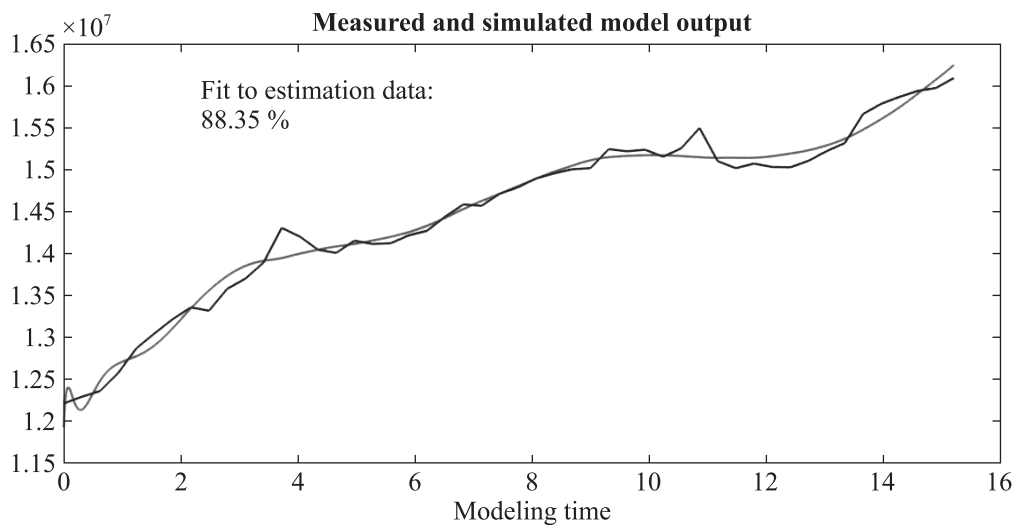


Figure 3. Evaluation of accuracy by using NRMSE for TF State-space model

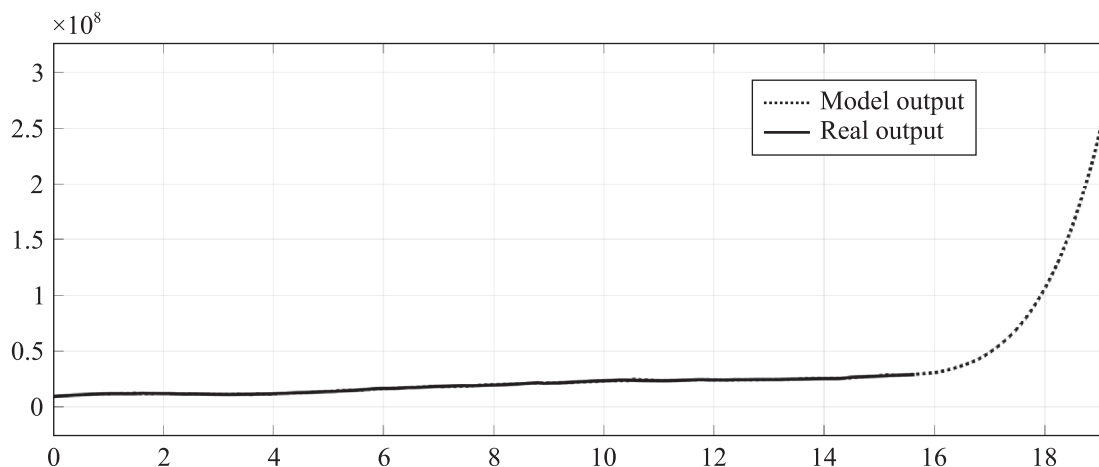


Figure 4. Results of a nonlinear forecasting modeling of credit debt for the time horizon of a year, Russian Rubles

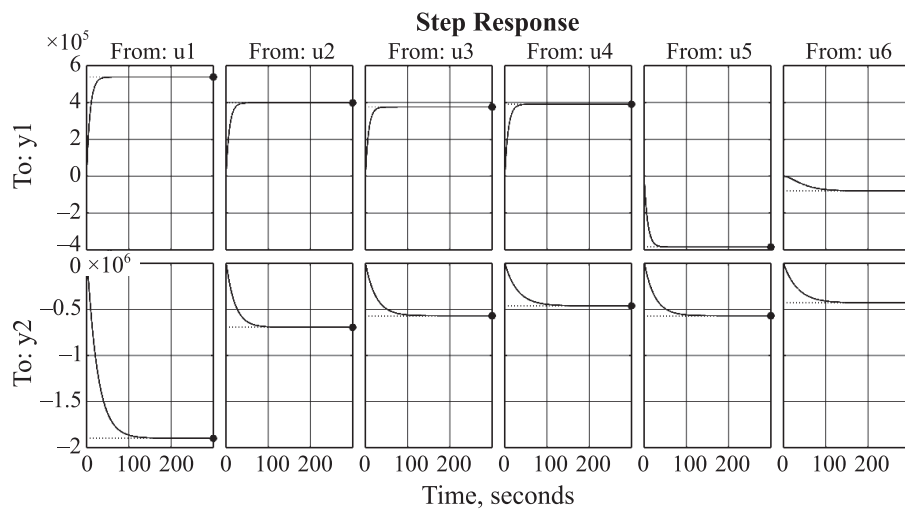


Figure 5. Step response for TF SSM

Of the remaining models, a linear discrete TF SSM model was chosen for forecasting the trend on credit debt, as the most accurately simulating the output signal (match with the reference function at the validation stage is 86.51% for loans for a period of more than a year, and 70.84% for loans for a period of less than a year). Validation was carried out on a part of the original data sample, NRMSE assessment is presented in Table 1.

Despite the fact that the first model was the most successful in the validation stage, in general, identification and validation performance was quite suitable, so we can assume good predictability of the system. To test this hypothesis, we calculate Hurst exponent for the output time series, which is an indicator of the presence of stable trends in the time series. This indicator is defined as follows:

$$E \left[\frac{R(n)}{S(n)} \right] = C \times n^H, n \rightarrow \infty, \quad (7)$$

$R(n)$ – the range of the first n cumulative deviations from the mean, and $S(n)$ is their standard deviation;
 $S(n)$ – standard deviation;
 $E[x]$ – the expected value;
 N – the time span of the observation (number of data points in a time series);
 C – constant.

For credit debt over a period of more than 1 year, the indicator assumes the value of 0.83, which indicates the persistence of the time series, which indicates good predictability. For credit debt over a period of more than 1 year, the Hurst coefficient assumes the value of 0.58, which also indicates the presence of trending trends. Since the value of the indicator is closer to 0.5, this can explain lower accuracy values for the models both at the identification and validation stage for this part of the debt structure.

As was mentioned at the end of the literature review, articles on modeling the monetary policy at the macro level are the closest to the task posed in our study. At the same time, it is difficult to compare the effectiveness of our forecast due to the lack of a completely identical task and assessment in these works. However, when studying the effect of monetary policy on macroeconomic parameters, researchers usually use aggregated parameters, i.e. main refinancing rate and total loan debt. The results of our study proves that there are relationships between interest rates for various loan terms and the structure of credit debt, which can be taken into account if it is necessary to clarify such large models, i.e. it is advisable to use the yield curve for forecasting, which is confirmed by the results of validation.

Modeling results

Further modeling was conducted in Matlab Simulink package, and we developed software to simulate various scenarios of the monetary policy of the regulator (in our case, the Bank of Russia), i.e. of changes in interest rates.

The Bank of Russia changes the key rate and affects the availability of credits and other segments of the financial market. The market reaction to these changes is an obvious result of the banking regulator decisions. In economics, this reaction is referred to as the “monetary transmission mechanism”.

The software is developed as a training simulation that describes the changes in the overall structure of credit debt depending on the regulatory impact of the Bank of Russia. It was developed as a convenient solution for such modeling task (the transition from interest rates to any related economic indicator), as part of the general task of developing simulations based on various socioeconomic blocks. The development of simulations for such processes allows us to decompose the economy into subsystems, and then move on to the meso- and microlevels, studying the change in the nature of the relationship between indicators at different levels. Software interface is shown in Figure 6. User sets the end date of the forecast modeling and the estimated values of the average weighted interest rates for various loan terms. Then the data is linearly interpolated and fed to the input of the loaded model.

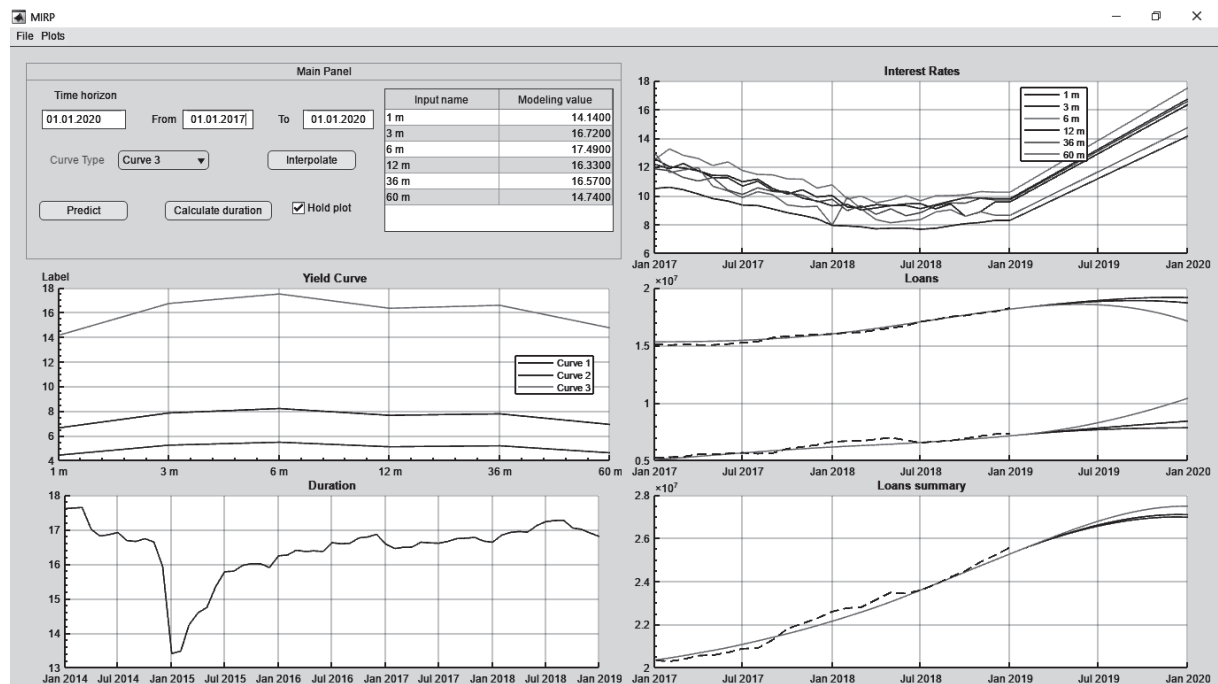


Figure 6. Modeling software interface

To test the accuracy of the remaining models, we conducted modeling for three different types of yield curve, in other words, for different scenarios of the monetary policy of the regulator (Fig. 7):

- the most likely scenario (basic) A, where no significant changes in interest rates on the forecast horizon are expected;
- mild scenario B, where the central bank lowers the key rate to 4%, guided by low inflationary expectations;

- hard scenario C, where the key rate rises to 16%, responding to speculative attacks on the ruble by currency speculators, which may be caused, in particular, by tightening financial sanctions.

Table 2 shows the last forecasting point for the modeling scenarios discussed above according to the initial statistical data for 5 previous years (from 2014 to 2019).

Table 2. Loans for the last forecasting point, tril. Russian Rubles

Scenario	TF SSM		SSM		ARMAX	
	> 1 year	< 1 year	> 1 year	< 1 year	> 1 year	< 1 year
Basic	18.7	8.4	18.5	11.2	19.0	11.8
Mild	19.1	7.8	18.4	11.3	18.8	11.9
Hard	17.0	10.4	19.1	11.0	19.5	11.3

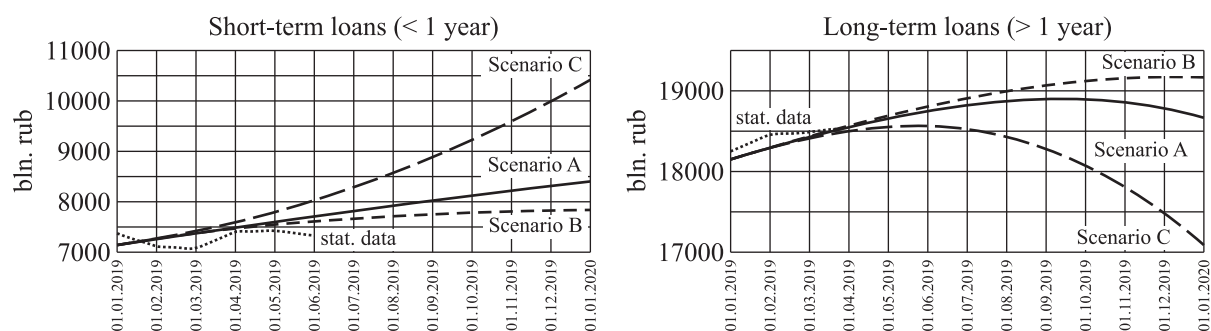


Figure 7. Modeling results by 01.01.2020 for TF SSM

Discussion

The obtained results and forecast scenarios allow us to conclude that on the forecast horizon of 1 year, the developed model predicts an increase in total credit debt for any of the proposed scenarios. Hard scenario that could be triggered by an external shock for Russia, for example, toughening sanctions or a sharp fall in crude oil prices, can have a significant influence at the structure of credit debts. At the same time, despite the highest interest rate compared to other scenarios, the growth of total credit debt will turn out to be the highest under this scenario. This can be explained by the fact that existing loans will be revalued at higher rates.

The mathematical apparatus used in the study made it possible to identify and explain the paradox of a more rapid growth of the money supply in the short-term time interval after a significant increase in interest rates. Tightening monetary policy has the strategic goal of reducing inflation and limiting the growth of money supply in the economy through the inaccessibility of credit. In reality, we see a faster growth in the money supply due to the revaluation of bank assets and liabilities at higher interest rates. On the long-term time horizon, a slowdown in the process of issuing new loans due to a drop in profitability in the economy is evident. The process of deleverage begins, when fewer and fewer borrowers prefer banks, and those who have a stable financial condition, prefer to avoid new loans. However, the overflow of already accumulated debts in the short term for unsuccessful borrowers with a high level of debt burden may cause the need for additional borrowing, despite the increase in interest rates.

Conclusion

Summing up, we can conclude that the tools and methods used in cybernetics and automatic control can be successfully used in the context of the task of predicting the structure of credit debt from the dynamics of interest rates on loans. The results showed that the TF SSM, which is a system of difference equations in the state space, was the most effective in predicting the structure of credit indebtedness from the dynamics of interest rates on loans, benefiting the other models in the NRMSD measurement. It was revealed that when constructing a nonlinear model of the Hammerstein-Wiener type to solve this kind of problems, the phenomenon of overfitting arises, when the model is effective only on the set of the training data. The directly identified SS and the classic ARMAX models give similar results in the prediction but lose in the accuracy estimation at the identification stage.

Since the developed TF State-space model is linear and based on difference equations, it will be effective only in forecasting the trend of credit debts. The maximum time horizon of the model for forecasting is 1 year.

References

- Варшавский Л. Е.* Использование методов теории управления для формирования рыночных структур // Компьютерные исследования и моделирование. — 2013. — Т. 6, № 5. — С. 839–859.
Varshavskii L. E. Ispolzovanie metodov teorii upravleniya dlya formirovaniya rynochnyh struktur [Control theory methods for creating market structures] // Computer Research and Modeling. — 2013. — Vol. 6, No. 5. — P. 839–859 (in Russian).
- Варшавский Л. Е.* Приближенные методы исследования динамики показателей рыночной структуры // Компьютерные исследования и моделирование. — 2012. — Т. 4, № 1. — С. 219–229.
Varshavskii L. E. Priblizhennyye metody issledovaniya dinamiki pokazatelej rynochnoj struktury [Approximate methods of studying dynamics of market structure] // Computer Research and Modeling. — 2012. — Vol. 4, No. 1. — P. 219–229 (in Russian).
- Варшавский Л. Е.* Моделирование динамики экономических систем с неопределенными параметрами // Компьютерные исследования и моделирование. — 2018. — Т. 10, № 2. — С. 261–276.
Varshavskii L. E. Modelirovanie dinamiki ekonomicheskikh sistem s neopredelennymi parametrami [Uncertainty factor in modeling dynamics of economic systems] // Computer Research and Modeling. — 2018. — Vol. 10, No. 2. — P. 261–276 (in Russian).
- Краснощечков П. С., Петров А. А.* Принципы построения моделей. — Изд. 2-е, перераб. и доп. — М.: Фазис, 2000. — 412 с.
Krasnoshchyokov P. S., Petrov A. A. Principy postroeniya modelej [Principles of Models Building]. — 2-e izd., peresm. i dop. — Moscow: Fazis, 2000. — 344 p. (in Russian).
- Красс И. А.* Математические модели эконометрической динамики. — М.: Советское радио, 1976. — 280 с.
Krass I. A. Matematicheskie modeli ehkonomicheskoy dinamiki [Mathematical Models of Economic Dynamics]. — Moscow: Sovetskoe radio, 1976. — 280 p. (in Russian).
- Кугаенко А. А.* Экономическая кибернетика. — М.: Вузовская книга, 2015. — 880 с.
Kugaenko A. A. Ehkonomicheskaya kibernetika [Economic Cybernetics]. — Moscow: Vuzovskaya kniga, 2015. — 880 p. (in Russian).
- Кугаенко А. А.* Методы динамического моделирования и управления экономикой. — М.: Вузовская книга, 2005. — 456 с.
Kugaenko A. A. Metody dinamicheskogo modelirovaniya v upravlenii ehkonomikoj [Methods of Dynamic Modeling and Regulation of Economics]. — Moscow: Vuzovskaya kniga, 2005. — 456 p. (in Russian).
- Макаров В. Л., Афанасьев А. А., Лосев А. А.* Вычислимая имитационная модель денежного обращения экономики // Экономика и математические методы. — 2011. — Т. 47, № 1. — С. 3–27.
Makarov V. L., Afanasyev A. A., Losev A. A. Vychislimaya imitacionnaya model' denezhnogo obrashcheniya ekonomiki [Computable simulation model for money circulation in the Russian economy] // Economics and Mathematical Methods. — 2011. — Vol. 47, No. 1. — P. 3–27 (in Russian).

- Малюгин В. И., Демиденко М. В., Калечиц Д. Л., Миксюк А. Ю., Цукарев Т. В.* Разработка и применение эконометрических моделей для прогнозирования и анализа вариантов денежно-кредитной политики // Прикладная эконометрика. — 2009. — Т. 2, № 14. — С. 24–38.
Malyugin V. I., Demidenko M. V., Kalechic D. L., Miksyuk A. Yu., Cukarev T. V. Razrabotka i primeneniye ekonometri-cheskih modelej dlya prognozirovaniya i analiza variantov denezhno-kreditnoj politiki [Development and application of econometric models for forecasting and analysis of monetary policy options] // Applied Econometrics. — 2009. — Vol. 2, No. 14. — P. 24–38 (in Russian).
- Петров А. А., Поспелов И. Г., Шананин А. А.* Опыт математического моделирования экономики. — М.: Энергоатомиздат, 1996. — 544 с.
Petrov A. A., Pospelov I. G., Shanenin A. A. Opyt matematicheskogo modelirovaniya ehkonomiki [Experience in Mathematical Modeling of Economics]. — Moscow: Ehnergoatomizdat, 1996. — 544 p. (in Russian).
- Романовский М. Ю., Романовский Ю. М.* Введение в эконофизику. Статистические и динамические модели. — М.–Ижевск: Институт компьютерных исследований, 2007. — 280 с.
Romanovskij M. Yu., Romanovskij Yu. M. Vvedeniye v ehkonofiziku. Statisticheskie i dinamicheskie modeli [Introduction to Econophysics. Statistical and Economic Models]. — Moscow–Izhevsk: Instilut kompiuternih issledovanliy, 2007. — 280 p. (in Russian).
- Billings S. A.* Identification of Nonlinear Systems: A Survey // IEE Proceedings D — Control Theory and Applications. — 1980. — Vol. 126, No. 6. — P. 272–285.
- Corréa J. M., Neto A. C., Júnior L. T., Franco E. M. C., Faria A. E.* Time series forecasting with the WARIMAX-GARCH method // Neurocomputing. — 2016. — Vol. 216. — P. 805–815.
- de Fiore F., Tristani O.* Optimal monetary policy in a model of the credit channel // The Economic Journal. — 2013. — Vol. 123, No. 571. — P. 906–931.
- Dorf R., Bishop R.* Modern control systems. — Upper Saddle River, NJ: Prentice Hall, 2001. — 1018 p.
- Durbin J., Koopman S. J.* Time Series Analysis by State Space Methods. — Oxford: Oxford University Press, 2001. — 240 p.
- Haber R., Keviczky L.* Nonlinear System Identification — Input Output Modeling Approach. — Vols. 1 and 2. — Kluwer Academic Publishers, 1999. — 823 p.
- Künsch H.* State space and hidden Markov models // Complex Stochastic Systems. — 2005. — P. 109–173.
- Ljung L.* Perspectives on System Identification // IFAC Proceedings Volumes. — 2008. — Vol. 49, No. 2. — P. 7172–7184.
- Ljung L.* System Identification: Theory for the User. — Upper Saddle River, NJ: Prentice-Hall PTR, 1999. — 609 p.
- Ma L., Liu X. A.* A nonlinear recursive instrumental variables identification method of Hammerstein ARMAX system // Nonlinear Dynamics. — 2015. — Vol. 79, No. 2. — P. 1601–1613.
- Migon H., Gamerman D., Lopez H., Ferreira M.* Bayesian dynamic models / D. Day and C. Rao (eds.) // Handbook of Statistics. — 2005. — Vol. 25, No. 19. — P. 553–588.
- Petris G., Petrone S., Campagnoli P.* Dynamic Linear Models with R // Journal of Applied Statistics. — 2009. — Vol. 38. — P. 31–84.
- Shumway M. H., Stoffer D. S.* Time Series Analysis and Its Applications With R Examples. — Springer; 3rd ed., 2011. — 596 p.
- Solgård S. B.* Modelling, simulation and control of macroeconomic systems. — Master's thesis. Norwegian University of Science and Technology, 2009. — 89 p.
- van Mulders A., Schoukens J., Vanbeylen L.* Identification of systems with localised nonlinearity: From state-space to block-structured models // Automatica. — 2013. — Vol. 49. — P. 1392–1396.

Yu F., Mao Zh., Jia M., Yuan P. Recursive Parameter Identification of Hammerstein-Wiener Systems With Measurement Noise // *Signal Processing*. — 2014. — Vol. 105. — P. 137–147.

Zhang X., Frey R. Improving ARMA-GARCH forecasts for high frequency data with regime-switching AR-MA-GARCH // *Journal of Computational Analysis and Applications*. — 2015. — Vol. 18, No. 1. — P. 727–757.