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Математическое моделирование неньютоновского потока крови в дуге аорты

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Целью проведенного исследования была разработка математической модели пульсирующего течения крови по участку аорты, включающему восходящий отдел, дугу аорты с ее ответвлениями и верхнюю часть нисходящего отдела. Поскольку при прохождении пульсовой волны деформации этой наиболее твердой части аорты малы, то при построении механической модели ее стенки считались абсолютно твердыми. В статье приводится описание внутренней структуры крови и ряда внутриструктурных эффектов. Этот анализ показывает, что кровь, которая по существу является суспензией, можно рассматривать только как неньютоновскую жидкость. Кроме того, кровь можно считать жидкостью только в кровеносных сосудах, диаметр которых намного больше характерного размера клеток крови и их агрегатных образований. В качестве неньютоновской жидкости была выбрана вязкая жидкость со степенным законом связи напряжения со скоростью деформации. Этот закон позволяет описывать поведение не только жидкостей, но и суспензий. При постановке граничного условия на входе в аорту, отражающего пульсирующий характер течения крови, было решено не ограничиваться заданием совокупного потока крови, который не дает представления о пространственном распределении скорости по поперечному сечению. В связи с этим было предложено моделировать огибающую поверхность этого пространственного распределения частью параболоида вращения с фиксированным радиусом основания и высотой, которая меняется во времени от нуля до максимального значения скорости. Для граничного условия на стенке сосуда предлагается использовать условие полупроскальзывания. Это связано с тем, что клетки крови, в силу своих электрохимических свойств, не прилипают к внутреннему слою сосуда. На внешних концах аорты и ее ответвлений задавалась величина давления. Для выполнения вычислений была построена геометрическая модель рассматриваемой части аорты с ответвлениями, на которую была нанесена тетраэдальная сетка с общим числом элементов 9810. Вычисления производились методом конечных элементов с шагом по времени 0.01 с с использованием пакета ABAQUS. В результате было получено распределение скоростей и давления на каждом шаге по времени. В областях ветвления сосудов было обнаружено вре́менное наличие вихрей и обратных течений. Они зарождались через 0.47 с от начала пульсового цикла и исчезали спустя 0.14 с.

Ключевые слова: математическое моделирование, течение крови, дуга аорты, распределение скорости и напряжения

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ANALYSIS AND MODELING OF COMPLEX LIVING SYSTEMS

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Mathematical modelling of the non-Newtonian blood flow in the aortic arc

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The purpose of research was to develop a mathematical model for pulsating blood flow in the part of aorta with their branches. Since the deformation of this most solid part of the aorta is small during the passage of the pulse wave, the blood vessels were considered as non-deformable curved cylinders. The article describes the internal structure of blood and some internal structural effects. This analysis shows that the blood, which is essentially a suspension, can only be regarded as a non-Newtonian fluid. In addition, the blood can be considered as a liquid only in the blood vessels, diameter of which is much higher than the characteristic size of blood cells and their aggregate formations. As a non-Newtonian fluid the viscous liquid with the power law of the relationship of stress with shift velocity was chosen. This law can describe the behaviour not only of liquids but also dispersions. When setting the boundary conditions at the entrance into aorta, reflecting the pulsating nature of the flow of blood, it was decided not to restrict the assignment of the total blood flow, which makes no assumptions about the spatial velocity distribution in a cross section. In this regard, it was proposed to model the surface envelope of this spatial distribution by a part of a paraboloid of rotation with a fixed base radius and height, which varies in time from zero to maximum speed value. The special attention was paid to the interaction of blood with the walls of the vessels. Having regard to the nature of this interaction, the so-called semi-slip condition was formulated as the boundary condition. At the outer ends of the aorta and its branches the amounts of pressure were given. To perform calculations the tetrahedral computer network for geometric model of the aorta with branches has been built. The total number of meshes is 9810. The calculations were performed with use of the software package ABACUS, which has also powerful tools for creating geometry of the model and visualization of calculations. The result is a distribution of velocities and pressure at each time step. In areas of branching vessels was discovered temporary presence of eddies and reverse currents. They were born via 0.47 s from the beginning of the pulse cycle and disappeared after 0.14 s.

Keywords: mathematical modelling, blood flow, aortic arc, distributions of stress and velocity

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1. Introduction

It should be noted that to date a large number of works devoted to mathematical modelling of blood flow in the arteries is published. Typically the authors identify one or two characteristic features of this complex problem and do not take into account others. Many authors consider the vessel walls as non-deformable. In particular, when solving the problem numerical simulation of blood flow in an artery with two successive bends [Hoogstraten, Kootstra, Hillen, Krijger, Wensing, 1996] the blood vessels are considered as non-deformable. The same assumption formulated also when investigating the influence of out-in-plane geometry on pulsative flow within a distal end-to-side anastomosis [Papaharilaou, Doorly, Sherwin, 2002].

Other works neglect the non-Newtonian properties of blood. In this regard, one can cite as example the well known monograph “Biomechanics circulation” [Fung, 1997]. The similar assumption was made in the numerical simulation and experimental validation of blood flow in arteries with structured-tree outflow conditions [Olufsen, Peskin, Kim, Pedersen, Alinadim, Larsen, 2000]. In addition to this the blood flow in the large systemic arteries was modelled using one-dimensional equations.

In the same time many authors emphasize the necessity of considering non-Newtonian properties of blood. First of all you need to mention the monograph of Siginer, Kee and Chhabra devoted to advances in the flow and reology of non-Newtonian fluids [Siginer, De Kee, Chhabra, 1999]. The influence of the non-Newtonian properties of blood on the flow in large arteries was investigated by Gijsen, Allanic and De Vosse [Gijsen, Allanic, De Vosse, 1999] but only for the vessels of simple geometry. The more complex geometry was used in the numerical investigation of the non-Newtonian blood flow in a bifurcation model [Chen, Lu, 2004]. You can also mention the investigation of effects of the non-Newtonian blood viscosity on flows in a diseased arterial vessel [Cho, Kensey, 1991] and investigation of non-Newtonian blood flow in human right coronary arteries [Johnson, Johnson, Corney, Kilpatrick, 2004] but both these investigations considered only stead flows.

Regarding modelling of blood flow in the aortic arc the situation is similar. The whole number of authors considers the blood as Newtonian liquid. In particular, the Newtonian model was used in the study of the influences of nonplanarity and bifurcation on the inflow and outflow dynamics in aortic arch [Liu, Fukasaku, Iwase, Mutsunaga, He, Yokoi, Himeno, 2003], in the three-dimensional numerical simulation of blood flow in the aortic arch with cardiopulmonary bypass [Tokuda, Song, Ueda, Usui, Akita, Yoneyama, Maruyama, 2008], where the vessel walls were considered to be rigid, and in the recently published blood flow analysis of the aortic arch using computational fluid dynamics [Numata, Itatani, Kanda, Doi, Yamazaki, Morimoto, Manabe, Ikemoto Yaku, 2016]. By the way, in this study the blood flow was considered to be stationary in the branches and the distribution of velocities and stresses was obtained only in the peak of systole. On the other hand it is necessary to point out the investigation of non-Newtonian characteristics of pulsatile human blood flows in vessels and in the aortic arch [Sultanov, Guster, Engelbrekt, Blankenbecler, 2008]. The authors of this investigation found significant disagreements between their results obtained with realistic non-Newtonian treatment of human blood and widely used a simple Newtonian approximation. A significant increase of the strain rate and, as a result, wall sear stress distribution, was found in the region of the aortic arch. The authors with good reason considered this result as theoretical evidence that supports existing clinical observations and those models not using non-Newtonian treatment underestimate the risk of disruption to the human vascular system.

In contrast to previous studies, it was decided to carry out mathematical modelling of blood flow in the aortic arch and its branches, which would include more characteristic features of the blood flow through the vessels than the above models. First, the model should reproduce the complex spatial geometry of blood vessels and their branches. Second, the developed model should take into account the complex blood rheology and internal structure of blood flow. Third, the model should describe the pulsating change of the velocity profile at the input cross-section of aorta. Finally, the developed model in the formulation of boundary conditions needs to take into account the interaction with the vessel walls not only the liquid component of blood, but also blood cells.

For this purpose above all it is necessary to perform the analysis of the structure and properties of blood and vessels.

2. Structure and properties of blood and blood vessels

The blood has a very complicated internal structure consisting of liquid plasma, composed of 93% water and 3% particles (electrolytes organic molecules and numerous proteins) and the set of blood cells (structural elements): red blood cells (erythrocytes), white blood cells (leucocytes) and platelets (thrombocytes), which make up 46 percent of the total blood volume. Erythrocytes are biconcave discs with a mean diameter of 6 to 8 μm and compose 99% by blood cell volume. In this regard, the percentage of erythrocyte volume to the whole blood volume, called hematocrit, is an important characteristic, influencing the blood apparent viscosity and hemorheology of the overall [Baskurt, Meiselman, 2003].

Besides erythrocytes are able to deform and can form aggregates in the form three-dimensional “coin columns”. In this regard, blood can be considered as a liquid only in large blood vessels where the characteristic sizes of the structural elements of blood and their aggregates are small compared to the diameter of the vessel cross-section. Otherwise the blood should be considered as a two-phase medium.

However, for rigid particles a vast amount of published literature exists (see e. g. [Roco, 1993]), but much less attention was paid to the study of suspensions of multiple, interacting and deformable particles such as blood [Bessonov, Sequeira, Simakov, Vassilevski, Volpert, 2016]. Leukocytes have a roughly form and are much larger than erythrocytes, their diameter ranges from 6 to 17 μm, but the total volume concentration leukocytes together with thrombocytes is only 1%. As to thrombocytes, it necessary to take in mind, that they play an important role in the blood clotting mechanism. In this regard, it is necessary to note, that moving along the vessel the blood cells migrate through the cross section. The erythrocytes, being the heavier particles, migrate towards the vessel axis and more light thrombocytes migrate towards the blood wall and their interaction with the endothelial cells should be taken into account when formulating the boundary conditions for blood flow. With regard to the processes of gelation and the formation of blood clots, they are more characteristic for slow flows [Ataullakhanov, Volkova, Guriia, Sarbash, 1995; Guria, Herrero, Zlobina, 2009].

This short analysis of the blood properties strengthens the necessity to consider it only as non-Newtonian liquid in order to take into account, at least partially, the influence of the complicated internal structure and rheology on the character of bloodstream.

With regard to blood vessels, they also have a very complicated three-layer structure and their properties are different for different parts. As to arteries, they carry out conductive and damping functions. The conducting function is responsible for the transport of blood and the damping function leads to a smoothing of the pressure pulses, so that at some distance from the aorta a blood flow becomes stationary. The main function of the aorta is transportation and its walls are sufficiently solid and rigid to withstand the impact of the pulse wave. In connection with this the blood vessels this part of aorta can be considered as deformable. Other arteries have both functions and are in varying degrees of deformable. In this case it is necessary to solve the very complicated problem of hydroelasticity (see, for example, [Ratkina, Tregubov, 2010]).

3. Mechanical models of the aortic arc and blood flow

The part of aorta including the ascending aorta, aortic arch with its branches (except the right common carotid artery) and upper part of the descending aorta were chosen for modelling.

As it is pointed above this part of aorta is sufficiently strong and it has undergone only minor deformation, which does not affect the blood flow. Taking this into account we considered the vessel walls as non-deformable. The space structure of the model was constructed by means of circular cylinders with curved axes and smooth connections between them (fig. 1). This operation was performed by means of the computation system ABAQUS, which is described below.

It is necessary to point out that the complicated spatial geometry of blood vessels and the presence of branching represent a substantial difficulty in modelling the blood flow. But on the other hand they are the areas of significant interest of physicians.

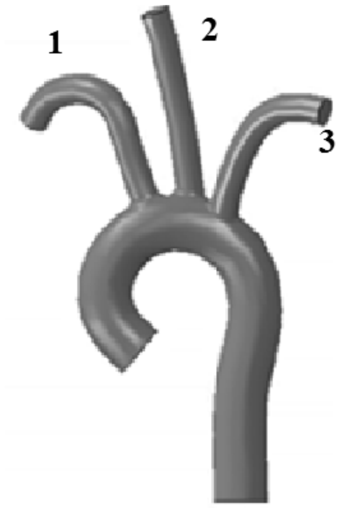

Fig. 1. The general view of the mechanical model of the aortic arc and its branches: 1 — *truncus brachiocephalicus*, 2 — *arteria carotis communis sinistra*, 3 — *arteria subclavia sinistra*

According to the analysis performed above the blood is considered as non-Newtonian fluid. In this case the concept of the so called apparent viscosity, which depends on the shear rate, is usually used. In the same time it is necessary to point out that there is the whole set of non-Newtonian liquids and each of which has its own law, expressing the relation between the stress and shear rate. In particular, dilatant liquid is one of the non-Newtonian liquid specific kinds. We used the power law for relationship between the stress and shear-strain rate. This law is able to describe the behaviour not only of liquid but also suspension with high content of solid particles, which can change their configuration, connect and disconnect to each other depending on a shear stress. These properties are most consistent with the properties of blood. The expression for the apparent viscosity for the one-dimensional flow is given by the formula $\mu_a = k\dot{\gamma}^{n-1}$, where $\dot{\gamma}$ is the shear-strain rate, k and n are the power law parameters that are typically considered to be constant. In the same time it was noted [Walburn, Schneck, 1976] that these constants depend on hematocrit, but its change is very small in the considered part of aorta.

In order to set the pulsing blood flow in the input cross-section of aorta the new mode was proposed for modelling of the velocity distribution in this cross-section by a paraboloid of revolution (fig. 2). The height of this paraboloid is variable from 0 to the maximal value and back so that the total blood flow, passing through this cross-section, would be equal to the blood flow ejected into the aorta from the left ventricle (fig. 3).

Since removing from the heart the blood flow ceases to be pulsating the following ordinary way may be used. The boundary conditions are set away from considered part of aorta and their branches, where the blood velocity and pressure are constant and equal to the average velocity $\boldsymbol{v}_{\text{aver}}$ and average pressure p_{aver}.

The particular attention has been given to the interaction between the blood and vessel wall. In the hydrodynamics of viscous liquids the *no-slip* boundary condition is usual adopted. However, this condition can not be applied for the blood as a hole but only for the blood plasma.

What actually happens is that the blood cells don't attach to the wall and slide along this wall or are repelled from it owing to their electro-chemical properties. This situation is sometimes is called as *full-slip condition*. In order to average boundary conditions for the plasma and blood sells the so called *semi-slip condition* can be used. In this case longitudinal velocity is reduced to preset value v_0. It is common practice to express the degree of this semi-slipping by means of the parameter b, which is as the distance between the vessel boundary line and the point c. In its turn c is the intersection point of the tangent to the envelop curve and the vertical axis (fig. 4). The parameter b can vary from zero to infinity for different liquids. That is why the semi-slip condition was used as the boundary condition at the vessel wall in distinction to all papers cited above.

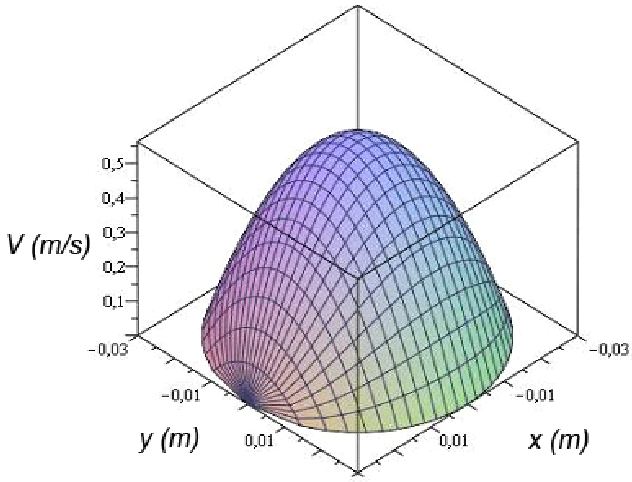

Fig. 2. The geometrical model of the velocity distribution V at the input cross-section of aortic arc, which is placed on the coordinate plane (X, Y). The height of paraboloid changes from 0 to the maximal value and back

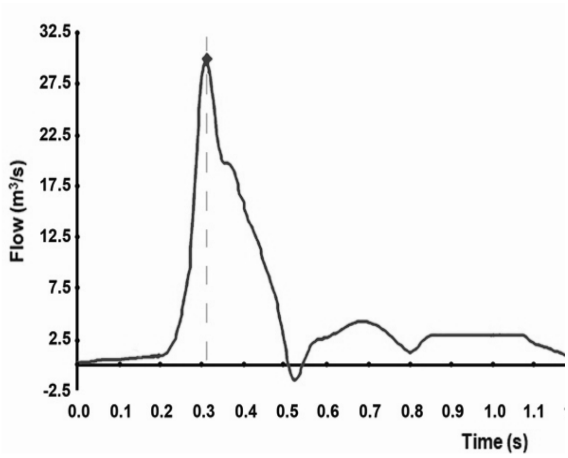

Fig. 3. The typical blood flow ejected from the left ventricle into aorta, which is usually measured during a medical examination of the patient

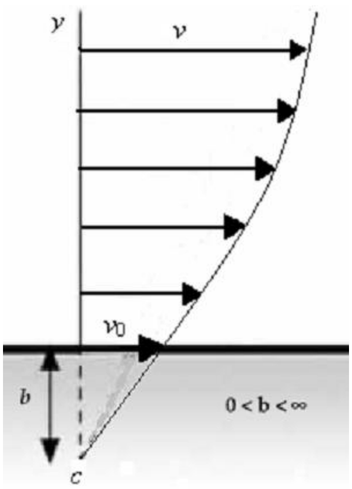

Fig. 4. The velocity profile and its envelope line near the vessel wall (thick black line) in the case of semi-slip condition; v_0 is the flow velocity along the wall, b is the distance between the vessel wall and point c, which is the intersection point of the tangent to the envelop curve and the vertical axis.

4. Mathematical description

Thus the proposed mathematical model needs to describe the motion of non-Newtonian fluid with a power law dependence of the stress tensor from the tensor of strain velocity tensor in the rigid curved vessels with pulsating boundary condition at the inlet, constant pressure at the outlet and semi-slip condition on the vessel wall.

In the three-dimension case the tensor representation of power law has the following form:

$$\mathbf{T} = 2k\left|I_2\right|^{\left(\frac{n-1}{2}\right)}\mathbf{D}, \tag{1}$$

where **T** is the viscous stress tensor, **D** is the strain velocity tensor and I_2 is its second principal invariant of **D**. Having the expression of this power law the motion equation for the blood flow can be written as follows:

$$\rho\frac{d\mathrm{v}}{dt} = \nabla\cdot(-p\mathbf{I}+\mathbf{T}), \tag{2}$$

where $\boldsymbol{v}$ is the velocity vector, ρ is the fluid density, p is the pressure and **I** is the identity tensor.

The equation of continuity has the form

$$\operatorname{div}\boldsymbol{v}\big|_{t=0} = 0. \tag{3}$$

Thus after substitution of the expression (1) in the equation (2) we will have one vector equation and one scalar equation to determine four unknown variables: the pressure p and three projections of velocity $\boldsymbol{v}$.

In addition to these equations it is necessary to formulate the initial and boundary conditions. In order to start a calculating procedure it was adopted that at the initial time instant the blood is at rest under constant pressure p_0. That is

$$\boldsymbol{v}\big|_{t=0} = 0 \quad \text{and} \quad p\big|_{t=0} = p_0\,. \tag{4}$$

Having these initial conditions the calculations continue until the process becomes strictly periodic. This condition may be in particular realized just after the start up of the heart during the medical operation.

At the input section of the considerable vessel part the pulsating change of the spatial velocity distribution may be set in the cylindrical coordinates r, φ, v as a paraboloid of revolution in the following way:

$$v(r,\,t)\big|_{\text{input}} = -v_{\max}(t)\left(\frac{r^2}{r_0^{\,2}}+1\right), \tag{5}$$

were v is the longitudinal blood velocity, r_0 is the vessel radius at the input, $v_{\max}(t)$ is the maximum speed value (apex of the paraboloid) in each time moment.

At the far output sections of aorta and its branches the boundary conditions may be written as

$$\boldsymbol{v} = \text{const} \equiv \boldsymbol{v}_{\text{aver}} \quad \text{and} \quad p = \text{const}. \tag{6}$$

The semi-slip condition at the wall is set as follows:

$$v_\tau\big|_{\text{wall}} \equiv v_0 = -b\left(\frac{\partial v_\tau}{\partial n}\right)\Bigg|_{\text{wall}}, \tag{7}$$

where v_τ is the tangent of velocity, n is the normal to the inner surface of vessel and b is the coefficient described above.

Thus, it is necessary to solve the closed system of differential equations (2)–(3) taking into account the relations (1) with the initial conditions (4) and boundary conditions (5)–(7).

5. Calculations and results

The software package ABAQUS was used for calculations. This choice was due to the fact that ABAQUS has powerful tools for creating model geometry, calculations and their visualization. In the first step the geometry of model was created by the junction of curved cylinders using the joint smoothing procedure. After this the computational grid with tetrahedral elements was applied (Fig. 5).

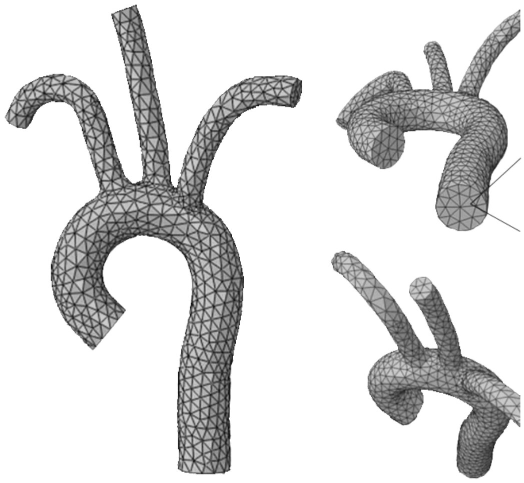

Fig. 5. The calculation grid of the blood-filled aortic arc model

The total number of the elements is equal 9810. The calculations were performed by use the Finite Element Method with time step 0.01 *s*. It was also chosen the following values of constants: $k = 0.016$, $n = 1.63$, $p_0 = 7000\ PA$.

As a result of calculation the distribution of blood velocity and pressure in the considerable aorta part were obtained at each time moment during the pulse period. As examples the blood velocity distribution in the aorta at the time t = 0.3 *s* are presented in fig. 6. This time point corresponds to the maximum flow ejected into aorta. It is necessary to point out that the maximum of blood velocity is shifted to the wall with a smaller radius of curvature.

We should also pay a particular attention to the appearance of the vortices and reverse blood flows in the inputs of branching area, which are shown in fig. 7. These vortexes and reverse flows are generated approximately at t = 0.47 *s* and are disappeared at t = 0.61 *s*. This is an expected phenomenon, however, only the mathematical model allows obtaining the velocity value in any point of the vortex within the discreteness of the computational mesh and at any time to within the given discrete time. The blood pressure distribution at the time moment t = 0.3 *s* are presented in fig. 8. The maximum pressure is equal 7179 *PA*, the minimum is equal 6652 *PA*.

6. Conclusion

First of all it should be noted that the proposed model combines the complex spatial geometry of blood vessels, the non-Newtonian blood properties, the pulsating distribution of blood velocity at the input cross-section of aorta and the unconventional semi-slip boundary condition at the vessel wall. This model has allowed establishing the characteristic features of blood flow in the upper part of aorta including the aortic arc, as well as obtaining the numerical characteristics of these features. The use of semi-slip boundary condition allowed us to establish the presence of a near wall flow, which in turn will give the opportunity to investigate the process of thrombus formation in this layer. In connection with this the obtained velocity profiles differ from the velocity profiles obtained in the no-slip condition.

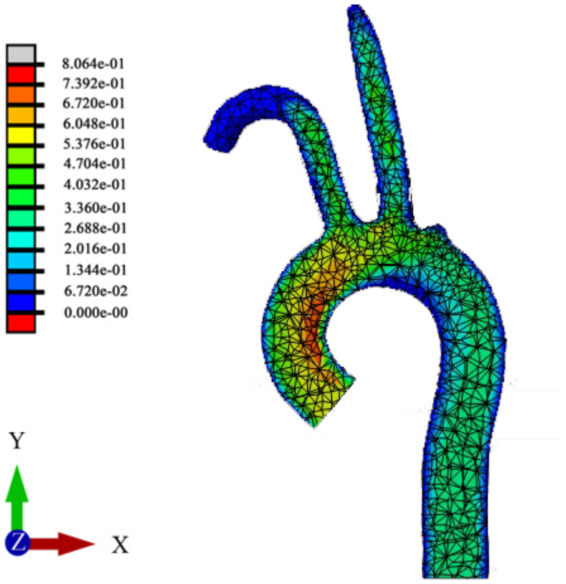

Fig. 6. The calculated blood velocity distribution at $t = 0.3$ s. The colour version of the figure is accessible in the electronic version of the paper on the journal website

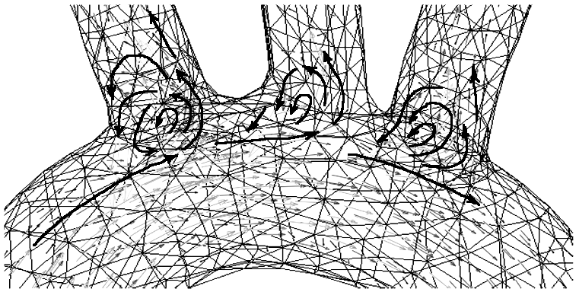

Fig. 7. The existence of the vortexes and reverse flows obtained as result of the blood flow calculation during the time interval from 0.47 s to 0.67 s. The manually plotted bold arrows indicate the main direction of blood flow

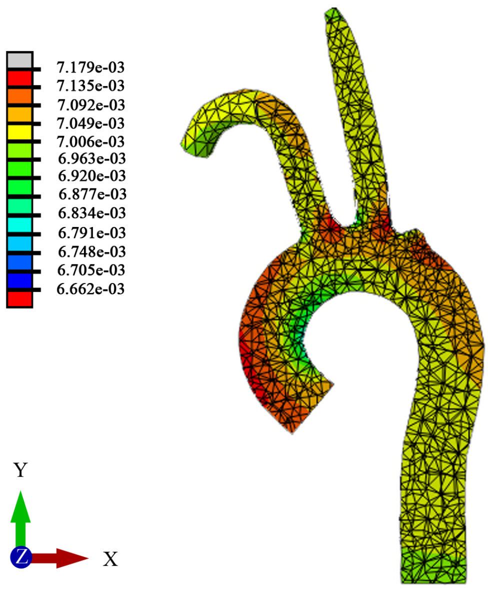

Fig. 8. The calculated pressure distribution at $t = 0.3$ s. The colour version of the figure is accessible in the electronic version of the paper on the journal website

In conclusion it is necessary to say that despite the fact that the forms of vessel mechanical models are somewhat idealized, the proposed method makes it possible to deviate from this idealization and to approximate the shape of the vessel to his real form for a particular patient, including pathological cases.

Finally, the proposed method makes it possible to simulate vascular changes of the shape as a result of surgery. This allows the surgeons to predict the consequences of surgical intervention.

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